Promt production of the $X(3872)$. — $X(3872)$ was first observed in $B$-meson decays by Belle and BaBar, in the $J/\psi\pi^+\pi^-$ invariant mass distribution at $\sim 3872$ MeV, with a decay width smaller than the experimental resolution of $\sim 2.3$ MeV. It was later confirmed by BaBar and in $p\bar{p}$ collisions by CDF and D0. The $J^{PC}$ assignment is still controversial: in what follows we will assume $J^{PC} = 1^{++}$. The vicinity of the mass of the $X$ to the $D^0\bar{D}^{0*}$ threshold suggested to identify the state with a $(D^0\bar{D}^{0*} + D^0\bar{D}^{0*})/\sqrt{2}$ $S$-wave molecule. This hypothesis is reinforced from the fact that $X$ decays to $J/\psi\rho$ and $J/\psi\omega$ with the same strength [1]. This meson-meson bound state would have a tiny binding energy $E_0 = (-0.25 \pm 0.4)$ MeV, namely compatible with zero.

In a recent paper [2] we tested the molecular assignation on the prompt production mechanism. It seems at odds with common intuition that such a loosely bound molecule could be produced promptly in a high energy hadron collision environment. A recent analysis by CDF [3] allows indeed to distinguish between the fraction of $X$ produced promptly and the one originated from $B$-decays: $\sigma(p\bar{p} \to X(3872) + \text{all})_{\text{prompt}} \times B(X(3872) \to J/\psi\pi^+\pi^-) = (3.1 \pm 0.7)$ nb, for $p_T \geq 5$ GeV and $|y| \leq 0.6$. Using some bounds on the branching fraction [4], one obtains $33$ nb $< \sigma(p\bar{p} \to X(3872) + \text{all})_{\text{prompt}} < 72$ nb. To estimate an upper bound for the theoretical prompt production cross section
we made use of a Schwartz inequality:

\[
\sigma(p\bar{p} \rightarrow X(3872)) \sim \left| \int d^3k \langle X|D\bar{D}^*(k)\rangle \langle D\bar{D}^*(k)|p\bar{p}\rangle \right|^2 \\
\simeq \left| \int_{\mathcal{R}} d^3k \langle X|D\bar{D}^*(k)\rangle \langle D\bar{D}^*(k)|p\bar{p}\rangle \right|^2 \\
\leq \int_{\mathcal{R}} d^3k |\psi(k)|^2 \int_{\mathcal{R}} d^3k |\langle D\bar{D}^*(k)|p\bar{p}\rangle|^2 \\
\leq \int_{\mathcal{R}} d^3k |\langle D\bar{D}^*(k)|p\bar{p}\rangle|^2 \sim \sigma(p\bar{p} \rightarrow X(3872))_{\text{max}},
\]

where \(k\) is the relative 3-momentum between the \(D\) mesons in their center-of-mass frame, \(\psi(k)\) is some bound state wave function and \(\mathcal{R}\) is the region where the wave function is appreciably different from zero. From the binding energy we can deduce the size of the molecule: \(r_0 \sim \sqrt{1/2\mu_0} \approx 8\) fm. Assuming a Gaussian wave function and using the minimal-uncertainty-principle relation we compute the spread of the relative 3-momentum \(\Delta k \sim 1/2r_0 \approx 12\) MeV around a central value \(k_0 = \sqrt{\lambda(m_x^2, m_D^2, m_{D^*}^2)/2m_x} \approx 27\) MeV. Thus \(\mathcal{R} = [0, k_0 + \Delta k] \sim [0, 35]\) MeV. We recover what we expected: the two mesons inside the molecule must be almost collinear to account for such a small binding energy.

The matrix element \(\langle D\bar{D}^*(k)|p\bar{p}\rangle\) can be computed using standard hadronization Monte Carlo programs like Herwig and Pythia, generating \(2 \rightarrow 2\) QCD events with loose partonic cuts. Tuning our MC tools on the CDF data for \(D^0\bar{D}^{*+}\) pair production, we obtain the differential distribution for the upper bound on the prompt production cross section of a \(D^0\bar{D}^{*+}\) pair as a function of their relative 3-momentum \(k\), see fig. 1. Integrating the distribution up to \(k \simeq 35\) MeV we obtain \(\sigma_{\text{prompt}}^{\text{th}} < 0.075\) nb with Herwig and \(\sigma_{\text{prompt}}^{\text{th}} < 0.11\) nb with Pythia, both \(
\sim 300\) times smaller than the lower experimental limit \(\sigma_{\text{prompt}}^{\text{exp}} > 33\) nb. This result challenges the molecular interpretation of the \(X\).

