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Pair photoproduction in a constant and homogeneous electromagnetic field

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Summary. — The process of pair photoproduction in an electromagnetic field of arbitrary configuration is investigated. At high energy the correction to the standard quasiclassical approximation (SQA) of the process probability has been calculated. In the region of intermediate photon energies where SQA is inapplicable the new approximation is used. The influence of weak electric field on the process in a magnetic field is considered. In particular, in the presence of this field the root divergence in the probability of pair creation on the Landau energy levels is vanished. For smaller photon energies the low energy approximations have been derived. At very low photon energy the found probability describes the absorption of soft photon by the particles created by the field. At low photon energy the electric field action dominates and the influence of the magnetic field on the process occurs because of its interaction with the magnetic moment of creating particles.

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1. – Introduction

The pair photoproduction in an electromagnetic field is the basic QED reaction which can play a significant role in many processes. This process was considered first in a magnetic field. The investigation was started in 1952 independently by Klepikov and Toll [1,2]. In Klepikov's paper [3], which was based on the solution of the Dirac equation, the probability of photoproduction had been obtained on the mass shell ($k^2 = 0, k$ is the 4-momentum of photon. We use the system of units with $\hbar = c = 1$ and the metric $ab = a^\mu b_\mu = a^0 b^0 - \boldsymbol{ab}$). In 1971 Adler [4] had calculated the photon polarization operator in a magnetic field using the proper-time technique developed by Schwinger [5] and Batalin and Shabad [6] had calculated this operator in an electromagnetic field using

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the Green's function found by Schwinger [5]. In 1975 the contribution of charged-particles loop in an electromagnetic field with n external photon lines had been calculated in [7]. For $n = 2$ the explicit expressions for the contribution of scalar and spinor particles to the polarization operator of photons were given in this work. Making use of the imaginary part of this operator for spinor particles the pair photoproduction probability was analyzed in the pure magnetic [8] and the pure electric [9] field.

The probability of pair photoproduction in a constant and homogeneous electric field in the quasi-classical approximation had been found by Narozhny [10] using the solution of the Dirac equation in the Sauter potential [11]. Nikishov [12] had obtained the differential distribution of this process also using the solution of the Dirac equation in the indicated field.

In the present paper we consider the integral probability of pair creation by an unpolarized photon in a constant and homogeneous electromagnetic field of an arbitrary configuration basing on the polarization operator [7]. In sect. **2** the exact expression for this probability has been received for the general case $k^2 \neq 0$. In sect. **3** the standard quasi-classical approximation (SQA) [13, 14] is outlined for the high-energy photon $\omega \gg m$ (m is the electron mass). The corrections to SQA, determined also the applicability region of SQA, have been calculated. The found expressions, given in the Lorentz invariant form, contain two invariant parameters. In sect. **4** the new approach has been developed for the relatively low energies where SQA is not applicable. This approach is based on the method proposed in [8]. The obtained probability is valid in the wide interval of photon energy, which is overlapped with SQA. In sect. **5** the case of the "non-relativistic" photon $\omega \ll m$ is analyzed. In particular, in the energy region $\omega \lesssim eE/m$ where the previous approach is inapplicable, the low energy and the very low energy approximations have been developed basing on the analysis in [9]. In turn the found results have an overlapping region of applicability with the previous approach and with each other. So just as in [9] we have four overlapping approximations which include all photon energies. At the photon energy $\omega \ll eEm/(m^2 + eE)$ the probability has been found for arbitrary values of electric E and magnetic B fields.

