

X-ray emission from a crystal undulator—Experimental results at channeling of electrons

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Summary. — Experiments have been performed at the Mainz Microtron MAMI to explore the radiation emission from a 4-period epitaxially grown strained layer $\text{Si}_{1-x}\text{Ge}_x$ undulator with a period length $\lambda_u = 9.9 \mu\text{m}$. Electron energies of 270 and 855 MeV have been chosen. In comparison with a flat silicon reference crystal, a broad excess yield around the theoretically expected photon energies of 0.069 and 0.637 MeV, respectively, has been observed for channeling at the undulating (110) planes. The results are discussed within the framework of the classical undulator theory.

PACS 41.60.-m – Radiation by moving charges.

PACS 61.85.+p – Channeling phenomena (blocking, energy loss, etc.).

PACS 87.56.bd – Accelerators.

1. – Introduction

The possibility to produce undulator-like radiation in the hundreds of keV up to the MeV region by means of positron channeling is well known and was discussed in a number of papers, see, *e.g.*, [1-3]. However, the demonstration and utilization of such devices hampers from the fact that high-quality positron beams in the GeV range are not easily available, in contrast to electron beams. Therefore, it was theoretically investigated [4, 5] whether by means of planar channeling of ultrarelativistic electrons in a periodically bent single crystal the production of undulator-like radiation would also be possible, or not. The basic idea of the latter work is that similar dechanneling lengths as for positrons can also be achieved with electrons if their beam energy is chosen a factor of about 20 larger as for positrons.

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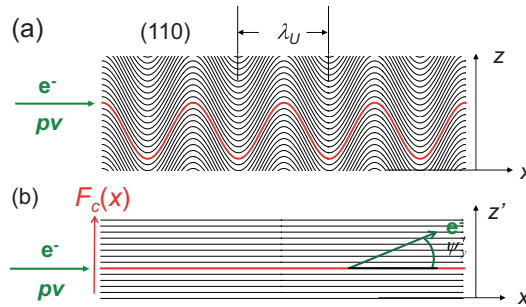


Fig. 1. – Undulator crystal in laboratory system (a), in undulating reference system (b) in which an additional centrifugal force $F_c(x) \hat{e}_{z'}$ in the z' direction acts. The available undulator should have four periods, period length $\lambda_U = 9.9 \mu\text{m}$, and amplitude $A = 4.0 \text{ \AA}$.

There are several possibilities to realize crystalline undulator targets. Our approach is based on the production of graded composition strained layers in an epitaxially grown $\text{Si}_{1-x}\text{Ge}_x$ superlattice [6]. Because of the slight difference in the lattice constants between Si and Ge, adding of a small content x of Ge to Si in a molecular beam epitaxy (MBE) growing process results in strain in the crystal and a bending of the crystal lattice. By varying the concentration x in the $\text{Si}_{1-x}\text{Ge}_x$ superlattice linearly and periodically [7], undulating (110) planes can be obtained. To the best of our knowledge, radiation production on the basis of this kind of periodic structures was first mentioned by Kephart *et al.* [8-10]. Although some channeling-radiation measurements using superlattice crystals were reported [11], neither the beam quality nor the crystal quality were good enough at that time about 25 years ago to obtain satisfactory results. In the mean time great progress has been made to grow epitaxially strained-layer $\text{Si}_{1-x}\text{Ge}_x$ superlattices. For instance, in the MBE laboratory of Aarhus there exists considerable experience [12,13]. Because of the availability of such crystal undulators, experiments were initiated the aim of which was to check their suitability for monochromatic X-ray radiation production employing both, electrons as well as positrons. First experiments were performed at the Mainz Microtron MAMI with a 4-period crystalline undulator with a period length of $50 \mu\text{m}$ at electron beam energies of 855 MeV and 1508 MeV [14]. In the mean time a 4-period crystalline undulator with a period length of $9.9 \mu\text{m}$ was produced at MBE Aarhus with which experiments at 195 MeV, 270 MeV, 350 MeV and 855 MeV were performed at MAMI. Preliminary results for 270 MeV and 855 MeV are described and discussed in the following.

