

Photon emission and photoproduction processes in bent single crystals

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Summary. — The process of photon emission by high energy electrons or positrons moving in a bent single crystal was considered. The spectrum of energy losses by particles and polarization of emitted photons were calculated. The corrections due to multiphoton emission were obtained. Photoproduction process via electron-positron pairs in a bent single crystal was also studied. Total cross-section at different bending radii was calculated for different initial polarization states of high energy photons.

PACS 61.85.+p – Channeling phenomena (blocking, energy loss, etc.).

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1. – Introduction

It is well known that an ordered location of atoms in single crystals creates periodic microscopic electric fields. The entering charged particle in a single crystal moves interacting with these fields. Processes arising at interaction of a particle with such electric fields are mostly coherent. The coherent processes take place in both straight and bent single crystals. In particular, the coherent scattering (volume reflection) of relativistic protons by bent crystallographic planes was observed in a recent experiment [1]. For light leptons (electrons and positrons) the volume reflection is accompanied by power photon emission. For the first time this process was calculated in [2] and then was measured in experiments [3,4]. The results of experiments are in satisfactory agreement with calculations.

These calculations (see [2-4]) were performed on the basis of the quasiclassical approach developed in [5]. There are many computing difficulties for solution of the problem under consideration with the help of this method. In this paper we propose a more simple method for calculation of photon emission in planar fields of a bent single crystal. This method may be applied for electron or positron beams with energies up to

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several hundreds GeV and a crystal thickness of several millimeters. The method has a common enough feature but we illustrate it under the conditions of the recent CERN experiment [6].

2. – Calculation of radiation intensity

It is well known that the process of radiation of a relativistic particle at quasiperiodic motion may be characterized with the help of parameter $\rho = 2\gamma^2\langle(v(t) - v_m(t))^2\rangle/c^2$, where γ is the Lorentz factor, $v(t)$, v_m are the current (as a function of time t) and mean transversal velocities of a particle, c is the velocity of light, the brackets $\langle\dots\rangle$ mean the averaging over time. The radiation process has a dipole (interference) character when $\rho \ll 1$ and a synchrotron-like one when $\rho \gg 1$. When $\rho \sim 1$ is an intermediate case.

Further consideration of the problem will be based on the results of the paper [7] which present the investigation of particle motion in bent planes of a single crystal. In this case, ultrarelativistic particles intersect a set of parallel crystallographic planes, and, because of this, their transversal velocities oscillate. The peculiarities of such motion are aperiodicity and amplitude variations of oscillations. However, on a short part of the particle trajectory these variations of period and amplitude are insignificant (for large enough bending radii). It allows us to calculate the ρ -parameter for every oscillation. These calculations (for 120 GeV positrons moving in the (110) silicon plane) show that the ρ -parameter is less than 2 and exceeds 1 only for several oscillations in the vicinity of a critical point. Further we will suppose that radiation process has a dipole character.

Our consideration is based on the assumption that the conditions for the coherent bremsstrahlung take place on a short part (about several periods) of the particle trajectory. It means that

$$(1) \quad \frac{d^2\mathcal{E}}{dE_q}(t) = \frac{dI}{dE_q}(\tilde{\theta}(t))cdt = \frac{dI}{dE_\gamma}(\tilde{\theta}(t))cdt,$$

where $d\mathcal{E}/dE_e$ is the differential energy losses of positrons at time t , $E_q = E_0 - E_e = E_\gamma$, E_0 and E_e are the positron energies before and after radiation, E_γ is the photon energy, $\tilde{\theta}$ is the averaged (see below) angle with respect to crystallographic planes, c is the velocity of light. For $dI/dE_\gamma(\theta) = nE_\gamma d\sigma_\gamma/dE_\gamma$ function (n is the number atoms per volume unit and σ_γ is the cross-section of the process) we take the corresponding relation from the theory of coherent bremsstrahlung [5, 8]. Besides, this equation is valid for large enough bending radii of a single crystal.

