

To be or not to be: Higgs impostors at the LHC

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Summary. — Consider the day when an invariant mass peak, roughly compatible with “the Higgs”, begins to emerge, say at the LHC, . . . and may you see that day. There will be a difference between discovery and scrutiny. The latter would involve an effort to ascertain what it is, or is not, that has been found. It turns out that the two concepts are linked: Scrutiny will naturally result in deeper knowledge—is *this* what you were all looking for?—but may also speed up discovery.

PACS 14.80.Bn – Standard-model Higgs bosons.

1. – Introduction

Let the single missing scalar of the Standard Model (SM) be called “the Higgs”, to stick to a debatable misdeed. Because the idea is so venerable, one may have grown insensitive to how special a Higgs boson would be. Its quantum numbers must be those of the vacuum, which its field permeates. The boson itself would be the vibrational quantum *of* the vacuum, not a mere quantum *in* the vacuum, or in some other substance. The couplings of the Higgs to quarks and leptons are proportional to their masses. So are its couplings to W^\pm and Z , a fact that, within the SM, is in a sense verified. A significantly precise direct measurement of the Higgs couplings to fermions is not an easy task. Even for the heaviest of them, the top quark, the required integrated luminosity is large, as illustrated by the ATLAS Collaboration on the left of fig. 1.

In the past, given a newly discovered particle, one had to figure out its J^{PC} quantum numbers (or its disrespect of the super-indexed ones) to have it appear in the Particle Data Book. Publication in the New York Times was not considered that urgent, nor was it immediate for bad news. Times have changed. Yet, two groups [1, 2] have thoroughly studied the determination of the quantum numbers and coupling characteristics of a putative signal at the LHC, that could be the elementary scalar of the SM, or an impostor thereof, both dubbed H here. The “golden channel” for this exercise is $H \rightarrow (ZZ \text{ or } ZZ^*) \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$, where $\ell_{1,2}^\pm$ is an e or a μ , and Z^* denotes that, for $M_H < 2M_Z$, one of the Z s is “off-shell”. For a review of previous work on the subject, see *e.g.* [3].

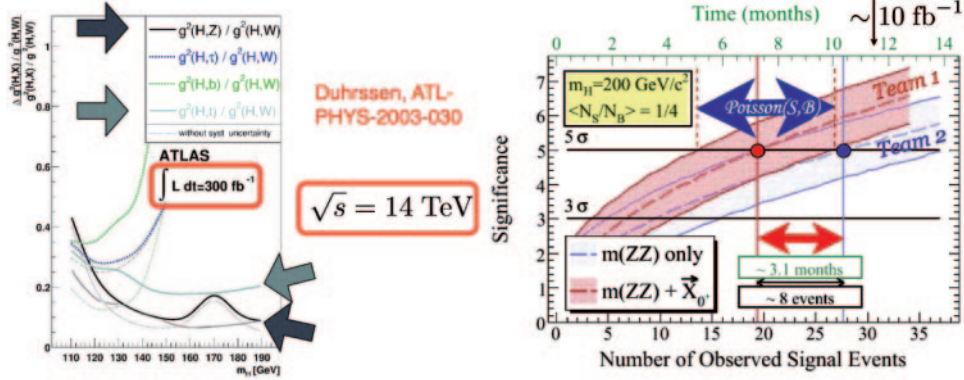


Fig. 1. – Left: fractional precision on the measurable ratios of branching ratios for SM H decays into W , Z , t and τ pairs as functions of M_H . Right: an example of discovery and scrutiny plot of a SM scalar with $M_H = 200 \text{ GeV}$, not specially chosen for effect.

To be realistic (?) let me consider two competing teams. They are working at a pp collider of energy $\sqrt{s} = 10 \text{ TeV}$, luminosity $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ and Snowmass factor of 3 (on average, things work well 1/3 of the time). The SM is correct, $M_H = 200 \text{ GeV}$ and the estimates of signals and backgrounds are reliable. As the number of events increases, Team 2 would then gather evidence for an M_{ZZ} peak at the rate shown on the right of fig. 1. Team 1 is additionally checking that, indeed, the object has $J^{PC} = 0^{++}$. T1 reaches “discovery” (5σ significance) some three months before T2. The horizontal error bars, dominated by fluctuations in the expected background, tell us that the two teams are *only* 1σ apart (iff from two different experiments!). But that means the probability of T1 (from experiment A) being 3 months ahead of T2 (from experiment $B \neq A$) is $\sim 66\%$ ($\sim 100\%$ for $B = A$). The odds for winning with dice, if your competitor lets you win for 4 out of the 6 faces are also 66%. If the stakes are this high, would you not play? It is interesting to compare the H -identity-determining integrated luminosities in figs. 1, more so since event numbers on its right refer to the chain $H \rightarrow ZZ \rightarrow e^+e^- \mu^+\mu^-$ and are approximately quadrupled when all 4ℓ channels are considered.