\(Y(4660)\) and \(Y(4350)\). – The \(Y\) resonances, \(Y(4260), Y(4350)\) and \(Y(4660)\), are \(J^{PC} = 1^{--}\) states produced via initial state radiation at the \(B\)-factories. The first
decays mainly into $J/\psi \pi^+ \pi^-$, while the other two into $\psi(2S)\pi^+ \pi^-$. Since the $Y(4660)$ is above the threshold for the baryon-antibaryon decay, a scan was performed in this channel and found a peak at $\sim 4630$ MeV. In [5] we reanalyzed the Belle data on $Y(4660) \rightarrow \psi(2S)\pi^+ \pi^-$ and $Y(4630) \rightarrow \Lambda_c^+ \Lambda_c^- [7]$. The two fits give consistent results, strongly supporting the hypothesis that the two structures are evidences of the same resonance, which we call $Y_{B}$, ($M_{Y_{B}} = (4661 \pm 9)$ MeV and $\Gamma_{Y_{B}} = (61 \pm 23)$ MeV).

From the same fits we extract also:

$$B(Y_{B} \rightarrow \Lambda_c^+ \Lambda_c^-)/B(Y_{B} \rightarrow \psi(2S)\pi^+ \pi^-) = 25 \pm 7.$$  

The $Y_{B}$ shows a strong affinity to the baryon-antibaryon decay mode, which is typical of a $[cq][\bar{c}q]$ tetraquark state. One can indeed describe the state as a diquark and an antidiquark attached at the end of a string which neutralizes their color, giving the “H”-shaped structure depicted in fig. 2 (A). The $\Lambda_c^+ \Lambda_c^-$ decay is favored because it is realized breaking the string in one single point, fig. 2 (B), while the others proceed through the breaking in multiple points, fig. 2 (C-D). Furthermore we performed an analysis on the dipion invariant mass distribution which indicates the presence of the $\sigma$ and $f_0(980)$ (famous tetraquark candidates in the light sector) as intermediate states in the decay $Y_{B} \rightarrow \psi(2S)\pi^+ \pi^-$. This result is another indication of the tetraquark structure of this resonance.

The two diquarks must be in an odd orbital excitation to give odd parity. The spectrum of these orbitally excited states can be obtained using the relativistic string model derived by Selem and Wilczek in [8]. Two masses $m_1$ and $m_2$ are connected by a relativistic string with constant tension $T$, rotating with angular velocity $\omega$. We consider the limit for equal and infinitely heavy masses $M$ and obtain an analytic relation between the energy and the orbital angular momentum:

$$E \simeq 2M + \frac{3}{16\pi^2M} (\sigma \ell)^{2/3},$$

where $\sigma = 2\pi T$. Using the string tension fitted from Regge trajectories $\sigma = 1.1$ GeV$^2$, and the value of the diquark mass computed in [9] $M = 1933$ MeV, one obtains $E = 4311$ MeV for the $\ell = 1$ state, which we identify with the $Y(4350)$. Taking into account the absence of the decay $Y(4660) \rightarrow J/\psi \pi^+ \pi^-$, we tentatively identify the $Y(4660)$ with a radial excitation of an $\ell = 1$ state, which should prefer to decay into charmonia with $n_r = 1 (\psi(2S))$, rather than $n_r = 0 (J/\psi)$. The mass gap due to radial excitations of $\ell = 1$ states can be estimated from the experimental data in the bottomonium sector: $M_{\chi_b(2P)} - M_{\chi_b(1P)} \simeq 360$ MeV. The mass of the the $\ell = 1$, $n_r = 1$ tetraquark would be $\simeq 4670$ MeV, really close to the $Y(4660)$ mass.
REFERENCES