2. – General expressions for the probability of process

Our analysis is based on the general expression for the contribution of spinor particles to the polarization operator obtained in a diagonal form (Baier, Katkov, Strakhovenko, 1975). The imaginary part of the eigenvalue κ_i of this operator on the mass shell ($k^2 = 0$) determines the probability per unit length W_i of the e^-e^+ pair creation by the real photon with the polarization e_i directed along the corresponding eigenvector. The consideration realizes in the frame where the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields are parallel and directed along the axis 3. The probability of pair creation by the unpolarized photon has the form

$$W = \frac{\alpha m^2 r}{2\pi \mathrm{i}\omega}\mu\nu \int_{-1}^{1} \mathrm{d}v \int_{-\infty-\mathrm{i}0}^{\infty-\mathrm{i}0} f(v,x)\exp[\mathrm{i}\psi(v,x)]x\mathrm{d}x; \tag{1}$$

$$r = \frac{\omega^2 - k_3^2}{4m^2}, \qquad \nu = \frac{eE}{m^2} = \frac{E}{E_0}, \qquad \mu = \frac{eB}{m^2} = \frac{B}{B_0}, \tag{2}$$

$$E_0 = 1.32 \cdot 10^{16}\,\mathrm{V/cm}, \quad B_0 = 4.41 \cdot 10^{13}\,\mathrm{G}.$$

Here

$$f(v,x) = \frac{\cosh(\nu x)(\cos(\mu x) - \cos(\mu x v))}{\sinh(\nu x)\sin^3(\mu x)} + \frac{\cos(\mu x)(\cosh(\nu x) - \cosh(\nu x v))}{\sin(\mu x)\sinh^3(\nu x)}, \tag{3}$$

$$\psi(v,x) = 2r\left(\frac{\cosh(\nu x) - \cosh(\nu x v)}{\nu\sinh(\nu x)} + \frac{\cos(\mu x) - \cos(\mu x v)}{\mu\sin(\mu x)}\right) - x. \tag{4}$$

After all calculations have been fulfilled we can return to a covariant form of the process description using the following expressions:

$$\nu^2 - \mu^2 = 2\mathcal{F} = \frac{\mathbf{E}^2}{E_0^2} - \frac{\mathbf{B}^2}{B_0^2}, \quad \nu\mu = \mathcal{G} = \frac{\mathbf{EB}}{E_0 B_0}. \tag{5}$$

The SQA is valid for ultrarelativistic created particles ($r \gg 1$) and can be derived from eqs. (1)-(4) by expanding the functions $f(v,x)$, $\psi(v,x)$ over x powers. Retaining the leading powers of x one obtains

$$W^{(\mathrm{SQA})} = \frac{\alpha m^2}{3\sqrt{3}\pi\omega}\int_0^1 \frac{9 - v^2}{1 - v^2}K_{2/3}(z)\mathrm{d}v, \quad z = \frac{8}{3(1 - v^2)\kappa}, \tag{6}$$
$$\kappa^2 = 4r(\mu^2 + \nu^2) = -\frac{e^2}{m^6}\left(F^{\mu\nu}k_\nu\right)^2.$$

To get the correction to the probability in SQA we shall keep the next powers of x. We have

$$W^{(1)} = -\frac{\alpha m^2\mathcal{F}}{30\sqrt{3}\pi\omega\kappa}\int_0^1 \frac{\mathrm{d}v}{1 - v^2}G(v,z), \tag{7}$$

where

$$G(v,z) = 2(1 + v^2 - 27z^2)K_{1/3}(z) + 3(7 - v^2)zK_{2/3}(z). \tag{8}$$

It is seen that in this order of decomposition the correction does not depend on the invariant parameter $\mathcal{G}$, because $\mathcal{G}$ is the pseudoscalar. The asimptotics of the integrals incoming in the correction terms have been given in Appendix C [8]. The asymptotic at $\kappa \ll 1$ will become necessary further

$$W^{(1)} = \frac{6\alpha m^2\mathcal{F}}{5\omega\kappa^2}\sqrt{\frac{2}{3}}\exp\left[-\frac{8}{3\kappa}\right], \qquad \frac{W^{(1)}}{W^{(\mathrm{SQA})}} = \frac{64\mathcal{F}}{15\kappa^3}. \tag{9}$$

