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Diagnostics of crystal-radiator of positrons by backward-going X-rays

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Summary. — In the production of positrons by ultra-relativistic electrons channeling in a crystal-radiator the incident electrons produce X-ray radiation in the crystal that is not used by now. In present paper we consider the properties of 3 kinds of X-ray radiation as the parametric X-ray radiation, the characteristic X-ray radiation, the diffracted transition radiation emitted in the backward direction from the crystalline radiator of positrons. We found that these kinds of quasi-monochromatic radiations can be registered by an X-ray spectrometer simultaneously with the production of positrons. We propose the observation of backward-going X-rays for the diagnostics of the crystal-radiator and electron beam status during production of positrons. Such on-line diagnostics can contribute to the optimization of the positron production. The same 3 kinds of radiation can be used for the diagnostics of the crystal-radiator during production of a coherent bremsstrahlung by the ultra-relativistic electron beam in a crystal and also for the diagnostics of bent crystals that are used for the steering of the high-energy particle beams trajectory. Besides, the diffracted transition radiation can be considered as a source of intense quasi-monochromatic tunable X-rays beam excited by the high-energy particle beam in a crystal.

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1. – Introduction

Positron beams now find a use in a lot of fields of the accelerator physics. The sources of the positron beam usually are based on the production of electron-positron pairs by γ-quanta in matter. A traditional way of production of γ-ray beams consists in the production of a bremsstrahlung by a relativistic electron beam in an amorphous converter. In order to increase the yield of positrons, Chehab *et al.* [1] proposed the application of a single crystal instead of the amorphous converter. The application of the crystalline

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converter allows to increase the production of γ-quanta due to channeling radiation and coherent bremsstrahlung and to increase the number of positrons per one incident electron, see, *e.g.*, [2]. In the present paper we turn our attention to the production of X-rays by the incident electrons in the crystalline radiator, that usually is ignored. Below we will consider the properties of the characteristic X-ray radiation, the parametric X-ray radiation, the diffracted transition radiation arising in a crystal-radiator and emitted at backward direction and propose their application for the online diagnostics of the crystal-radiator state. As a concrete example, we estimated the spectral and angular properties of the radiation emitted from a thick Si single crystal whose $\langle 111 \rangle$ axis is aligned along the incident electron beam with energy up to 8 GeV.

2. – Characteristic X-ray radiation

The characteristic X-ray radiation (CXR) arises due to the ionization of the inner shells of atoms. Below we will mean the ionization of crystal atoms by incident ultra-relativistic electrons. Consider the yield of the CXR excited by the electrons in a crystal without account of orientation effects. The yield in the backward direction of the CXR arising due to the ionization of atoms K-shell in a thick target with the attenuation of the CXR taken into account is [3]

$$Y_{\mathrm{CXR}} = \left(\frac{\mathrm{d}N}{\mathrm{d}\Omega}\right)_{\mathrm{CXR}} = \frac{n_0 T_e}{4\pi} w_K \sigma_K, \tag{1}$$

where Y_{CXR} is the number of characteristic quanta $\mathrm{d}N$ emitted at the solid angle $\mathrm{d}\Omega$ per one incident electron, n_0 is the atomic density of the target, $T_e = T_e(\omega)$ is the length of the absorption by a factor e of X-rays with frequency ω in the target, w_K is the fluorescent yield, σ_K is the K-shell ionization cross-section. There are a lot of theoretical calculations of the K-shell ionization cross-sections but results of such calculations are rather different (for example, see calculations in [3, 4]).

