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On the analogies between the processes of coherent radiation at collisions of relativistic particles with bunches of relativistic particles and crystals

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Summary. — At scattering of relativistic bunches of charged particles the effect of a significant amplification of radiation in the region of low radiated frequencies, when the spectral density is proportional to the square number of particles in the bunch, is possible. This effect holds both for head-scattered bunches, and bunches which scattered at a small angle between their axes of motion. At the same time with an increasing number of the particles in bunches the radiation is almost independent of the number of particles. The analogy of these effects and the effects of radiation at the passage of charged particles in matter is discussed. The analogy of the mechanisms of these radiation processes not only for coherent, but also for incoherent radiation, is shown. The possibility of usage of the coherent effect in radiation for monitoring charged particle beams is noted.

PACS 61.80.-x – Physical radiation effects, radiation damage.
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1. – Introduction

At collisions of bunches of relativistic charged particles the coherent effect in radiation, when the spectral density of radiation is proportional to the number of particles in the incident bunch, is possible. This effect occurs at low radiated frequencies, the formation length of which is large enough. The possibility of this effect was pointed out in [1-3], where the analysis was performed in the frame of the Born perturbation theory of QED, when the condition $Ne^2/\hbar c \ll 1$ holds. However, with increasing number of particles in the incident bunch, that may be equal to $N \sim 10^{10}$, as shown in [4], the applicability conditions of perturbation theory are violated, $Ne^2/\hbar c \gg 1$, and there is need to consider this process beyond the applicability frame of perturbation theory.

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At high energies of particles of colliding bunches, bremsstrahlung process can be considered on the basis of the Weizsacker-Williams method as the process of Compton scattering of equivalent photons of the charge Coulomb field on the incident bunch. In [5] they showed that the process of backward Compton scattering of low-frequency photons (when it is possible to neglect the recoil effect in radiation) on the relativistic charge can be considered from the viewpoint of classical electrodynamics as a process of radiation of a charge at its accelerated motion in the field of the incident plane electromagnetic wave. In other words, the classical and quantum theory in this case yields identical results. Physically, the process of radiation of a relativistic charge in the field of the incident plane wave is identical to the process of radiation of a charge when it is moving in an undulator.

The above results show that the bremsstrahlung of relativistic electrons in the field of incident bunch in the region of sufficiently small frequencies of radiation, when the recoil effect in radiation may be neglected, could be considered classically.

The most interesting is the case of low frequencies of radiation ω, when the formation length $l_{\mathrm{coh}} = 1/\delta \approx 2\gamma^2/\omega$ (γ is the Lorenz factor of radiated electron) is greater than the longitudinal size of the incident bunch L. In this case correlations in the electron scattering on separate charges of the bunch are significant, so that the total electron scattering angle is determined by the total number of particles in the bunch N. Spectral density of radiation at the same time is determined by the total electron scattering angle. For sufficiently large impact parameters of scattering ρ the conditions of applicability of the dipole approximation of the classical theory of radiation may be held, and the spectral density of radiation is proportional to N^2 [4]:

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = N^2 \frac{32e^4Q^2}{3\pi m^2\rho^2}\,. \tag{1}$$

For example, for electrons with energy $\varepsilon = 5\,\mathrm{GeV}$, impact parameter $\rho = 0.01\,\mathrm{cm}$, bunch length $L \sim 0.1\,\mathrm{cm}$ and number of particles in the bunch $N \sim 10^{10}$, the coherent effect in radiation is possible for radiated frequencies up to 50 keV.

At the decrease of impact parameter the effects of multiple scattering of electrons on the bunch particles are significant (it becomes necessary to take into account the bunch structure) and the quadratic dependence of the spectral density on the number of particles becomes logarithmic:

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = \frac{4e^2}{\pi} \ln\left(N\frac{4eQ}{m\rho}\right). \tag{2}$$

In this case, the radiation has essentially a nondipolar origin, and the mechanism of its appearance is similar to mechanisms of the effects such as Landau-Pomeranchuk-Migdal as well as Ternovskii-Shul'ga-Fomin, *i.e.* the suppression of bremsstrahlung by fast particles in an amorphous medium [6-8] (the last effect was recently confirmed in CERN experiment NA63).

Thus, there is a profound analogy among the many processes of radiation at collisions of relativistic bunches of charged particles and at the passage of particles in crystals and amorphous media. This analogy allows us, for the description of the radiation at the collision of bunches, to use many methods developed for studying the radiation by particles in matter. The essential distinction of these processes is in the potentials. Namely, the charge in a matter interacts with the screened potential of atoms, while at collision of bunches the scattering occurs in the long-range Coulomb field of charges. This leads to the fact that the longitudinal distances, responsible for the radiation process, are the same as in the matter l_{coh}, but the transverse distances at γ-times are greater than the longitudinal and can be huge. For example, for electrons with energy $\varepsilon \sim 50\,\text{MeV}$ and radiated photons with $\omega = 1\,\text{keV}$, the formation length is $l_{\text{coh}} \sim 1\,\text{cm}$, while the transverse distances are of about 1 m (accounting of this phenomenon is important in the design of radiation detectors in accelerators). Availability of this feature leads to the necessity of a better understanding of this analogy. In particular, it is important to study the spectral density not only in the low-frequency region of radiation, but in the case of high frequencies (but such that the recoil effect could still be ignored), when a substantial contribution to the radiation is due to the incoherent part.

