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Matrix model and β-deformation of $\mathcal{N} = 4$ Yang-Mills theory(*)

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Summary. — We present the result of the determination of the effective coupling constant for the low-energy (abelian) degrees of freedom in the so-called β-deformed $\mathcal{N} = 4$ model, by means of the deep connection between supersymmetric gauge field theories and matrix models.

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In recent years, there has been a renewed interest in a particular $\mathcal{N} = 1$ model, known as the β-deformation of the $\mathcal{N} = 4$ Super Yang-Mills. Since Leigh and Strassler found this model in the nineties [1], various aspects have been studied [2, 3], but only after its gravity dual was found [4], it was extensively investigated in numerous papers [5-7].

This model is obtained by deforming the $\mathcal{N} = 4$ superpotential, enlarging its parameter space, through the introduction of a new coupling constant and a (generically complex) parameter β via the definition

$$W_{\mathcal{N}=4}(\Phi_i) \equiv g\,\mathrm{tr}\,\Phi_1[\Phi_2, \Phi_3] \to W^{\beta}_{\mathcal{N}=4}(\Phi_i) \equiv h\,\mathrm{tr}\,\Phi_1[\Phi_2, \Phi_3]_{\beta}. \quad (1)$$

If the scalars have a vanishing vacuum expectation value (vev), $\langle \Phi_I \rangle = 0$, the theory is confining and conformally invariant. In the opposite case, for generic values of the deformation parameters, it is possible to identify branches in the moduli space where the gauge group gets spontaneously broken. For the sake of illustration we will concentrate in this paper on particular branches, where only one of the three chiral superfields (which we take to be Φ_1) develops a vev,

$$\langle \Phi_1 \rangle = \mathrm{diag}(\varphi_1, \varphi_2, \ldots, \varphi_{N_c}). \quad (2)$$

Although at some special point of the moduli space (corresponding to sets of coinciding φ_a's) some subgroups of the original $U(N_c)$ gauge group can remain unbroken, at a

(*) This work is dedicated to Kensuke Yoshida. He was a friend besides being a teacher.

generic point, *i.e.* on what we will be calling the "Coulomb branch" [8], the gauge group will be broken spontaneously to $U(1)^{N_c}$. In this case, the massless spectrum consists of N_c "photons", corresponding to the diagonal elements of the gauge field, and their gluino superpartners. The two fields combine to form N_c Abelian vector supermultiplets of $\mathcal{N}=1$ SUSY. We will denote the corresponding field strength by $w_{a\alpha}$, $a=1,\ldots,N_c$. Obviously, there are also N_c massless chiral multiplets, corresponding to the fluctuations around the non-vanishing eigenvalues φ_a.

In this situation, although no superpotential can be generated, the kinetic term for the massless fields may receive quantum corrections. The effective low-energy action for the gauge fields is a *holomorphic* function which must be of the form

$$W_{\mathrm{eff}} \propto \sum_{a,b=1}^{N_c} \tau_{ab} w^{\alpha,a} w^b_\alpha. \tag{3}$$

The complex $N_c \times N_c$ matrix τ_{ab} encodes the effective gauge couplings and vacuum angles. Restricting to the particular branch we considered, τ_{ab} will obviously depend only on the diagonal part of the chiral superfield Φ_1, which we shall compactly rewrite as $\langle\Phi_1\rangle$.

At the classical level, τ_{ab} is proportional to the identity, $\tau^{\mathrm{cl}}_{ab}=\delta_{ab}\tau_0$. Standard non-renormalization theorems guarantee that perturbative quantum effects are limited to one-loop corrections, while non-perturbative instanton-like terms are expected at any order. In other words we will have for $\tau_{ab}(\langle\Phi_1\rangle)$ an expansion of the type

$$\tau_{ab}(\langle\Phi_1\rangle) = \tau_0\delta_{ab} + \tau_{ab}^{1-\mathrm{loop}}(\langle\Phi_1\rangle) + \sum_{k=1}^{\infty} \tau_{ab}^{(k)}(\langle\Phi_1\rangle) \equiv \tau_0\delta_{ab} + \hat{\tau}_{ab}(\langle\Phi_1\rangle). \tag{4}$$

The one-loop perturbative correction has already been computed in [8, 9].