Standard signal and background cross sections times branching ratios were used in fig. 1. In discussing H impostors we accept that they should not be distinguished from a SM H on these grounds, which, for all impostors, are hugely model dependent.

2. – Methodology

The technique to be used to measure J^{PC} for a putative H signal has some pedigree. Its quantum-mechanical version (called nowadays the “matrix element” method) capitalizes on the entanglement of the two Z polarizations and dates back at least to the first (correct) measurements of the correlated γ polarizations in parapositronium (0^{-+}) decay [4]. The technique is even older, as it actually consists in comparing theory and observations. The art is in exploiting *a maximum of the information* from both sides.

The event-by-event information on the channel at hand is very large, some of it is illustrated in fig. 2, for the decay chain $H \rightarrow ZZ \rightarrow e^+e^- \mu^+\mu^-$, with H brought to rest. The angular variables $\vec{\Omega}$ describe Z -pair production relative to the annihilating gg or $q\bar{q}$ pair. The variables $\vec{\omega}$ are the Z -pair decay angles. For fixed $\vec{\Omega}$, $\vec{\omega}$, and M^* (the mass of

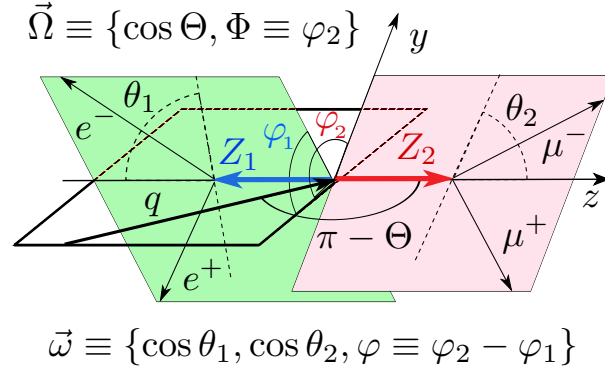


Fig. 2. – The angles of ZZ pair-production and leptonic decay.

a lepton pair if its parent Z is off-shell) that is all there is: none less than six beautifully entangled variables ($M[4\ell]$ is also measured event by event, M_H is traditionally extracted from a fit to the $M[4\ell]$ distribution).

Real detectors have limited coverage in angles and momenta, they “mis-shape” the theoretical distributions in the quantities just described. An example for a realistic detector and an unrealistic flat expectation is illustrated on the right of fig. 3. For an

