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Determination of quark masses from lattice QCD

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Summary. — In this paper we present a determination of the average up/down, strange and charm quark masses, performed in lattice QCD with $N_f = 2$ twisted mass Wilson fermions, obtained by comparing the calculations of pseudoscalar mesons masses with their experimental values. By using four different lattice spacings and pion mass as low as 280 MeV we performed an accurate chiral and continuum extrapolation.

PACS `11.15.Ha` – Lattice gauge theory.
PACS `12.38.Gc` – Lattice QCD calculations.
PACS `12.39.Fe` – Chiral Lagrangians.

1. – Introduction

Quark masses are fundamental parameters of the Standard Model. Their values are needed for many calculations in Quantum Cromodynamics, but their are not directly measurable due to confinement. In lattice QCD they can be obtained by computing some hadronic quantities to be compared to experimental measurements. In particular we have focused on the determination of m_q from the pseudoscalar meson masses. This work is set in the ETM Collaboration and make use of the $N_f = 2$ degenerate configurations from it produced, and update a series of older works. Regarding the average up/down quark mass, this work is very similar to a recent paper by ETMC [1] with which we find good agreement. The main differences are the simultaneous use of all the four lattice spacings, and the updated values for the renormalization constants. The strange quark mass has been already determined in [2], using only one lattice spacing. Having added continuum limit extrapolation we find a value of m_s about 10% lower than our previous result, but still compatible with it. The charm quark mass has been calculated by ETMC in a previous paper [3], on which a slightly lower value for m_c was found, with a larger error. The more precise determination of m_s and m_c from lattice QCD is given by HPQCD Collaboration [4] which extract them from a perturbative analysis of high momenta of current correlation functions. The control over nonperturbative aspects of the procedure of this method needs to be better clarified.

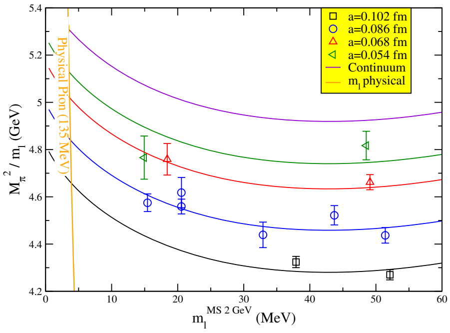

Fig. 1. – M_π^2/m_l as a function of m_l.

All the results shown in this paper are preliminary. Final results will be presented in a forthcoming publication [5].

2. – Lattice methods

At large enough Euclidean time, the time correlation function $C_{PS}(\tau)$ of the operator $O_{PS} = V^{-1}\int \mathrm{d}\vec{x}\bar{\psi}(x)\gamma_5\psi(x)$ behaves as $C_{PS}(\tau) \simeq |\langle 0|O_{PS}|PS\rangle|^2 e^{-m_{PS}\tau}/2m_{PS}$ where PS is the lowest mass particle with quantum number of the operator O_{PS}. By interpolating among/extrapolating from calculation of M_{PS} at different values of m_q, one can determine the value m_q^{phys} which reproduce the physical value of M_{PS}. We have computed M_{PS} as a function of m_q in lattice QCD. For computational reasons it is yet not possible to perform calculations at the physical value of the light quarks mass keeping at the same time large volume and small lattice spacings. In order to have a good statistics all computations are performed relaxing these requirements, and treating the outcoming systematics effects in different ways: we will discuss them in details.

Finite cutoff effects: to get rid of unphysical *discretization effects* we have calculated M_{PS} at four different lattice spacings in the range 0.050–0.100 fm, and extrapolated it

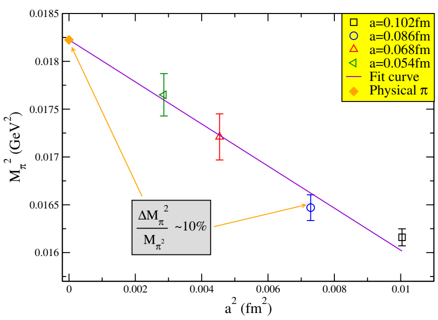

Fig. 2. – M_π^2 at m_l^{phys} as a function of a^2.

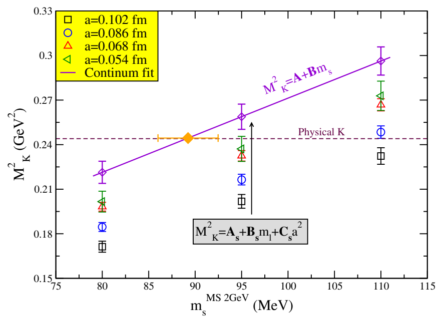

Fig. 3. – $SU(2)$ fit of M_K^2 as a function of m_s.

to the limit $a \to 0$. Having used the improved twisted mass regularization at maximal twist, the discretization effects are proportional to a^2, ranging from the order of 5% for pion mass up to 20% for the case of η_c meson.