2. – Experiments with a 4-period $\lambda_U = 9.9 \mu\text{m}$ undulator with electrons

With the undulator schematically shown in fig. 1 various experiments have been performed with the cw-electron beam of MAMI. At beam energies of 270 and 855 MeV the dechanneling length for a flat crystal was measured to be in the range between 30 and $40 \mu\text{m}$ [15] which matches about the thickness of the 4-period $\lambda_u = 9.9 \mu\text{m}$ undulator of $39.6 \mu\text{m}$. The experimental setups are shown in fig. 2.

2.1. The experiment at 855 MeV. – Due to the low emittance of MAMI, a beam with small angular divergence can be prepared. The horizontal and vertical emittances amount to $\varepsilon_h = 0.01 \text{ mm mrad}$ and $\varepsilon_v = 0.0003 \text{ mm mrad}$, respectively. Typical beam spots in

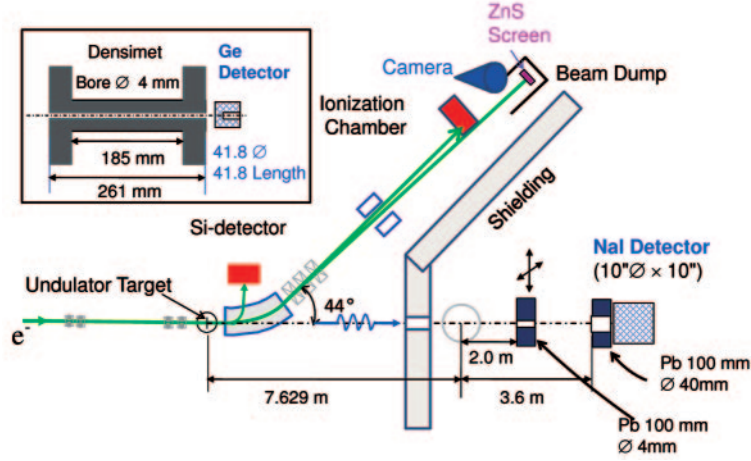


Fig. 2. – Experimental setup at MAMI. Downstream the undulator target the beam is deflected horizontally by a 44° bending magnet and thereafter vertically by a second 7.2° -bending magnet. The ionization chamber and the Si detector are monitor counters for the detection of channeling. At 855 MeV photon spectra are recorded with a $10'' \text{ } \varnothing \times 10''$ NaI detector, at 270 MeV with a Ge(i) detector, see inset (bore: 7.8 mm \varnothing and 30 mm length). The photon beam from the target is collimated with Densimet 176 (density of 17.6 g/cm^3 , 92.5% tungsten, 5% nickel, 2.48% iron). The diameters are 52 mm for the inner part and 96 mm for the outer parts. Photons proceed 8.109 m in vacuum and 0.362 m in air just in front of the Ge(i) detector.

our experiments had standard deviations $\sigma_h \approx 0.30 \text{ mm}$, horizontally, and $\sigma_v \approx 0.20 \text{ mm}$, vertically, resulting in standard deviations of the beam divergences of $\sigma'_h \approx 0.033 \text{ mrad}$ and $\sigma'_v \approx 0.0015 \text{ mrad}$, respectively.

The undulator crystal was mounted on goniometers with which rotations around three axes can be accomplished. Details on the goniometers, the procedure to align the crystal, etc. are described in ref. [16]. Results of the scans for a flat reference crystal and the undulator crystal are shown in fig. 3. It is worthwhile to notice that for the (110) plane of the undulator crystal the base width $\Delta\varphi_u$ is about a factor of two broader than the base width $\Delta\varphi_f$ for the flat one. Interpreting this broadening as a shift which originates from the maximum slope of the sinusoidal function of the undulator crystal, this slope can be calculated to be $\psi_{\max} = |\alpha_u| \Delta\varphi_u / 2 - |\alpha_f| \Delta\varphi_f / 2 = 0.0995 \cdot 4.84 \text{ mrad} - 0.0981 \cdot 2.34 \text{ mrad} = 0.251 \text{ mrad}$. From this number the amplitude $A = \psi_{\max} \lambda_u / 2\pi = 4.0 \text{ \AA}$ follows by means of eq. (A.1), and the undulator parameter $K = 0.425$ by means of eq. (A.3) in appendix A. Under ideal conditions and on-axis observation a peak with a width of 145 keV is expected from calculations on the basis of the classical undulator theory [17]. According to eq. (A.4), the photon energy is $\hbar\omega = 644 \text{ keV}$ ($n = 1$). A $10'' \text{ } \varnothing \times 10''$ NaI crystal with a resolution of 9% is therefore well suited as a photon detector. This detector with a thickness of 10 radiation lengths has the advantage that the peak-to-total ratio is large, and thus tedious deconvolution procedures can be avoided in a first analysis. The photons from the target were collimated by two lead walls with bores as specified in fig. 2.