In 2002, Dijkgraaf and Vafa [10] conjectured that holomorphic quantities in the low-energy regime are related to the free-energy of an auxiliary Matrix Model (MM). Since then, the correspondence has passed various non-trivial tests [11] and it has become a powerful tool in the study of the low-energy limit of a vast class of $\mathcal{N}=1$ gauge theories.

The Dijkgraaf-Vafa correspondence states that, given an $\mathcal{N}=1$ model, we must consider the superpotential as the action for Hermitian random matrices $\hat{N}\times\hat{N}$, $\hat{\Phi}$, replacing the original chiral superfields. Having built the corresponding partition function,

$$Z = \exp\left[-\frac{\hat{N}^2}{g_m^2}\mathcal{F}\right] = \int \mathrm{d}\hat{\Phi} \exp\left[\frac{\hat{N}}{g_m} \operatorname{tr} W_{\mathrm{tree}}(\hat{\Phi})\right],$$

the correspondence gives the relation that determines the low-energy effective holomorphic superpotential and the coupling for the massless degrees of freedom, once the large-$\hat{N}$ limit is considered. In the same limit, the MM enjoys of a 't Hooft expansion,

$$-\log Z = \frac{\hat{N}^2}{g_m^2}\mathcal{F} = \sum_{h\geq 0}\left[\frac{g_m}{\hat{N}}\right]^{2h-2}\mathcal{F}_h(\mathcal{S}_i), \qquad \mathcal{S}_i \equiv \lim_{\hat{N}\to\infty} g_m \frac{\hat{N}_i}{\hat{N}}, \tag{5}$$

such that the leading term is given by the planar contribution.

For simplicity we shall consider the gauge group breaking $U(2) \mapsto U(1)^2$. Then, the low-energy coupling is simply given by

$$\hat{\tau}_{ab} = \tau \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad \tau = \frac{\partial^2 F_m(\mathcal{S}_i)}{\partial \mathcal{S}_1 \partial \mathcal{S}_2} = \tau^{1-\text{loop}} + \tau^{(1)} + \tau^{(2)} + \dots, \tag{6}$$

where the series expansion is naturally introduced by the *perturbative* expansion of the MM. Stopping at the first order(1) in the MM formulation, the complete expression of $\hat{\tau}$ is

$$\begin{aligned} \tau^{1-\text{loop}} + \tau^{(1)} + \dots = &-\log \frac{g^2(\varphi_1-\varphi_2)^2}{h^2(\mathrm{e}^{\imath\beta/2}\,\varphi_1 - \mathrm{e}^{-\imath\beta/2}\,\varphi_2)(\mathrm{e}^{-\imath\beta/2}\,\varphi_1 - \mathrm{e}^{\imath\beta/2}\,\varphi_2)} \\ &+\frac{8h^4A^2\sin^2\beta/2}{g^4\Delta^4(0)\Delta(\beta)\Delta(-\beta)}\Big[(-3+4\cos\beta-\cos 2\beta)\varphi_1^6 \\ &+2(-5+6\cos\beta-\cos 2\beta)\varphi_1^5\varphi_2 \\ &+(-13+16\cos\beta-3\cos 2\beta)\varphi_1^4\varphi_2^2 \\ &+8(-1+2\cos\beta-\cos 2\beta)\varphi_1^3\varphi_2^3 \\ &+(-13+16\cos\beta-3\cos 2\beta)\varphi_1^2\varphi_2^4 \\ &+2(-5+6\cos\beta-\cos 2\beta)\varphi_1\varphi_2^5 \\ &+(-3+4\cos\beta-\cos 2\beta)\varphi_2^6\Big] + \dots, \end{aligned} \tag{7}$$

where $\Delta(x) \equiv \exp[\imath x/2]\varphi_1 - \exp[-\imath x/2]\varphi_2$. As expected, eq. (7) is organized as a power series expansion in the instanton action, $A^2 \equiv \exp[2\pi\imath\tau_0] \propto \exp[-8\pi^2/g^2]$, so that this computation —*perturbative* from the MM point of view— gives *non-perturbative* information on the physics on the gauge theory side. In the $h^2 \to g^2$ and $\beta \to 0$ limit, tantamount to recovering the pure $\mathcal{N}=4$ Super Yang-Mills theory, all corrections vanish, leaving only the overall diagonal contribution, τ_0. The result confirms thus the interpretation of the physics described by the formalism as a spontaneous symmetry breaking phenomenon.

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(1) The next order and more details on the derivation can be found in [12].