Let us estimate the yield of the K-CXR from a Si crystal. In the estimation we will use the formula for σ_K that has been obtained as a result of the approximation of the experimental data for Si in [4]:

$$\sigma_K(\mathrm{barn}) = 134 \ln \gamma + 1025, \tag{2}$$

where γ is the relativistic factor of the incident particles. The density effect is not taken into account in this formula. The influence of the density effect should lead to saturation of the cross-section at great depths in the target at a relativistic factor of the incident particles above the value $\gamma > \frac{E_K}{\hbar\omega_p}$, where E_K is the energy of the absorption K-edge, ω_p is the plasma frequency. However, the density effect should be masked for the CXR emitted in the backward direction because of the ionization of near-surface atoms by a transition radiation arising at the target surface [5]. Besides, the orientation effects should be suppressed at high incident particle energy. Therefore we will use the formula (2) in our estimations at high incident electron energy. For a Si single crystal, the energy of K-CXR photons is $\hbar\omega_{\mathrm{CXR}} = 1.74\,\mathrm{keV}$, $T_e = 13.3\ \mu\mathrm{m}$, $w_K = 0.047$, $n_0 = 5.0 \cdot 10^{22}\,\mathrm{cm}^{-3}$, $\hbar\omega_p \approx 31.1\,\mathrm{eV}$. The results of the calculations of the K-CXR yield from a Si single crystal *versus* the incident electron energy E_e are given in table I.

Thus, the CXR of Si atoms with energy 1.74 keV is emitted in the backward direction from a Si near-surface layer of thickness $13.3\,\mu\mathrm{m}$. The CXR yield slowly increases with

TABLE I. – *The CXR yield* $Y_{\rm CXR}$ *in the backward direction from a Si single crystal as a function of the incident electron energy* E_e. *Calculations were performed by formulae* (1), (2).

E_e, MeV	$Y_{\rm CXR}$, quanta/sr
250	$4.62 \cdot 10^{-4}$
500	$4.85 \cdot 10^{-4}$
1000	$5.08 \cdot 10^{-4}$
2000	$5.31 \cdot 10^{-4}$
4000	$5.54 \cdot 10^{-4}$
8000	$5.77 \cdot 10^{-4}$

increasing the incident electron energy from 250 to 8000 MeV. The spectral peak of the PXR can be observed by a spectrometric X-ray detector installed at an arbitrary observation angle θ close to π.

3. – Reflection of parametric X-ray radiation in the backward direction

The parametric X-ray radiation (PXR) arises due to the interaction of a relativistic charged particle with the periodic structure of a crystal. Information about the main properties and research of the PXR may be found, *e.g.*, in reviews [6-8]. The yield of the PXR has maxima (the PXR reflection) in the vicinity of the Bragg direction relative to a crystallographic plane. The Ter-Mikaelian theory close fits to the experimentally observed properties of radiation in the PXR reflection. In the case under consideration in the present paper the crystallographic planes are aligned at the right angle to the particle velocity vector $\vec{V}$ and the PXR reflections are emitted in the backward direction along the vector $-\vec{V}$. The angular distribution of the PXR yield, $Y_{\rm PXR}$, in such PXR reflection with the absorption in the crystal taken into account is (see eqs. (15), (22) in [7])

$$Y_{\rm PXR} = \left(\frac{{\rm d}N}{{\rm d}\Omega}\right)_{\rm PXR} = \frac{T_e \cdot g \cdot |\chi_{\vec{g}}|^2}{2 \cdot \pi \cdot 137 \cdot \varepsilon_0^{5/2} \cdot \xi^2 \cdot (\xi^{-1} + \cos\rho)^2} \cdot \frac{\rho^2}{\left[\rho^2 + \gamma_{\rm eff}^{-2}\right]^2}, \tag{3}$$

where ${\rm d}N$ is the number of the PXR quanta emitted at the solid angle ${\rm d}\Omega$ per one incident electron, $\vec{g}$ is the reciprocal lattice vector, $g = |\vec{g}| = \frac{2\pi}{a}$, a is the distance between the crystallographic planes, $\chi_{\vec{g}} = \chi_{\vec{g}}(\omega)$ is the Fourier component of the dielectric susceptibility, $T_e = \frac{c}{\omega|\chi_0''|}$ is the absorption length, ε_0 is the average permittivity, $\gamma \gg 1$, $\xi = \frac{V\sqrt{\varepsilon_0}}{c}$, $\varepsilon_0^{5/2} \cdot \xi^2 \cdot (\xi^{-1} + \cos\rho)^2 \approx 4$, $\gamma_{\rm eff} = [\gamma^{-2} + |\chi_0|]^{-1/2}$, $|\chi_0| = (\frac{\omega_p}{\omega})^2$ at ω exceeding atomic frequencies in the crystal, ρ is the angle between the observation direction and the vector $-\vec{V}$, $\rho \ll 1$, $\theta = \pi - \rho$ is the observation angle relative to the vector $\vec{V}$, the PXR frequency is