3. – Region of intermediate photon energies

In the field, which is weak in comparison with the critical field $E/E_0 = \nu \ll 1$, $B/B_0 = \mu \ll 1$ and at the relatively low photon energies $r \lesssim \nu^{-2/3}$, the standard quasi-classical approximation is non-applicable. At these energies, if the condition $r \gg \nu^2$ is fulfilled, the saddle-point method can be applied to integration over x. In this case the small values of v contribute to the integral over v. So one can expand the phase $\psi(v,x)$ over v and extend the integration limit to the infinity. We get

$$W=\frac{\alpha m^2 r}{2\pi \mathrm{i}\omega}\mu\nu\int_{-\infty}^{\infty}\mathrm{d}v\int_{-\infty}^{\infty}f(0,x)\exp\left[-\mathrm{i}\left[\varphi(x)+v^2\chi(x)\right]\right]x\mathrm{d}x, \tag{10}$$

where

$$\varphi(x)=2r\left(\frac{1}{\mu}\tan\frac{\mu x}{2}-\frac{1}{\nu}\tanh\frac{\nu x}{2}\right)+x, \tag{11}$$

$$\chi(x)=rx^2\left(\frac{\nu}{\sinh(\nu x)}-\frac{\mu}{\sin(\mu x)}\right). \tag{12}$$

From the equation $\varphi'(x_0)=0$ we find the saddle point x_0

$$\tan^2\frac{\nu s}{2}+\tanh^2\frac{\mu s}{2}=\frac{1}{r},\qquad x_0=-\mathrm{i}s. \tag{13}$$

At $r\gg 1$ we have

$$W=\frac{3\alpha m^2\kappa}{16\omega}\sqrt{\frac{3}{2}}\exp\left[-\frac{8}{3\kappa}+\frac{64\mathcal{F}}{15\kappa^3}\right]. \tag{14}$$

This expression is valid at $\kappa\ll 1$ and coincides with eq. (9) for $\mathcal{F}\ll\kappa^3$. So the overlapping region of both approximations exists.

It is interesting to consider the photon energy region $|r-1|\ll 1$ in the presence of a weak electric field ($\nu\ll\mu$) where in the absence of an electric field the approach under consideration is valid if the condition $r-1\gg\mu$ is fulfilled. In this case the following approximate equations are valid:

$$\frac{\xi^2 y_0^2}{16}\simeq\exp[-y_0]+\frac{1-r}{4},\quad y_0=\mu s,\ \xi=\frac{\nu}{\mu}; \tag{15}$$

$$y_0\simeq 2\ln\frac{2}{\xi\ln\frac{4}{\xi}}\left(1-\frac{r-1}{2\xi^2\ln\frac{2}{\xi}\ln^3\frac{4}{\xi}}\right),\quad |r-1|\lesssim\xi^2; \tag{16}$$

$$y_0\simeq\ln\frac{4}{r-1}\left(1-\frac{\xi^2}{4(r-1)}\ln\frac{4}{r-1}\right),\quad r-1\gg\xi^2; \tag{17}$$

$$\xi y_0=\nu s\simeq 2\sqrt{1-r},\quad 1-r\gg\xi^2. \tag{18}$$

The applicability of the using saddle-point method is connected with the large value of the coefficient to the second power $(y-y_0)^2$ of the decomposition in the phase $\varphi(x)$. In the energy region under consideration we have

$$\mathrm{i}\varphi''(x_0)(x-x_0)^2/2\simeq\frac{\xi^2}{4\mu}\left[y_0+\frac{y_0^2}{2}+\frac{2(r-1)}{\xi^2}\right](y-y_0)^2. \tag{19}$$

So, in the case $\nu/\mu=\xi\ll 1$, $|r-1|\lesssim\xi^2$ this approximation is valid if the condition $\xi^2/\mu\gg 1$ is fulfilled. In the case $1\gg r-1\gg\xi^2$ the condition $r-1\gg\mu$ has to be available for that. And in the case $1 \gg 1 - r \gg \xi^2$ the condition $\sqrt{1-r}\xi/\mu = \sqrt{(\xi^2/\mu)(1-r)/\mu} \gg 1$ is necessary.