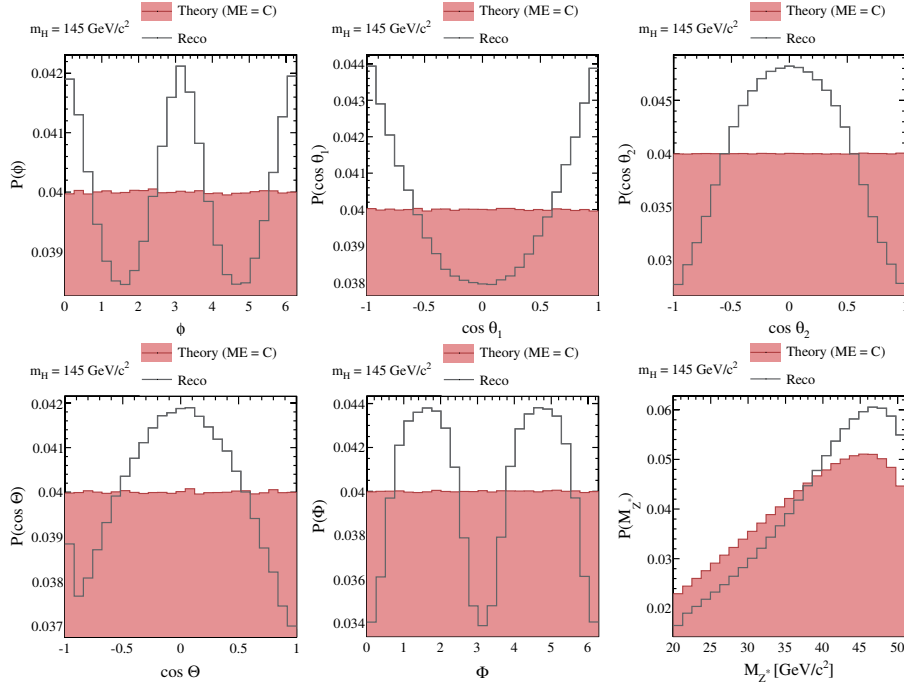


Fig. 3. – Detector-shaping effects at $M_H = 145$ GeV, for all relevant angles and M^* . The trigger and energy thresholds, resolutions and angular coverage are those of a “typical” detector.

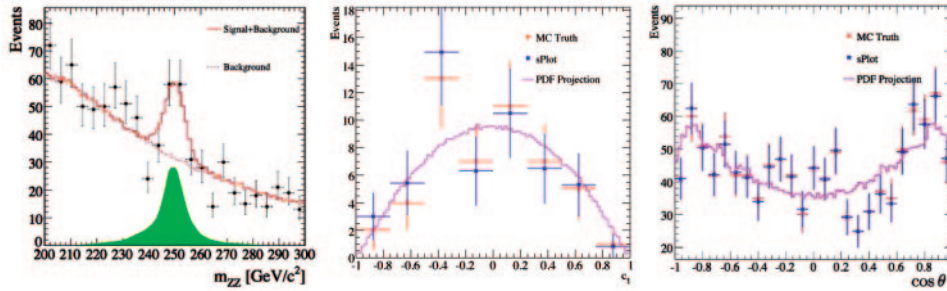


Fig. 4. – Left: a signal on an $M(ZZ)$ distribution. Middle: sPlot of the $\cos\theta$ distribution of the “signal” events, compared with the Monte Carlo truth and the (detector-shaped) expected distribution, for $J^{PC} = 0^{++}$. Right: same as middle, for the “background” events.

H with $J = 0$, the distribution in $\vec{\Omega}$ is flat, so that its inclusion (in this case) would seem like an overkill. Not so! detector-shaping effects and the correlations between the angular variables conspire to make the use of the full machinery a necessity [2].

There is a wonderful “s-weighing” method for (much of) the exercise of ascertaining the LHC’s potential to select the preferred hypothesis for an observed H candidate. Consider an $M[4\ell]$ distribution with an H peak at 250 GeV, constructed with the standard expectations for signal and background, as in fig. 4. Performing a maximum-likelihood fit to this distribution one can ascertain the probability of events in each $M[4\ell]$ bin to be signal or background. Next one can astutely (and even statistically optimally) reweigh the events into “signal” and “background” categories, to study their distributions in other variables [5], such as $\cos\theta = \cos\theta_1$ or $\cos\theta_2$ in fig. 4. In this pseudo-experiment one knows the “Monte Carlo truth”, compared in the figure with the impressive s-outcomes and the detector-shaped expectation. We use the full (correlated) distributions in all mentioned variables, but M_H , to confront “data” with different hypotheses.

The astute reader has noticed that I have not mentioned the η and p_T distributions of the ZZ or ZZ^* pair (be it an H signal or the irreducible background). Event by event, one can undo the corresponding boost but, to ascertain the detector-shaping effects, as in fig. 3, for all the various SM or impostor H objects, one has to use a specific event generator. We have done it [2], but we chose to “pessimize” our results in this respect, *not* exploiting the (η, p_T) distributions as part of the theoretical expectations (which for impostors would be quite model dependent). One reason is that the relevant parton distribution functions (PDFs) will be better known by the time a Higgs hunt becomes realistic. Another is that one can use the s-weigh technique to extract and separately plot the (η, p_T) distribution for signal and background. The production of a SM H —but not that of most conceivable impostors—is dominated by an extremely theory-laden process: gluon fusion via a top loop. As a first step it is preferable *to see* whether or not the (η, p_T) distribution of the s-sieved signal events is that expected for gg fusion, as opposed to $q\bar{q}$ annihilation⁽¹⁾. The answer would be fascinating.