For usage of eq. (1) one needs to know the dependence of $\tilde{\theta}$ on time, or, in other words, we should find the equation of motion in bent single crystals. In particular, this problem was considered in [7]. As was shown, in our case the problem is reduced to one-dimensional motion in the transversal direction. One can describe the variation of the transversal coordinate x by the following equation:

$$(2) \quad t = \sqrt{\frac{E_0}{2c^2}} \int_{x_0}^x \frac{dx}{\sqrt{E - U(x) - E_0x/R}},$$

where $U(x)$ is the periodic interplanar potential, R is the bending radius and $E = E_0\theta_0/2 + U(x_0) + E_0x_0/R$ is the total transversal energy and $t = 0$ corresponds to x_0 . Here we put the relative total particle velocity (in comparing with c) equal to 1.

The transversal coordinate x_c and time t_c corresponding to the point of particle reflection satisfy the equation

$$(3) \quad E - U(x_c) - E_0 x_c / R = 0.$$

The particle angle (with respect to the crystallographic planes) θ is equal to $\frac{1}{c} \frac{dx(t)}{dt}$. At particle motion this angle performs oscillations and accordingly to the theory of dipole radiation we should insert in eq. (1) its value averaged over the oscillation period, or, in other words, the $\tilde{\theta}$ -angle. One can describe the variation of transversal averaged over oscillations coordinate of the moving particle by the following equation:

$$(4) \quad x = x_0 + \tilde{v}_0(x_0)t - \frac{c^2 t^2}{2R},$$

where

$$(5) \quad \tilde{v}_0(x_0) = c \sqrt{\theta_0^2 + 2(U(x_0) - U(x_c))/E_0},$$

and x_0, θ_0 are the initial transversal coordinate and the angle.

From here, the averaged mean angle $\tilde{\theta}$ is approximately equal to

$$(6) \quad \tilde{\theta} = \frac{\tilde{v}_0}{c} - \frac{ct}{R}.$$

However, this approximation works from $t = -\infty$ until $t = t_c$. As was considered in [7] at $t = t_c$ particle undergoes the volume reflection and the direction of its transversal velocity is changed. For $t > t_c$ (using the symmetry properties of the volume reflection process) we can write

$$(7) \quad \tilde{\theta} = \frac{2ct_c}{R} - \frac{\tilde{v}_0}{c} - \frac{ct}{R}.$$

From here, we see that the averaged angle $\tilde{\theta}$ is a linear function of time from $t = -\infty$ until t_c and from t_c until $+\infty$, and it is changed by leap at $t = t_c$. Then the absolute value of angle variation is equal to $\theta_{\min} = |\tilde{v}_0/c - ct_c/R|$. From [7] one can find that $\theta_{\min} = |\alpha(E)|/2$, where $\alpha(E)$ is the angle of volume reflection of a particle with the transversal energy E . As was shown in [7] α -value depends weakly on E at large enough bending radii, and one can take the mean value α (averaged over one period of E) for calculations. Taking this and eq. (1) into account we can calculate the radiation energy losses of particle from t_1 until t_2 ($t_1 < t_c < t_2$):

$$(8) \quad \frac{d\mathcal{E}}{dE_q}(E_q) = R \int_{\theta_1}^{-\theta_{\min}} \frac{dI}{dE_q}(\tilde{\theta})d\tilde{\theta} + R \int_{\theta_{\min}}^{\theta_2} \frac{dI}{dE_q}(\tilde{\theta})d\tilde{\theta},$$

where $\theta_1 < -\theta_{\min}$ and $\theta_2 > \theta_{\min}$ are the entrance and exit angles. In the case when $t_2 < t_c$ ($t_1 > t_c$) one plain integral should be used. Note one can find the function $I(\theta)$ in the literature, see, for example [5, 8].

In straight single crystals (if we do not take into account multiple scattering) the energy losses of ultrarelativistic particles are equal to $I(\theta)L$. It means that the form

of spectrum is independent of the thickness of a single crystal. Our calculations show that in bent single crystals the form of spectrum depends on the thickness and bending radius.

It should be noted that at fixed angle θ_0 the particle trajectories depend on initial x_0 (see term $U(x_0)$ in eq. (5)), but for large enough bending radii this dependence is very weak, and it is possible not to take it into account.