At energy $r \ll 1$ ($\nu^2 \ll r \ll \nu^{2/3}$) we have $\nu s \simeq \pi - 2\sqrt{r}$,

$$W = \frac{\alpha m^2 \mu}{4\omega\sqrt{r}} \coth(\pi\eta) \exp\left[-\frac{1}{\nu}\left(\pi - 4\sqrt{r} + \frac{2r}{\eta}\tanh\frac{\pi\eta}{2}\right)\right]. \tag{20}$$

At $\eta \gg 1$ the probability W has been increased by the factor $\eta\pi\exp[\pi r/\nu]$ in comparison with the case of the absence of magnetic field.

4. – Approximations at low photon energy

At $r \sim \nu^2$ the above approximation becomes non-applicable and another approach has to be applied. We close the integration over x contour in the lower half-plane and represent eq. (1) in the following form:

$$W = \frac{\alpha m^2 r}{2\pi i\omega}\mu\nu \int_{-1}^{1} \mathrm{d}v \sum_{n=1}^{\infty} \oint f(v,x)\exp[i\psi(v,x)]x\mathrm{d}x, \tag{21}$$

where the path of integration is any simple closed contour around the point $-i\pi n/\nu$. Let us choose the contour near this point in the following way $\nu x = -i\pi n + \xi_n$, $|\xi_n| \sim \sqrt{r} \sim \nu$ and expand the function entering in over the variables ξ_n. In the case $\nu \ll 1$, because of the appearance of the factor $\exp[-i\pi n/\nu]$, the main contribution to the sum gives the term $n = 1$. Near the point $-i\pi/\nu$ the main terms of expansion ($\xi \equiv \xi_1$) are

$$f = \frac{2i}{\xi^3}\coth(\pi\eta)\cos^2\frac{\pi v}{2}, \qquad \psi = \frac{4r}{\xi\nu}\cos^2\frac{\pi v}{2} - \frac{\xi}{\nu} + \frac{i\pi}{\nu}. \tag{22}$$

Using eq. (7.3.1) and eq. (7.7.1 (11)) in [15] we find after integration over ξ and v

$$W = \frac{\alpha m^2}{\omega}\eta\pi\coth(\pi\eta)\exp\left[-\frac{\pi}{\nu}\right] I_1^2(z), \quad z = \frac{2\sqrt{r}}{\nu}, \tag{23}$$

where $I_n(z)$ is the Bessel function of the imaginary argument. The found probability is applicable for $r \ll \nu$.

For $r \gg \nu^2$ the asymptotic representation $I_n(z) \simeq \exp[z]/\sqrt{2\pi z}$ can be used. As a result one obtains eq. (20) if in the exponent of the last one leaves out the term $\propto r/\nu$. At very low photon energy $r \ll \nu^2$, using the expansion of the Bessel functions for the small value of argument, we have

$$W = \frac{\alpha m^2 r}{\omega\nu^2}\eta\pi\coth(\pi\eta)\exp\left[-\frac{\pi}{\nu}\right]. \tag{24}$$

The probability under consideration draws the interest of theoreticians for arbitrary values μ and ν. For $r \ll \nu^2/(1+\nu^2)$ one can conserve in the phase $\psi(v,x)$ the term $-x$

only. After integrating over v we get the following equation for the probability of photon absorption:

$$W = \frac{\alpha m^2 r}{\mathrm{i}\pi\omega}\sum_{n=1}^{\infty}\oint F(y_n)\exp\left[-\mathrm{i}\frac{y_n}{\nu}\right]\mathrm{d}y_n, \quad y_n = -\mathrm{i}n\pi + y, \tag{25}$$

$$F(y) = \frac{\cosh(y)\left(\eta y\cos(\eta y) - \sin(\eta y)\right)}{\sinh y \sin^3 \eta y} + \frac{\eta\cos(\eta y)(y\cosh y - \sinh y)}{\sinh^3 y\sin(\eta y)}. \tag{26}$$

Summing the residues in the points $y_n = -\mathrm{i}n\pi$ one obtains

$$W = \frac{\alpha m^2 r}{\omega}\sum_{n=1}^{\infty}\exp\left[-\frac{\pi n}{\nu}\right]\Phi(z_n), \quad z_n = \eta\pi n, \tag{27}$$

$$\Phi(z_n) = \frac{z_n}{\nu^2}\coth z_n + \frac{2}{\sinh^2 z_n}\left[\frac{\eta z_n}{\nu} + \left(1+\eta^2\right)z_n\coth z_n - 1\right]. \tag{28}$$