The result of the measurement is shown in fig. 4, left panel. The background of about 50% at 1 MeV photon energy, taken with the crystal tuned to an off-channeling position, has already been subtracted. Below a photon energy of 1.2 MeV a steep increase of the

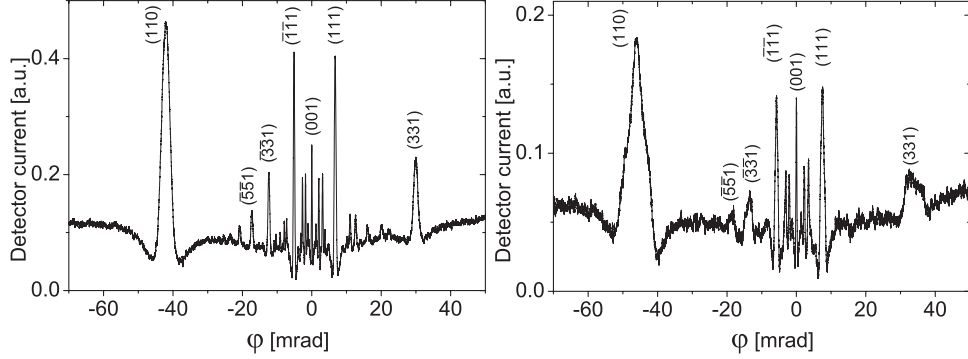


Fig. 3. – Signals of the Si detector, see fig. 2, detecting electrons with an energy of about half the beam energy of $E_e = 855$ MeV. Shown are φ scans around the vertical z -axis. For definition of the angles see fig. 2(a) of ref. [16]. All three angles α , θ , and φ are defined to be positive for a right-handed rotation around the x' , y' , and z axis, respectively. Starting from an original alignment of the crystal with the $[110]$ direction coinciding with the beam direction, the x -axis, and the (110) planes aligned parallel to the horizontal y -axis, the flat crystal (left panel) was aligned with angles $\alpha_f = -5.62^\circ$, $\theta_f = -0.235^\circ$, and the undulator crystal (right panel) with $\alpha_u = -5.70^\circ$, $\theta_u = -0.262^\circ$.

intensity is observed for the undulator crystal. However, the expected broad peak at 0.644 MeV, indicated by the arrow, with a width of about 0.37 MeV as calculated under more realistic conditions is absent. The reason might be found in the centrifugal force

$$(1) \quad \vec{F}_c(x) = pvk_u^2 A \cos(k_u x) \hat{e}_{z'}$$

acting in the z' direction on the channeled electrons. In a classical limit a centrifugal potential $U_c(x, z') = -F_c(x) \cdot z'$ may be defined which superimposes the crystal potential