$$\omega_{\rm PXR} = \frac{cgV}{c + \sqrt{\varepsilon_0} V \cos\rho}. \tag{4}$$

TABLE II. – *Properties of the radiation at observation angle* $\theta_{\mathrm{PXR}} = \pi - 2\sqrt{|\chi_0|}$ *in the PXR reflection from the Si crystallographic planes (111), (333), (444), (555) aligned at the right angle to the incident particle velocity vector* $\vec{V}$*. The energy of the PXR spectral peak is* $\hbar\omega_{\mathrm{PXR}}$*, the PXR is emitted from the Si near-surface layers of thicknesses* T_e*. Calculations were performed by formulae* (4), (6), (7).

	PXR (111)	PXR (333)	PXR (444)	PXR (555)
$\rho = 2\sqrt{|\chi_0|}$, mrad	29.5	10.6	7.9	6.3
$\hbar\omega_{\mathrm{PXR}}$, keV	1.977	5.931	7.908	9.886
T_e, μm	1.51	29.0	66.6	128
Y_{PXR}, quanta/sr	$0.67 \cdot 10^{-4}$	$2.0 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$	$1.15 \cdot 10^{-4}$
E_e^{crit}, MeV	33	97	130	162

Let us consider the PXR yield (3) at high incident electron energy, when the PXR yield is saturated due to the influence of the density effect [9] that occurs at the condition $\gamma^2 \gg |\chi_0|^{-1}$. This condition means that the energy of the incident particle of mass m should exceed the critical energy E_e^{crit}

$$E_e > E_e^{\mathrm{crit}}, \tag{5}$$

where

$$E_e^{\mathrm{crit}} = \frac{mc^2}{\sqrt{|\chi_0|}}. \tag{6}$$

If the PXR frequency exceeds the atomic frequencies, $E_e^{\mathrm{crit}} = \frac{\omega_{\mathrm{PXR}}}{\omega_p} mc^2$. Under condition (5), $\gamma_{\mathrm{eff}}^{-2} = |\chi_0|$ and eq. (3) can be written in the form

$$Y_{\mathrm{PXR}} \approx \frac{|\chi_{\vec{g}}|^2}{4 \cdot \pi \cdot 137 \cdot |\chi_0''| \cdot |\chi_0|} \cdot \frac{\rho_\chi^2}{\left[\rho_\chi^2 + 1\right]^2}, \tag{7}$$

where $\rho_\chi = \frac{\rho}{|\chi_0|^{1/2}}$ is the angle between the observation direction and the vector $-\vec{V}$ in units of $|\chi_0|^{1/2}$.

The pure PXR yield (7) under condition (5) is independent of the incident electron energy due to influence of the density effect. However, Backe *et al.* in [10] showed that the PXR can destructively interfere with the diffracted transition radiation. But there is a possibility to observe almost pure PXR without sufficient interference at observation angles $\rho_\chi > 1$. In order to estimate the possibilities of observation and application of the PXR we calculated the PXR yield (7) for a Si crystal at $\rho_\chi = 2$. At $\rho_\chi = 2$ the interference should be insignificant at least for higher-order reflections but the PXR yield is reduced relative to its maximum value by factor $\frac{16}{25}$ only. The results of the calculations of some properties of radiation in the PXR reflections excited by electrons moving along the $\langle 111 \rangle$ axis of a thick Si crystal are shown in table II.