3. – Taking into account multiplicity of photon emission

It is obvious that eq. (8) is valid only for thin enough single crystals, when the probability of emission of two and more photons by one electron is small in comparison with one. The simulations for conditions of the experiment [6] by Monte Carlo method based on the above-mentioned description of the radiation process have shown that the mean number of photons with energy more than 1 GeV per positron was about 1.5. This fact requires to correct our description and to take into account this possibility. As a result for radiation energy losses we get

$$(9) \quad \frac{d\mathcal{E}}{dE'}(\epsilon, E', z_0) = (E_0 - E') \left(\frac{dN_{1e}}{dE'}(\epsilon, E', z_0) + \frac{dN_{2e}}{dE'}(\epsilon, E', z_0) \right),$$

where E' is the positron energy on the exit of a single crystal with thickness equal to z_0 and ϵ is the photon cutting energy (see below). The N_{1e} , N_{2e} functions are

$$(10) \quad \frac{dN_{1e}}{dE_+}(\epsilon, E_+, z_0) = \int_0^{z_0} \frac{d^2N_e}{dzdE_+}(\epsilon, E_+, z) \exp \left[-n \int_z^{z_0} \sigma_\gamma(\epsilon, E_+, z) dz \right] dz,$$

$$(11) \quad \begin{aligned} \frac{dN_{2e}}{dE'}(\epsilon, E', z_0) = & \\ = \int_0^{z_0} \int_{E'+\epsilon}^{E_0-\epsilon} \frac{d^2N_e}{dzdE_+}(\epsilon, E_+, z) & \left\{ 1 - \exp \left[-n \int_z^{z_0} \sigma_\gamma(\epsilon, E_+, z) dz \right] \right\} \\ & \times \rho(\epsilon, E_+, E', z) dE_+ dz, \end{aligned}$$

where E_+ and E' are the positron energies after the first and second photon emission (with the photon energy E_γ in the range from ϵ up to E_0), n is the number atoms per unit of volume and $\sigma_\gamma(\epsilon, E_+, z)$ is the cross-section of coherent bremsstrahlung [5, 8] on the current thickness $z \leq z_0$ of a single crystal and it is coupled with the intensity on the same coordinate by the equation $d\sigma_\gamma/dE_\gamma = \frac{1}{E_\gamma} dI/dE_\gamma$ and the angle θ is determined by the current coordinate $z = ct$. One can find the N_e and ρ -functions from the relations

$$(12) \quad \frac{d^2N_e}{dzdE_+}(\epsilon, E_+, z) = \exp \left[-n \int_0^z \sigma_\gamma(\epsilon, E_0, z) dz \right] n \sigma_\gamma(\epsilon, E_0, z) \rho(\epsilon, E_0, E_+, z),$$

with the function

$$(13) \quad \rho(\epsilon, E_0, E_+, z) = \frac{1}{n \sigma_\gamma(\epsilon, E_0, z)} \frac{d\sigma_\gamma(\epsilon, E_0, E_+, z)}{dE_+}.$$

For small thickness (when $n \int_0^{z_0} \sigma_\gamma(\epsilon, E', z) dz \ll 1$) eq. (9) gives the result

$$(14) \quad \frac{d\mathcal{E}}{dE'}(\epsilon, E', z_0) = (E_0 - E')n \int_0^{z_0} \frac{d\sigma_\gamma}{dE'}(\epsilon, E_0, E', z) dz.$$

Under the conditions of the experiment $n \int_z^{z_0} \sigma_\gamma(\epsilon, E', z) dz \sim 1$ and hence we should use eqs. (9)–(12) for calculations. This consideration allows one to take into account: 1) the nonlinear (exponential decrease) character of the process as a function of thickness; 2) the multiplicity of photon emission by one positron. Our consideration works for the case when only not larger than two high energy photons may be emitted. Of course, it is easy to obtain similar relations for more multiplicity but there are difficulties of calculations of multidimensional integrals.