⁽¹⁾ The only impact of the difference between the two processes is on the detector-shaping effects. But these are not large enough for the ensuing differences to affect our results.

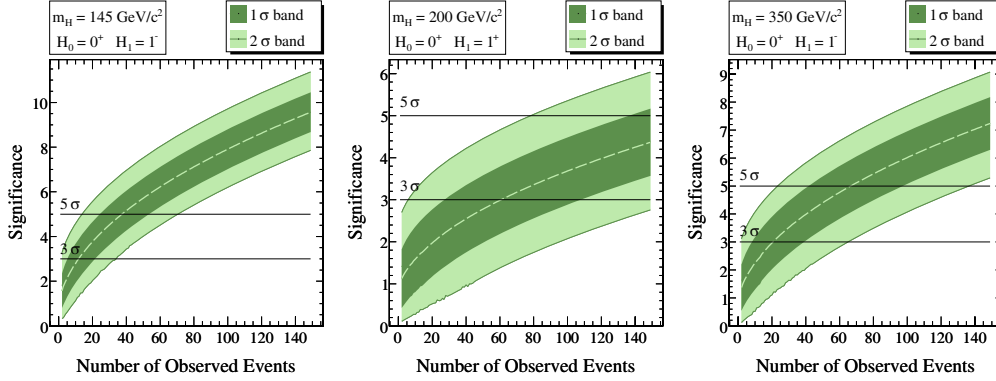


Fig. 5. – Expected confidence levels, as functions of the number of events, to reject the wrong hypothesis (H_0 , the SM in this case) in favour of the right one (H_1). Left and right: H_1 is 1^- , for $M_H = 145$ and 350 GeV. Middle: H_1 is 1^+ , $M_H = 200$ GeV.

3. – Theory

The most general Lorentz-invariant couplings of $J = 0, 1$ particles to the polarization vectors ϵ_1^μ and ϵ_2^α of two Z s of four-momenta p_1 and p_2 are given by the expressions:

$$\begin{aligned} -i L_{\mu\alpha} &= X_0 g_{\mu\alpha} + (P_0 + i Q_0) \epsilon_{\mu\alpha\sigma\tau} p_1^\sigma p_2^\tau / M_Z^2 - (Y_0 + i Z_0) (p_1 + p_2)_\alpha (p_1 + p_2)_\mu / M_Z^2, \\ -i L^{\rho\mu\alpha} &= X_1 (g^{\rho\mu} p_1^\alpha + g^{\rho\alpha} p_2^\mu) + (P_1 + i Q_1) \epsilon^{\rho\mu\alpha\sigma} (p_1 - p_2)_\sigma. \end{aligned}$$

The vertex for $J = 2$ is cumbersome. The quantities $X_i, P_i \dots$ can be taken to be real, but for small absorptive effects. The expressions can be used to derive the distribution functions $\text{pdf}(J^{PC}; M^*, \cos \Theta, \Phi, \cos \theta_1, \cos \theta_2, \varphi)$ allowing one to determine the spin of an H and the properties of the HZZ coupling. To give some $J = 0$ examples: in the SM only $X_0 = g M_Z / \cos \theta_W$ is nonvanishing. For $J = 0^-$ only $Q_0 \neq 0$. If X_0 and Q_0 (or P_0) $\neq 0$, the HZZ vertex violates P (or CP). For a “composite scalar” $X_0, Y_0 \neq 0$.

4. – Some results

While Team 1 members are trying to establish the significance of the discovery of an object of specified properties (as in fig. 1, right), they may, with a few extra lines of code, be extracting much more information from the same data set, by asking leading questions, NLQs,>NNLQs. . . , whose answers are decreasingly statistically significant.

The quintessential LQ is which of two hypotheses describes the data best, assuming that one of them is right. If the hypotheses are “simple” (contain no parameters to be fit) the Neyman-Pearson lemma guarantees that the test is *universally most powerful*. Three examples are given in fig. 5. On its left and right it is seen that it is “easy” (it takes a few tens of events) to rule out the SM, if the observed resonance is an $M_H = 145$ or 350 GeV vector. On its middle, we see that, if the object is an axial vector, it would be much harder. This it is not due to the differing J^P , but to the choice $M_H = 200$ GeV. For masses close to the $H \rightarrow ZZ$ threshold, the level arm provided by the lepton three-momenta is short, and the differences between pdfs is diminished. In fact, as an answer to a NLQ, we have shown that it is “easy” to tell any $J = 0$ from