Thus, four PXR spectral peaks with energies in the range 2–10 keV with comparable intensities are generated from the crystal near-surface layers of thicknesses in the range 1.5–129 μm at $E_e > E_{\mathrm{crit}}$. The properties of the PXR spectral peaks listed in the table II are independent of the incident electron energy at $E_e > 250$ MeV due to the influence of the density effect for PXR [9]. The PXR spectral peaks can be observed by a spectrometric X-ray detector installed at angles $\rho = 2\sqrt{|\chi_0|}$ relative to the vector $-\vec{V}$. These peaks are due to the PXR reflections emitted in the vicinity of the Bragg direction.

4. – The plane effect for parametric X-ray radiation

The PXR spectrum should contain spectral peaks arising due to the plane effect for PXR described in [11]. Every peak arises from the crystallographic planes whose reciprocal lattice vectors end on a definite plane in the reciprocal space perpendicular to the vector $\vec{V}$, under the condition $\vec{g}\vec{V} = \mathrm{const}$. The frequency of the PXR peak from the plane at the observation angle $\theta = \pi$ is

$$\omega_{\mathrm{PXR}} = \frac{c|\vec{g}\vec{V}|}{c + \sqrt{\varepsilon_0}V}. \tag{8}$$

The yield of the radiation from a single crystallographic plane can be found from eqs. (8), (12) [7]

$$Y_{\vec{g}} = \frac{|\chi_{\vec{g}}|^2 \cdot T_e \cdot \hbar\omega}{8 \cdot \pi \cdot 137 \cdot c \cdot \hbar} \tan^2 \phi, \tag{9}$$

where ϕ is the angle between the crystallographic plane and the vector $\vec{V}$. The total yield from the plane is

$$Y_{\mathrm{plane}} = \sum_{\mathrm{plane}} Y_{\vec{g}}, \tag{10}$$

where the sum includes the yields from all reciprocal lattice vectors that end on the plane. In the above-considered example the vector $\vec{V}$ is parallel to the Si $\langle 111 \rangle$ axis and the plane effect should provide spectral peaks with energies 0.695, 1.977, 2.636, 3.295, 4.613, 5.272, 5.931 keV and so on. Every peak is from the plane in the reciprocal space that is determined by the ends of next reciprocal lattice vectors, respectively, $[(11\bar{1}), (1\bar{1}1), (\bar{1}11)]$, $[(\bar{3}33), (3\bar{3}3), (33\bar{3})]$, $[(400), (040), (004)]$, $[(311), (131), (113)]$, $[(511), (151), (115)]$, $[(422), (242), (224)]$, $[(711), (171), (117)]$ and so on. In principle, they can be observed at any observation angles close to π as the angular distribution of the yield is smooth around $\theta = \pi$. Our estimations show that the yields of radiation in some of these low-energy peaks can reach values about 10^{-6} quanta per steradian that is much less than in those listed in the table II, because these peaks are due to PXR reflections emitted at significant angular distances from the observation direction.

5. – Reflection of diffracted transition radiation in the backward direction

The transition radiation (TR) is generated when a charged particle crosses the boundary of a target. If the target is a crystal, some X-ray components of the TR with Bragg frequency ω_B cannot penetrate into the crystal depth because of the Bragg diffraction in the near-surface layer of the crystal. The diffracted TR (DTR) forms the DTR reflection.

Authors of the work [10] have adapted general expressions from [12] to the special case of the backward emission of X-rays from a crystal. The spectral-angular distribution Y_{DTRs} of radiation in the isolated DTR reflection from the crystallographic plane aligned at the right angle to the vector $\vec{V}$ (see eqs. (2.35)–(2.36) in [10]) under condition (5) can be written in the form

$$Y_{\mathrm{DTRs}} = \frac{\mathrm{d}^2 N}{\mathrm{d}\Omega \frac{\mathrm{d}\hbar\omega}{\hbar\omega}} \approx \frac{|R_A|^2 \rho^2}{137 \cdot \pi^2 \cdot (\rho^2 + \gamma^{-2})^2}, \tag{11}$$

where ρ is the angle between the vector $-\vec{V}$ and the observation direction, $|R_A|^2$ is the Darwin-Prince curve, for centrosymmetric crystals $|R_A|^2 = |-y \pm \sqrt{y^2-1}|^2$, $y = \frac{2\varepsilon + i\chi_0''}{\chi_H' + \chi_H''}$, $\varepsilon = \frac{\Delta\omega}{\omega}$. The term $|R_A|^2$ provides the quasi-monochromaticity of the DTR spectral peak with Bragg frequency

$$\omega_{\mathrm{DTR}} = \omega_B = \frac{cg}{2\sqrt{\varepsilon_0}\cos\rho}. \tag{12}$$