Finite volume effects: pseudoscalar meson masses are calculated at finite volume and so affected by *finite volume effects*, which being proportional to $\exp[-M_{PS}L]$, are visible only for kaons and mainly for pions and are of the order of permill. It is possible to calculate [6] a correction factor $r_{PS}(L, m_q) \equiv M_{PS}(L, m_q)/M_{PS}(L \to \infty, m_q)$ analytically, and so obtain infinite volume results for calculated data.

Chiral extrapolation: we have calculated M_{PS} in a range of m_q between 10 and 50 MeV, which correspond to $M_\pi \in \{280\text{–}500\}$ MeV, and extrapolated them to m_l^{phys}.

Renormalization constants: the quark mass renormalization constants $Z_m = Z_P^{-1}$ have been determined non-perturbatively with the so-called RI-MOM method [7].

For continuum and chiral extrapolation we have tried different variations of χPT formulas, truncated at different orders and with various kind of discretization terms, putting the spread as final systematic effects. Here we will discuss in detail the procedure used.

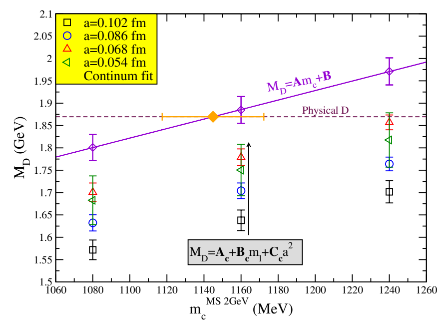

Fig. 4. – M_D fit as a function of m_c.

Light quark: in the case of the light quark we have performed a global fit of all data at different lattice spacings and quark masses with an $SU(2) - \chi PT$ formula $m_\pi^2 = 2B_0 m_l[1+m_l \log(2B_0 m_l/\Lambda_3)+D_m a^2+T^{\mathrm{NNLO}}]$ where the NNLO term T is a complicated function of m_q and various low energy constants of χPT. We have tried to put or not the NNLO terms and the term D_m describing the discretization effects, in order to check the effects of ignoring higher-order terms. Figure 1 shows M_π^2/m_l as a function of m_l: points are lattice data and lines are $SU(2) - NLO$ fit. The ascissa of the intercept between continuum and physical pion lines gives m_l^{phys}. In fig. 2 we show M_π^2 extrapolated to m_l^{phys} as a function of a^2: discretization effects are about 10%.

Strange quark: for the kaon we have performed a preliminary chiral and continuum fit for each separate strange quark masses, trying $SU(2)$ NLO formula for kaons $M_K^2 = A_s + B_s m_l + C_s a^2$ and $SU(3)$ formulas with some but not all higher-order terms, $M_K^2 = B_0/(m_l + m_s)[1 + B_0 m_s/(2\pi^2 f_0^2) \log m_s + A_s m_l + B m_s + C m_s^2 + D_s a^2]$ followed by a linear fit of extrapolated data in terms of the strange quark. In fig. 3 we the continuum point are extrapolated separately for each m_s, and fitted as a function of m_s. We have also determined m_s from a fictious $s\bar{s}$ meson, similarly to what done in [4].

Charm quark: for the D, D_s and η_c meson we have done the same, using for each simulated charm quark mass the formulas: $M_{D/\eta_c} = A_c + B_c m_l + C_c a^2$ for D and ηc, and $M_{D_s} = A_c + B_c m_l + C_c m_s + D m_s m_l + (E_c + F m_s) a^2$ for D_s meson, followed by a linear fit in terms of the charm mass. Figure 4 is similar to fig. 3 but shows the M_D.

Keeping into account statistic error and systematics due to the spread between different assumptions for the extrapolations, our results for the quark masses in the $\overline{MS}$ scheme read: $\overline{m}_{u/d}(2\,\mathrm{GeV}) = 3.5(3)\,\mathrm{MeV}$, $\overline{m}_s(2\,\mathrm{GeV}) = 91(5)\,\mathrm{MeV}$, $\overline{m}_c(\overline{m}_c) = 1.27(3)\,\mathrm{GeV}$.

REFERENCES

[1] Baron R. *et al.* (ETM Collaboration), *JHEP*, **1008** (2010) 097, arXiv:0911.5061.
[2] Blossier B. *et al.* (ETM Collaboration), *JHEP*, **0804** (2008) 020.
[3] Blossier B. *et al.*, *JHEP*, **1004** (2010) 049.
[4] McNeile C., Davies C. T. H., Follana E., Hornbostel K. and Lepage G. P., *Phys. Rev. D*, **82** (2010) 034512, arXiv:1004.4285.
[5] Dimopoulos P., Frezzotti R., Lubicz V., Martinelli G., Rossi G., Sanfilippo F., Simula S. and Tarantino C., *Phys. Rev. D*, **82** (2010) 114513.
[6] Colangelo G., Durr S. and Haefeli C., *Nucl. Phys. B*, **721** (2005) 136.
[7] Constantinou M. *et al.*, *JHEP*, **1008** (2010) 068, arXiv:1004.1115 [hep-lat].