Besides, for a correct work of the method the value ϵ should be defined correctly. Our choice of this value can be understood from the following simple arguments. In the experiment only the more than 2 GeV energy losses were fixed. Besides, the momentum spread of the positron beam was about 1 percent. It allows us to put ϵ equal to 1.2 GeV. Thus, we do not take into account the emission of low energy photons. Results of calculations should be practically independent of ϵ -value. Really, for variations of ϵ in the range from 0.12 up to 1.2 GeV the calculated spectra of energy losses are very close in between. Besides, under the conditions of the experiment [6] the energy losses are determined mainly by a coherent part of the cross-section, which is a finite value. Calculations with zero incoherent cross-section (and $\epsilon = 0$) demonstrate insignificant differences of results.

There was additional amorphous material in the beam line and this material gave radiation energy losses of positrons which are not coupled with the studied process. It is a usual situation in similar experiments. Due to this fact, we introduce approximate calculations of the influence of an additional material on energy losses spectra. As already noted, the total differential cross-section of the investigated process is the sum of coherent $d\sigma_c$ and incoherent $d\sigma_a$ terms. We believe that the additional material may be taken into account by a corresponding increasing of $d\sigma_a$.

Note our calculations were performed on the basis of atomic form factors taken from X-ray measurements [9, 10].

4. – Calculation and comparison with measurements

Here we illustrate the developed method of calculations for conditions of the recent CERN experiment [6]. The experiment was performed at positron energy equal to 120 GeV. The silicon single crystal of the (110) orientation, 2 mm of thickness was used in measurements at two values of bending radii (4.7 and 11 meters). Figure 1 illustrates the calculated spectra ((a), (b) for bending radii 11 m and 4.7 m, correspondingly) and degree of linear polarization ((c), the curves 1 and 2 for bending radii 11 m and 4.7 m, correspondingly). The curves 1 and 2 correspond to the two cases. The first case is calculation for a pure crystal in the beam and the second one is for the sum of crystal and background from an additional substance in the beam line. Measurements (without crystal and with nonoriented crystal) give the value equal to ≈ 0.7 for ratio of energy losses. The circles and squares are the results of measurements for oriented and nonoriented positions. The curves 1 and 2 in fig. 1(a,b) are the result of the averaging over the angle divergence of positron beam, which was ± 50 and $\pm 173 \mu\text{rad}$ relative to the central coming angle, correspondingly. Note our calculations show that averaged spectra are close to the spectra calculated for the central angle. This is true also for the polarization

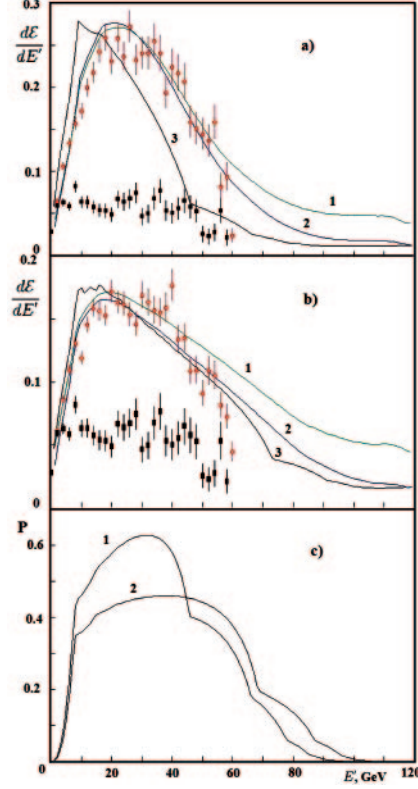


Fig. 1. – Calculations of radiation energy losses (a, b) and linear polarization (c) of 120 GeV positrons in the (110) plane of a silicon single crystal (2 mm of thickness) as functions of energy. In (a, b) the curves 1 present results of calculation of the energy losses in the experiment (crystal + background), while the curves 2 correspond to a pure crystal. In (c) the curves 1 and 2 present linear polarization as a function of photon energy for bending radii 11 and 4.7 meters, respectively. For additional information, see the text.

dependences. The curves 3 (in fig. 1(a,b)) are (multiplied on 0.65 and 0.85, correspondingly) the energy losses calculated with the help of eq. (14) and hence do not take into account multiphoton production.

We have employed the proposed here method for calculation of energy losses of 180 GeV positrons and electrons [4]. Figure 2 illustrates these calculations.