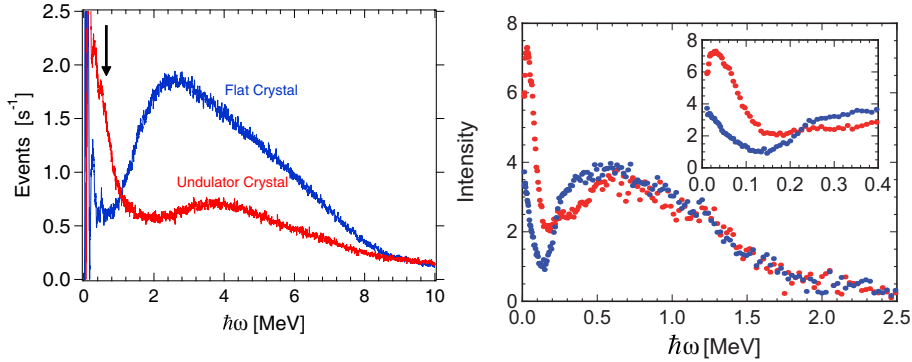


Fig. 4. – (Colour on-line) Left panel: raw photon spectra at (110) planar channeling of 855 MeV electrons for the flat (blue) and the undulator crystal (red) with effective thicknesses of $49.5 \mu\text{m}$ and $39.6 \mu\text{m}$, respectively. Data were taken with the NaI detector, see fig. 2. Right panel: deconvoluted photon spectra at (110) planar channeling of 270 MeV electrons for the flat (blue) and the undulator crystal (red). Data were taken with the Ge(i) detector shown in the inset of fig. 2.

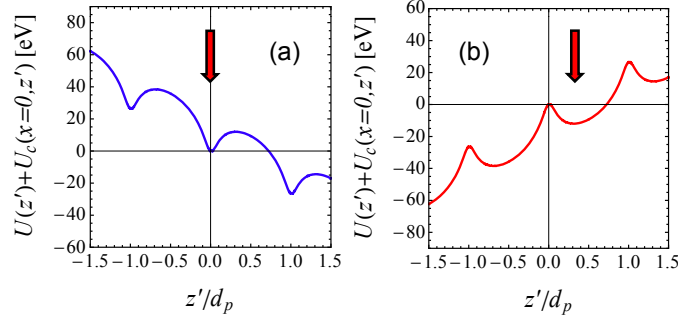


Fig. 5. – Sum of maximum centrifugal potential $U_c(x = 0, z') = E_e \cdot A \cdot k_u^2 \cdot d_p \cdot (z'/d_p)$ and crystal potential $U(z')$ at $pv \simeq E_e = 855$ MeV for the (110) plane; $A = 4.0$ Å, $\lambda_U = 9.9$ μm, and $d_p = 1.92$ Å, the distance between (110) planes, for electrons (a), and for positrons (b).

$U(z')$. It can be seen from fig. 5(a) that the barrier height at maximum curvature is quite substantially reduced. This fact enhances dechanneling which manifests itself also in a suppression of the channeling radiation in the energy range between 1 and 8 MeV for the undulator crystal. To check this conjecture we performed another experiment at 270 MeV for which, according to eq. (1), the centrifugal force is reduced by more than a factor of three.

2.2. The experiment at 270 MeV. – The undulator parameter is reduced to $K = 0.134$ and the peak is expected at 69.6 keV with a width of 15.6 keV under ideal conditions. For such parameters the $10'' \text{ } \varnothing \times 10''$ NaI crystal is not anymore suited as a detector. Therefore, we used a Ge(i) detector the dimensions of which are quoted in the inset of fig. 2. Since the response function for such a detector is rather complicated for photons with energies exceeding about 100 keV, the experimental spectra must be deconvoluted. To accomplish this task, simulation calculations for various photon energies have been performed with the program package GEANT4 which includes not only the geometry of the Ge(i) detector but in addition also support materials as well as the aperture system. The result of the already deconvoluted measurement is shown in fig. 4, right panel. The background of about 55% at 0.11 MeV photon energy, taken with the crystal tuned off-channeling, has been subtracted.

3. – Discussion

3.1. Experimental results with electrons. – Three features are apparent from fig. 4 which will be discussed in the following.

First of all we observe that at a beam energy of 270 MeV the channeling radiation distributions for the flat and the undulator crystal above about 0.7 MeV are about the same, see fig. 4, right panel. By lowering the beam energy the centrifugal force has obviously been reduced to such an extent that much more electrons remain captured in the channel. The gradual reduction of the channeling radiation between 0.250 and 0.7 MeV might be explained by a loss of weakly bound electrons during the steering process which have long oscillation periods resulting in the emission of low-energy photons.