At $\eta \to 0$, $z_n \to 0$ we have (compare with eq. (28) in [9])

$$\Phi = \frac{1}{\nu^2} + \frac{2}{\nu\pi n} + \frac{2}{\pi^2 n^2} + \frac{2}{3}, \tag{29}$$

$$W = \frac{\alpha m^2 r}{\omega}\left[\left(\frac{1}{\nu^2} + \frac{2}{3}\right)\frac{1}{e^{\pi/\nu} - 1} - \frac{2}{\pi\nu}\ln\left(1 - e^{-\pi/\nu}\right) + \frac{2}{\pi^2}\mathrm{Li}_2\left(e^{-\pi/\nu}\right)\right], \tag{30}$$

where $\mathrm{Li}_2(z)$ is the Euler dilogarithm. In the opposite case $\eta \to \infty$, $z_n \to \infty$ one obtains

$$\Phi = \frac{\pi\eta n}{\nu^2}, \qquad W = \frac{\alpha m^2 r}{\omega\nu^2}\frac{\pi\eta}{4}\sinh^{-2}\frac{\pi}{2\nu}. \tag{31}$$

5. – Conclusion

The probability of the process has been calculated using four different overlapping approximations. In the region of SQA applicability the particles created by a photon have ultrarelativistic energies. The role of fields in this case is to transfer the required transverse momentum and the electric and magnetic field actions are equivalent. But even in this case it is necessary to note the special significance of the weak electric field $E = \xi B$ ($\xi \ll 1$) in the removal of the root divergence of the probability when the particles of the pair are created on the Landau levels with the electron and positron momentum $p_3 = 0$. In the frame used, $k_3 = 0$.

Generally speaking, at $\xi \ll 1$ the formation time t_c of the process under consideration is $1/\mu$. Here we use units $\hbar = c = m = 1$. At this time the particle of the pair gets the momentum $\delta p_3 \sim \xi$ because of the electric field. If the value ξ^2 becomes more larger than the distance apart Landau levels 2μ ($\nu^2 \gg \mu^3$) all levels have been overlapped. Under this condition the divergence of the probability is vanished and the new quasi-classical approach is valid even in the energy region $r - 1 \lesssim \mu$ where it has been inapplicable in the absence of the electric field. In the opposite case $\nu^2 \ll \mu^3$ for the small value of $p_3 \ll \sqrt{\mu}$ in the region where the influence of the electric field can be neglected, the formation time of the process t_f is $1/p_3^2$ and $\delta p_3 \sim \nu/p_3^2 \ll p_3$. It follows from the above that in this case the condition $\nu^{1/3} \ll p_3 \ll \sqrt{\mu}$ has to be satisfied. At this condition the value of discontinuity is $\sqrt{t_f/t_c} \sim \sqrt{\mu}/p_3$. For $\nu^{1/3} \gg p_3$ the time t_f is determined by the self-consistent equation $\delta\varepsilon^2 \sim 1/t_f \sim \nu^2 t_f^2$, $t_f \sim \nu^{-2/3}$ and the value of discontinuity becomes $\sqrt{\mu t_f} \sim (\mu^3/\nu^2)^{1/6}$ instead of $\sqrt{\mu}/p_3$. In the region $\omega \lesssim 2m$ ($r \lesssim 1$) the energy transfer from the electric field to the created particles becomes appreciable and for $\omega \ll m$ it determines mainly the probability of the process. At $\omega \ll eE/m$ the photon assistance in the pair creation comes to an end and the probability under consideration defines the probability of photon absorption by the particles created by electromagnetic fields. The influence of the magnetic field on the process is connected with the interaction of the magnetic moment of the created particles and magnetic field. This interaction, in particular, has appeared in the distinction of the pair creation probability by field for scalar and spinor particles [5].

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