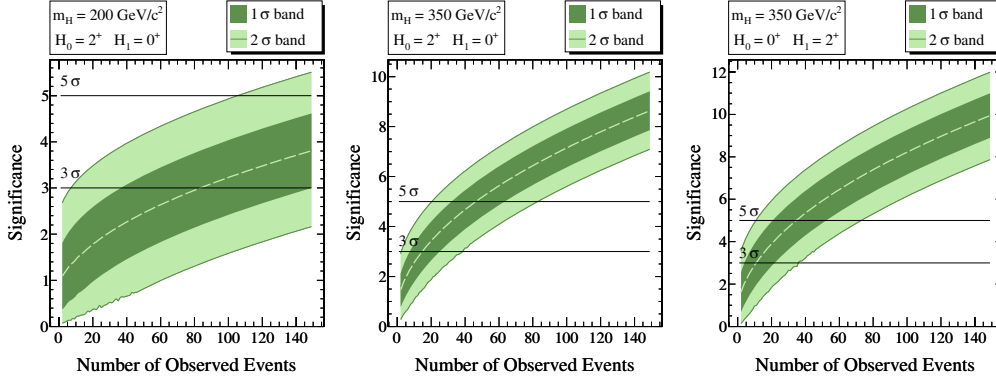


Fig. 6. – The same as fig. 5, with the hypotheses $J^P = 2^+$ and 0^+ , once interchanged.

any $J = 1$ object, no matter how general their HZZ couplings are [2]. In fig. 6 we see that it is easy, if the SM is right, to exclude $J = 2^+$ at $M_H = 350$ GeV, but not at 200. We also see that the interchange of right and wrong hypotheses leads to very similar expectations.

On the right of fig. 7 is the answer to a NNLQ. We have assumed that a composite $J^{PC} = 0^{++}$ Higgs has been found and parametrized its ZZ coupling by an angle $\xi_{XY} = \arctan(Y_0/X_0)$. The measured value of ξ_{XY} is seen to be the input one, but for 50 events the uncertainties on what the input was, to be read horizontally, are large. For this case of a specific J^{PC} , but a complicated coupling, the various terms in the pdf are not distinguishable on grounds of their properties under P and CP . They do strongly interfere for specific values of ξ_{XY} , and the results of fig. 7 are not easy to obtain, requiring a full Feldman-Cousins belt construction [2].

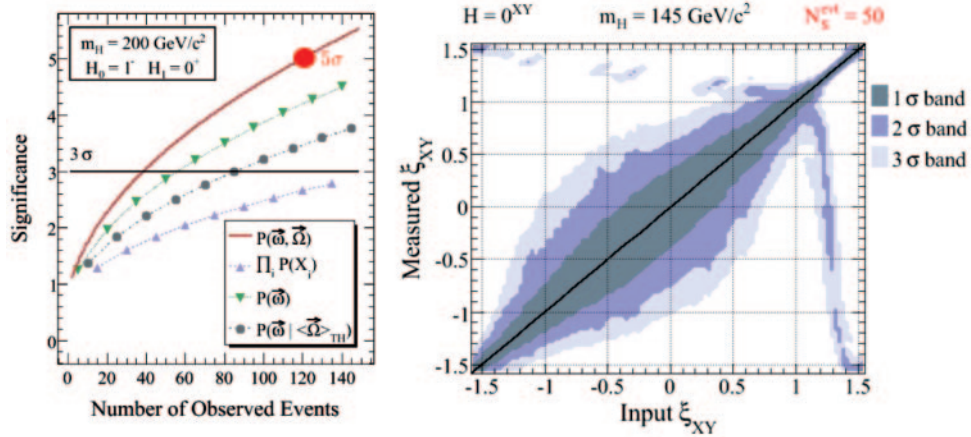


Fig. 7. – Left: various choices of pdfs, employing different sub-optimal choices of likelihood functions, are compared with a pdf containing all angular variables and their correlations. Right: true and measured values of the mixing angle describing a composite scalar, for $M_H = 145$ GeV.