Secondly, for the flat crystal an increase of the intensity is observed below about 0.12 MeV. This phenomenon is not yet fully understood. Transition radiation can be excluded as an explanation. A quasi-channeling contribution, which is an above bar-

rier motion of the electron with small transverse energy, may be a possibility for an explanation. For a photon energy of 0.070 MeV the angle between the electron direction and the (110) crystal plane amounts to only $\psi = d_p/\lambda_U = 0.19$ mrad. The low transverse energy $E_{\perp} = pv\psi^2/2 = 4.9$ eV may be small enough to cause an undulating trajectory in the interaction with the crystal potential of 22.6 eV depth. Notice that the rather large multiple scattering angle for amorphous matter of 0.36 mrad within one period will be reduced in the above barrier motion. Another possible explanation could be that during the rearrangement of the initial angular distribution of the electrons at the entrance interface between vacuum and crystal very strong accelerations act which lead to the emission of this radiation component.

Thirdly, and this is the most striking feature, a broad peak-like structure appears for the undulator crystal at low energies. The peak is expected at 0.070 MeV with a width of about 0.032 MeV if several line broadening effects are taken into account properly⁽¹⁾. However, the peak seems to be shifted to an energy of about 0.030 MeV. A possible reason for this fact might be found in a non-perfect undulator structure. In fig. 3, right panel, one not only observes for the (110) plane the already discussed broadening in the wings but, in addition, also a superimposed peak at the center. Certainly, scattered electrons which rechannel may contribute to this peak. However, the rms values for multiple scattering through small angles in amorphous matter amounts for a $39.6/2 \mu\text{m}$ thick silicon layer already to $\theta_0 = 0.50$ mrad [19] which is twice of the maximum undulator slope $\psi_{\text{max}} = 0.25$ mrad. But it also cannot be excluded that part of the undulator crystal behaves more or less like a flat one. Accepting this as a possible explanation, the number of periods would be reduced resulting in a broadening. It might well be that on top of such a broadened structure peaking at 0.07 MeV another structure is superimposed which peaks at 0.03 MeV. The latter might be a subharmonic of the former one which could well originate from inaccuracies of the germanium admixture during the crystal growing process.

We finally discuss the possibility that the low-energy radiation component for the undulator crystal originates from an interference of coherent bremsstrahlung which is produced by electrons moving on rectilinear trajectories. Such a possibility which does not rely on channeling at all has been described in a paper of Shul'ga and Boiko [20]. Maxima which are associated with the modulation of the crystallographic atomic planes should appear at energies given by the relation $\hbar\omega = n4\pi\gamma^2\hbar c/\lambda_U$, a formula which is quite similar as eq. (A.4) for undulator radiation at channeling. However, we would like to stress that even a broad peak might not be expected by this effect since the coherence length $\ell_c = 2\gamma^2\hbar c/\hbar\omega$ [21] results, substituting the above photon energy, in $\ell_c = \lambda_U/(n2\pi) = 1.57 \mu\text{m}$ which is only a small fraction of the undulator period⁽²⁾. In addition, the condition $4A/\lambda_U > \psi_{\text{crit}}$ for which the considerations in ref. [20] hold is not fulfilled for our experiments. For instance, even at a beam energy of 855 MeV one obtains for $4A/\lambda_U = 0.162$ mrad which is smaller than the critical angle $\psi_{\text{crit}} = 0.227$ mrad. Finally, the multiple scattering angle over one period amounts to 0.113 mrad (rms) which implies that also the assumption of a rectilinear trajectory is only approximately fulfilled.

⁽¹⁾ These are broadenings caused by the finite crystal thickness of $39.6 \mu\text{m}$, the dechanneling length $\ell_{\text{dech}} = 30 \mu\text{m}$, the coherence loss by phase fluctuations due to multiple scattering $\ell_{\text{phase}} = 33 \mu\text{m}$ as calculated by eq. (30) of ref. [18] with $2D = (10.6 \text{ MeV}/\gamma m_e c^2)^2/X_0$ and X_0 the radiation length, and the relaxation of the oscillation amplitude as function of the crystal thickness for which $\ell_{\text{relax}} = 40 \mu\text{m}$ was assumed.

⁽²⁾ Incidentally, this statement holds for any beam energy since γ cancels.

For a beam energy of 270 MeV the same considerations result in an even worse picture. Therefore, it must be concluded that the mechanism discussed by Shul'ga and Boiko [20] most likely does not apply to our experiment.