2. – Efficiency of coherent and incoherent radiation

Below we consider the electron incident on a short bunch of fast charges. Let us assume that the conditions of the dipole radiation are hold. Spectral density of radiation in this case [9]

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = \frac{e^2\omega}{2\pi}\int_\delta^\infty \frac{\mathrm{d}q}{q^2}\left(1 - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right)|W(q)|^2 \tag{3}$$

is determined by the transverse component of electron acceleration when electron is scattered in the field of a bunch

$$W(q) = \int_{-\infty}^{+\infty} \mathrm{d}t e^{iqt}\dot{v}_\perp(t) = -\frac{2e}{E}\int_{-\infty}^{+\infty}\mathrm{d}t e^{iqt}\frac{\partial U(r,t)}{\partial\rho}\,, \tag{4}$$

where $U(r,t) = \sum_n u(\rho - \rho_n, vt - z_n)$ is the potential of all particles of the bunch, $u(r)$ is the potential of individual particle and r_n is the bunch particle position. Using Fourier transform $U(r) = 1/(2\pi)^3\int U_k e^{-ikr}\mathrm{d}^3k$ expression (4) can be reduced to

$$W(q) = -\frac{4\pi e}{E}\sum_n\int \mathrm{d}^2k_\perp u_{k_\perp,q}k_\perp e^{iqz_n}e^{-ik_\perp(\rho-\rho_n)}. \tag{5}$$

Particles in the incident bunch have some variation in the longitudinal and transverse direction, so expression (3) should be averaged over the bunch particles positions. For definiteness, let us assume that in longitudinal direction the bunch is homogeneous with a length L, while in transverse direction it has a Gaussian distribution $f(\rho) = \frac{1}{\pi\bar{u}^2}\exp[-\rho^2/\bar{u}^2]$. Averaging procedure affects only the square modulus of the Fourier components (4) and can be written as

$$\begin{aligned}\left\langle\left|\vec{W}(q)\right|^2\right\rangle = &\int\frac{\mathrm{d}^2\rho_1}{\pi\bar{u}^2}\dots\frac{\mathrm{d}^2\rho_N}{\pi\bar{u}^2}\frac{\mathrm{d}z_1}{L}\dots\frac{\mathrm{d}z_N}{L}e^{-\rho_1^2/\bar{u}^2}\dots e^{-\rho_N^2/\bar{u}^2}\times\\ &\times\int\left(\frac{4\pi e}{E}\right)^2\sum_{n,m}\int\mathrm{d}^2k_\perp\mathrm{d}^2k'_\perp u_{k_\perp,q}u_{k'_\perp,q}k_\perp k'_\perp\times\\ &\times e^{iq(z_n-z_m)}e^{-ik_\perp(\rho-\rho_n)}e^{ik'_\perp(\rho-\rho_m)}.\end{aligned} \tag{6}$$

The dependence of this expression on the impact parameter ρ of the scattered electron is difficult enough. However, in many problems of radiation we have to consider the scattering of the divergent beam, so the challenge is to determine the radiation per unit flux density. Usually, for this purpose, the radiation efficiency $\mathrm{d}K/\mathrm{d}\omega = \int \mathrm{d}^2\rho \mathrm{d}E/\mathrm{d}\omega$ is used [9], which in fact corresponds to the averaging of the radiation spectral density by a homogeneous distribution of particles in the radiated bunch.

Performing this averaging in expression (6) greatly simplifies the calculations, so that in its final form, we obtain the expression

$$\int \left\langle |W(q)|^2 \right\rangle \mathrm{d}^2\rho = \left(\frac{8\pi^2 e}{E}\right)^2 N \int \mathrm{d}^2k_\perp u^2(k_\perp, q) k_\perp^2 \left[1 - \left(\frac{\sin qL/2}{qL/2}\right)^2 e^{-\frac{k_\perp^2 \bar{u}^2}{2}}\right] + \\ + \left(\frac{8\pi^2 e}{E}\right)^2 N^2 \left(\frac{\sin qL/2}{qL/2}\right)^2 \int \mathrm{d}^2k_\perp u^2(k_\perp, q) k_\perp^2 e^{-\frac{k_\perp^2 \bar{u}^2}{2}}. \tag{7}$$