The angular distribution of the DTR yield at the integration of (11) is

$$Y_{\mathrm{DTR}} = \left(\frac{\mathrm{d}N}{\mathrm{d}\Omega}\right)_{\mathrm{DTR}} = \frac{\rho^2}{137 \cdot \pi^2 \cdot (\rho^2 + \gamma^{-2})^2} \frac{\int_{-\infty}^{\infty} |R_A|^2 \mathrm{d}\hbar\omega}{\hbar\omega_B}. \tag{13}$$

At low absorption, the right factor in (13) can be estimated as $\frac{\int_{-\infty}^{\infty} |R_A|^2 \mathrm{d}\hbar\omega}{\hbar\omega} \approx |\chi_{\vec{g}}|$. In this case eq. (13) takes the form

$$Y_{\mathrm{DTR}} = \frac{\gamma^2 \cdot |\chi_{\vec{g}}|}{\pi^2 \cdot 137} \frac{(\rho\gamma)^2}{[(\rho\gamma)^2 + 1]^2}, \tag{14}$$

where $\rho\gamma$ is the angle ρ in units of γ^{-1}. The maximum yield in the DTR reflection (14) is

$$Y_{\mathrm{DTR}}^{\max} = \frac{\gamma^2 \cdot |\chi_{\vec{g}}|}{4 \cdot \pi^2 \cdot 137} \tag{15}$$

at the angle $\rho = \gamma^{-1}$. The maximum yield increases $\sim E_e^2$ with increasing the electron energy. Some calculated properties of the DTR excited by electrons of different energies incident on the thick Si crystal along the $\langle 111 \rangle$ axis are given in table III.

Thus, four DTR spectral peaks with energies in the range 2–10 keV are generated from the crystal near-surface layers of thicknesses in the range 0.9–14.5 μm. The DTR spectral peaks can be observed in the maximum of intensity by spectrometric X-ray detectors installed at angles $\rho = \gamma^{-1}$ relative to the vector $-\vec{V}$. Note that the DTR, in contrast to the PXR, do not provide any radiation due to the row or the plane effect.

TABLE III. – *Some properties of the DTR reflections in the backward direction excited by electrons incident along the* $\langle 111 \rangle$ *axis of a thick Si crystal. Calculated values of the differential yield* $Y_{\mathrm{DTR}}^{\max}$ *in the maximum of the DTR reflection at* $\rho_{\max} = \gamma^{-1}$ *are given for every reflection and incident electron energy. Values* L_{ext}*, are given for every DTR reflection in the bottom row of the table. Calculations were performed by eqs.* (12), (15), (17).

		The yield of the DTR in the maximum, $Y_{\mathrm{DTR}}^{\max}$, quanta/sr			
E_e, MeV	$\rho_{\max} = \gamma^{-1}$, mrad	DTR (111)	DTR (333)	DTR (444)	DTR (555)
250	2.04	$4.96 \cdot 10^{-3}$	$3.65 \cdot 10^{-4}$	$2.03 \cdot 10^{-4}$	$6.12 \cdot 10^{-5}$
500	1.02	$1.98 \cdot 10^{-2}$	$1.46 \cdot 10^{-3}$	$8.13 \cdot 10^{-4}$	$2.45 \cdot 10^{-4}$
1000	0.51	$7.94 \cdot 10^{-2}$	$5.85 \cdot 10^{-3}$	$3.25 \cdot 10^{-3}$	$9.80 \cdot 10^{-4}$
2000	0.255	$3.18 \cdot 10^{-1}$	$2.34 \cdot 10^{-2}$	$1.30 \cdot 10^{-2}$	$3.92 \cdot 10^{-3}$
4000	0.127	$1.27 \cdot 10^{0}$	$9.35 \cdot 10^{-2}$	$5.20 \cdot 10^{-2}$	$1.57 \cdot 10^{-2}$
8000	0.064	$5.08 \cdot 10^{0}$	$3.74 \cdot 10^{-1}$	$2.08 \cdot 10^{-1}$	$6.27 \cdot 10^{-2}$
The extinction depth					
L_{ext}, μm		0.890	4.03	5.43	14.42