3'2. Advantages of positrons. – It has been shown in sect. 2 that an undulator crystal has been prepared at MBE Aarhus which exhibits in experiments with electrons some of the expected features. However, the absence of the expected photon distribution in combination with the suppression of the channeling radiation for K parameters in the order of 0.4 for the experiment at 855 MeV raises the question whether electrons are as well suited as positrons to produce undulator radiation [4, 5]. To discuss this question we refer once more to fig. 5(a). For the situation shown, the barrier height is reduced from $U_m = 22.6$ eV to about 12.6 eV, *i.e.* by nearly a factor of two. This fact already enhances the dechanneling probability considerably. But even worse, a still channeled electron resides rather close to an atomic plane, indicated by the arrow in fig. 5(a). Here it experiences a much stronger transverse energy increase than a weakly bound electron in a flat crystal at $E_{\perp}/U_m \approx 1$. As a consequence, a dramatically enhanced dechanneling probability is expected. In other words, deeply bound electrons experience a rather strong transverse energy heating and rapidly boil off from the potential pocket at the curved parts of the undulator.

A completely different scenario is observed for positrons. Of course, the barrier reduction is similar as for electrons with the difference that the potential shown in fig. 5(a) must be reflected at the horizontal z'/d_p axis, see fig. 5(b). Also in this case positrons are lost due to the reduction of the barrier height. In addition, the local potential minimum shifts to $0.3 d_p$ and the oscillating positrons come at one of their turning points rather close to the thermally vibrating atoms comprising the atomic planes. This effect as well as the rather small barrier width close to the local maximum must be taken into account in a more detailed consideration. Nevertheless, at least part of the bound positrons experience a more than two orders of magnitude reduced transverse energy heating as electrons. This fact can be concluded from fig. 2 of ref. [22]. Therefore, the dechanneling length as obtained for planar crystals is expected to be maintained also in the undulator crystal for a certain fraction of channeled positrons.

For positron channeling a suitable machine in Europe is missing. The Beam Test Facility BTF at INFN Frascati, Italy, would be a good candidate [23, 24] if a number of modifications are performed. It should be mentioned that a comparable facility was in operation about 25 years ago at the Lawrence Livermore National Laboratory Electron-Positron Linear Accelerator, USA [25].

4. – Conclusions

An epitaxially grown strained-layer $\text{Si}_{1-x}\text{Ge}_x$ 4-period $\lambda_U = 9.9 \mu\text{m}$ undulator has been produced with which a clear enhancement of radiation has been observed for the first time at electron beam energies of 270 MeV and 855 MeV. The combined discussion which includes also an angular scan through the (110) planar channeling region of the plane and undulator crystal, suggests that the undulator crystal probably has not been produced with the design parameters. From a channeling experiment with a suitable 500 MeV positron beam results are expected that would round out the picture from the undulator as gained with electrons.

Finally, it should be stressed that efforts must be intensified to produce high-quality crystalline undulators.

APPENDIX A.

Characteristics of the crystal undulator

For the characterization of the undulator shown in fig. 1 the (110) planes are assumed to be deformed according to the function

$$(A.1) \quad z = A \cos(k_u x); \quad dz/dx = -A k_u \sin(k_u x); \quad k_u = 2\pi/\lambda_u$$

with dz/dx the first derivative of the vertical coordinate z with respect to x , the beam direction, A the oscillation amplitude, and λ_u the period length. The maximum slope of the sinusoidal undulating function is given by $\psi_{\max} = A k_u$. The critical angle for channeling is

$$(A.2) \quad \psi_{\text{crit}} = \sqrt{\frac{2 \cdot U_0}{\gamma \cdot m_e c^2}}$$

Here $U_0 = 22.6$ eV is the potential depth of the (110) planes, $\gamma = E_e/m_e c^2$ the relativistic factor, and m_e the rest mass of the electron. The undulator parameter is given by

$$(A.3) \quad K = \gamma \cdot A \cdot k_u$$

and the photon energy reads

$$(A.4) \quad \hbar\omega = n \frac{4\pi \cdot \gamma^2 \hbar c}{\lambda_u (1 + K^2/2 + \gamma^2(\theta_y^2 + \theta_z^2))}$$

with n the radiation order.

* * *

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