Note that, if in the longitudinal direction the bunch also has a Gaussian distribution $\frac{1}{L}\exp[-z^2/L^2]$, this expression takes the form

$$\int \left\langle |W(q)|^2 \right\rangle \mathrm{d}^2\rho = \left(\frac{8\pi^2 e}{E}\right)^2 N \int \mathrm{d}^2k_\perp u^2(k_\perp, q) k_\perp^2 \left[1 - e^{-\frac{q^2L^2}{2}} e^{-\frac{k_\perp^2 \bar{u}^2}{2}}\right] + \\ + \left(\frac{8\pi^2 e}{E}\right)^2 N^2 e^{-\frac{q^2L^2}{2}} \int \mathrm{d}^2k_\perp u^2(k_\perp, q) k_\perp^2 e^{-\frac{k_\perp^2 \bar{u}^2}{2}}. \tag{8}$$

Further, for obtaining the efficiency of radiation it is necessary to substitute (7) or (8) in (3) and integrate over q. The main contribution to the integral yields values $q \sim \delta$. This means that in the low-frequency region of radiation, when the condition $L\delta \sim L/l_{\mathrm{coh}} \ll 1$ holds, the main contribution to the spectral density gives the term in (7) and (8), that is proportional to N^2, and thus the coherent effect in radiation takes place. In the region of high frequencies ($L\delta \gg 1$) the coherent part decays exponentially and the radiation efficiency is determined mainly by terms, that are proportional to N, *i.e.* radiation has an incoherent character.

Note that expressions (7) and (8) contain the potential of the particles of the general form, therefore, they can be applied to scattering by a charge and to scattering by atoms having the screening Coulomb potential. At the same time spread of particles of the bunch in the transverse direction is similar to the deviation of atoms in the chain of the crystal from the equilibrium positions due to thermal vibrations [9].

3. – Radiation at collision on the charges with Coulomb potential

In the case where it is necessary to allow dependence of the spectral density (3) on the impact parameter, it is necessary initially to use a particular form of the potential of the particles. For the incident electric charge the potential of one particle is

$$u_1(\rho, t) = \frac{e}{\sqrt{(2t - z_1)^2 + \frac{(\rho - \rho_1)^2}{\gamma^2}}}. \tag{9}$$

(The appearance of two in the denominator is due to the fact that the relativistic particles move towards each other.) Summing up this potential for all particles of the bunch, and substituting it in (4) we obtain

$$W(q) = -\frac{2e}{E}\int_{-\infty}^{+\infty} \mathrm{d}t e^{iqt}\frac{\partial}{\partial\rho}\sum_n \frac{e}{\sqrt{(2t-z_n)^2+\frac{(\rho-\rho_n)^2}{\gamma^2}}} = -\frac{e^2}{2E\gamma}\sum_n q e^{iqz_n/2}K_1\left(\frac{q}{2\gamma}|\rho-\rho_n|\right)\frac{\rho-\rho_n}{|\rho-\rho_n|}\,, \tag{10}$$

where $K_1(x)$ is the Macdonald function. Next, one must also average the squared modulus of (10) by a homogeneous distribution of particles in the longitudinal direction and a Gaussian distribution $f(\rho)$ in the transverse. After simple transformations we obtain the following expression:

$$\langle |W(q)|^2\rangle = \left(\frac{e^2 q}{2\epsilon\gamma}\right)^2 N\left[\int \mathrm{d}^2 x f(\vec\rho+\vec x)K_1^2\left(\frac{q}{2\gamma}x\right) - \left(\frac{2\gamma}{q}\right)^2\left(\frac{\sin qL/4}{qL/4}\right)^2\left\{\int \mathrm{d}^2 x\frac{\partial f(\vec\rho+\vec x)}{\partial\vec\rho}K_0\left(\frac{q}{2\gamma}x\right)\right\}^2\right] + \left(\frac{e^2 q}{2\epsilon\gamma}\right)^2 N^2\left(\frac{\sin qL/4}{qL/4}\right)^2\left(\frac{2\gamma}{q}\right)^2\left[\int \mathrm{d}^2 x\frac{\partial f(\vec\rho+\vec x)}{\partial\vec\rho}K_0\left(\frac{q}{2\gamma}x\right)\right]^2. \tag{11}$$

In this expression terms corresponding to the coherent and incoherent radiation have already been grouped. This expression can be used to calculate the spectral density for various distributions of particles in the incident bunch.

4. – Conclusions

Expressions (7) and (8), which define the coherent and incoherent parts of the spectral density of radiation in dipole approximation, are very close to similar expressions obtained for the spectral density of radiation of the fast electron, scattered on a chain of atoms in a crystal. This means that the processes of radiation in both these cases are identical. In particular, except the correspondence between the coherent effects in these processes, the incoherent part in (7) at high radiated frequencies behaves close to the well-known quantum Bethe-Heitler result [9], when the recoil effect can be neglected.

Easily seen, in the region of low-frequency radiation the coherent term in (11) has a sharp maximum for impact parameters close to the transverse size of the incident bunch. This fact allows the use of coherent radiation effect for monitoring of bunches of relativistic particles by detecting the maximum of coherent radiation at different impact parameters of scattered electrons.

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