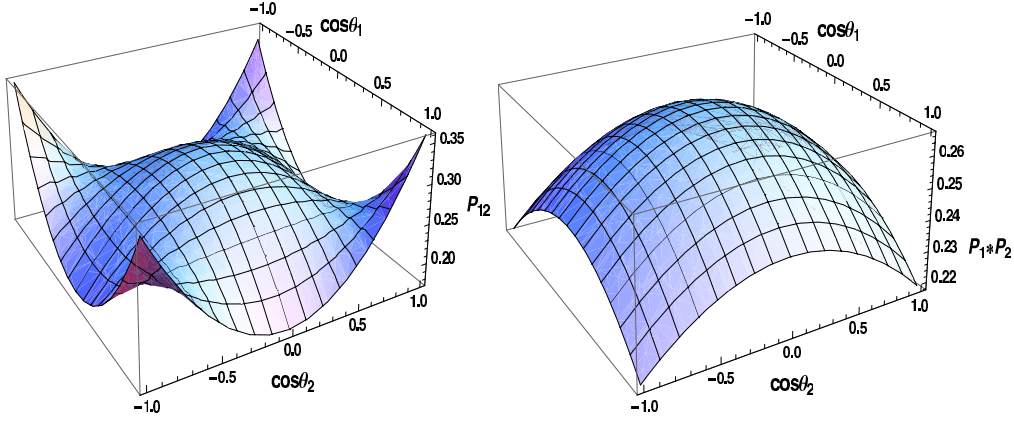


Fig. 8. – The pdfs of the SM at $M_H = 200$ GeV, integrated in all variables but $\cos \theta_1$ and $\cos \theta_2$. Left: the correct $P(\cos \theta_1, \cos \theta_2)$. Right: the “approximation” $P(\cos \theta_1) \times P(\cos \theta_2)$.

Given a small data set constituting an initial discovery, one might settle for a stripped-down analysis. The cost of such a sub-optimal choice is shown on the left of fig. 7 for $M_H = 200$ GeV, illustrating the discrimination between the 0^+ and 1^- hypotheses for likelihood definitions that exploit different sets of variables. N -dimensional pdfs in the variables $\{a_1, \dots, a_N\}$ are denoted $P(a_1, \dots, a_N)$, while $\prod_i P(X_i)$ is constructed from one-dimensional pdfs for all variables, ignoring (erroneously) their correlations. $P(\vec{\omega} | \langle \vec{\Omega} \rangle_{\text{TH}})$ are pdfs including the variables $\vec{\omega}$ and their correlations, but with the hypothesis 1^- represented by a pdf in which the variables $\vec{\Omega}$ have been integrated out. The likelihood $P(\vec{\omega} | \langle \vec{\Omega} \rangle_{\text{TH}})$ performs badly even relative to $P(\vec{\omega})$, which uses fewer angular variables. The two differ only in that the first construction implicitly assumes a uniform 4π coverage of the observed leptons (an assumption customary in the literature) as if the muon p_T and η analysis requirements did not depend on the $\vec{\Omega}$ angular variables.

Treating the correlated angular variables as uncorrelated, as in the $\prod_i P(X_i)$ example of fig. 7, not only degrades the discrimination significance but would lead to absurdly time-dependent conclusions. Assume, for example, the SM with $m_H = 200$ GeV. Let the data be fit to either a fully correlated pdf or an uncorrelated one. The projections of the corresponding theoretical pdfs, involving only the variables $\cos \theta_1$ and $\cos \theta_2$, are illustrated in fig. 8. On the left (right) of the figure we see $P[\cos \theta_1, \cos \theta_2]$ ($P[\cos \theta_1] \times P[\cos \theta_2]$). With limited statistics—insufficient to distinguish between the correlated and uncorrelated distributions—the correct conclusion will be reached: the data are compatible with the SM. But, as the statistics are increased, the data will significantly deviate from the $P[\cos \theta_1] \times P[\cos \theta_2]$ distribution, and a false rejection of the SM hypothesis would become increasingly supported.

The difference between $P[\cos \theta_1, \cos \theta_2]$ and $P[\cos \theta_1] \times P[\cos \theta_2]$ is *precisely* what an unbelieving Einstein called *spooky action at a distance*. But, mercifully for physicists, the Lord is subtle *and* perverse.

5. – Conclusions

I have alleged, by way of example, that for a fixed detector performance and integrated luminosity (and no extra Swiss Francs) it pays to have *ab initio* an analysis combining discovery and scrutiny. This is arguably true for many physics items other than $H \rightarrow 4\ell$. They readily come to mind.

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