In a whole we see good enough agreement between calculations and measurements. Some disagreement in fig. 2 can be explained as at 180 GeV the condition for dipole radiation is more violated than at 120 GeV. Calculations presented here for crystals of 1-2 mm of thickness show also importance taking account the multiphoton character of emission. From this point of view it is easy to understand why discrepancy between measurements and calculations carried out in one-photon approximation (curves 3 in fig. 1 and curves 2 in fig. 2) is less for smaller bending radii which correspond to smaller photon multiplicity. The process of coherent bremsstrahlung in bent single crystals was also studied in the papers [11-13] but these results do not take into account the conditions for volume reflection. Because of this, one can expect enhancement of a soft part of photon spectrum (see, for example [13], where the calculated soft part exceeds strongly corresponding experimental data).

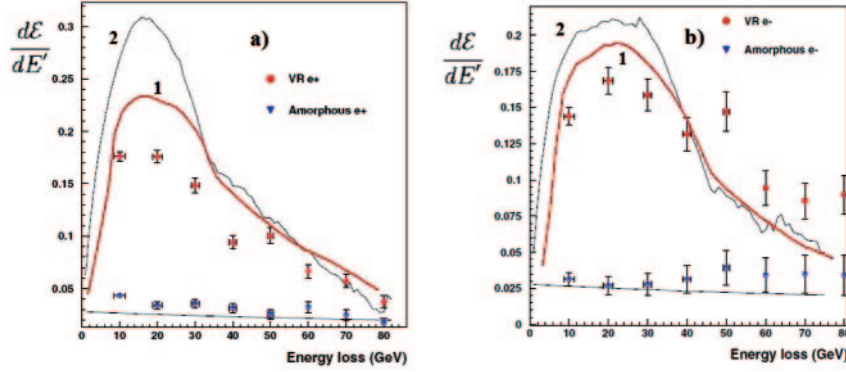


Fig. 2. – Calculations of radiation energy losses of positrons (a) and electrons (b) in the (111) planes of silicon single crystals. The curves marked as 1 are new calculations, the curves 2 are previous ones [4]. Symbols are measurements.

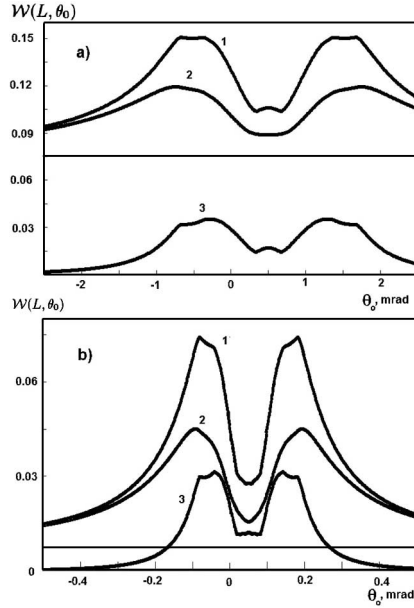


Fig. 3. – The probabilities of pair production in the (110) bent planes of silicon single crystals as a function of the entrance angle θ_0 . Curves 1 and 2 are the probabilities for photons with the linear polarization along and perpendicular with respect to the planar electric field and curve 3 is their difference. The thin straight line is the probability in a nonoriented single crystal. The bending radius is 10 m. The photon energies and the thickness of the crystal are 120 GeV and 1 cm (a) and 1000 GeV and 0.1 cm (b).

5. – Photoproduction of electron-positron pairs in bent single crystals

The process of photoproduction of electron-positron pairs in bent single crystals was investigated in the paper [14]. In many respects this consideration is similar to the coherent radiation process. Figure 3 illustrates the probability of e^\pm pair production for photons with energies equal to 120 and 1000 GeV.

6. – Conclusions

The results obtained here allow us to draw the following conclusions:

- 1) A simple method of calculations of photon emission by electrons (positrons) moving in a planar field of bent single crystal was proposed.
- 2) The results of simulations and experimental data are in good agreement.
- 3) The consideration may be expanded on the axial case of orientations.
- 4) The process of photoproduction of e^\pm -pairs in bent single crystals was investigated.
- 5) The polarization dependences for both the processes were presented.

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