6. – Comparison of the DTR and PXR

The angular distributions of the yield in the PXR (3) and DTR (13) reflections have very similar cone-like shapes around the vector $-\vec{V}$. The angular size of the reflection can be estimated as the angular distance between the vector $-\vec{V}$ and the maximum in the yield. The PXR reflection is wider than the DTR reflection because the PXR reflection angular size $\sqrt{\chi_0}$ always exceeds the DTR one γ^{-1} under condition (5).

The frequencies of radiation in both PXR and DTR reflections are very close one to another and to value $\frac{cg}{2}$ but the DTR frequency ω_B always exceeds the PXR frequency ω_{PXR}. On can find the normalized difference of frequencies [13] comparing the expressions for frequencies (4) and (12)

$$\frac{\omega_{\mathrm{DTR}} - \omega_{\mathrm{PXR}}}{\omega_{\mathrm{PXR}}} \approx \frac{\gamma^{-2} + |\chi_0| + \rho^2}{4}. \tag{16}$$

The thickness of the near-surface layer of the crystal that provides the formation of the DTR reflection can be estimated as the extinction depth L_{ext} (see eq. (111) in [14])

$$L_{\mathrm{ext}} = \frac{c}{\omega_B |\chi_{\vec{g}}|}. \tag{17}$$

The extinction length usually is much less than the absorption lengths if the X-ray frequency is out of an absorption edge

$$T_e = \frac{c}{\omega |\chi_0''|} \tag{18}$$

(as an example, see data is tables II and III). The normalized width of the PXR spectral peak is determined by the absorption length in the crystal

$$\frac{\Delta\omega_{\mathrm{PXR}}}{\omega_{\mathrm{PXR}}} = \frac{|\chi_0''|}{2} = \frac{c}{2\omega_{\mathrm{PXR}} T_e}. \tag{19}$$

The normalized width of the DTR spectral peak is determined by the width of the Darwin-Prince curve $|\chi_{\vec{g}}|$ or by the extinction length

$$\frac{\Delta\omega_{\mathrm{DTR}}}{\omega_{\mathrm{DTR}}} = |\chi_{\vec{g}}| = \frac{c}{\omega_{\mathrm{DTR}} L_{\mathrm{ext}}}. \tag{20}$$

It easy to find the relation of the widths (19), (20) as the frequencies ω_{DTR} and ω_{PXR} are very close to one another:

$$\frac{\Delta\omega_{\mathrm{PXR}}}{\Delta\omega_{\mathrm{DTR}}} = \frac{|\chi_0''|}{2|\chi_{\vec{g}}|} = \frac{L_{\mathrm{ext}}}{2T_e}, \tag{21}$$

that is determined by the relation of the absorption and extinction lengths.

Comparing yields of the DTR (13) and the PXR (7) at angles $\rho_\chi > 1$ one can find that their relation is

$$\frac{Y_{\mathrm{DTR}}}{Y_{\mathrm{PXR}}} = \frac{4(\rho_\chi^2+1)^2}{\pi\rho_\chi^4}\frac{|\chi_0''|}{|\chi_{\vec{g}}|} = \frac{4(\rho_\chi^2+1)^2}{\pi\rho_\chi^4}\frac{L_{\mathrm{ext}}}{T_e}, \tag{22}$$

where $\rho_\chi = \frac{\rho}{|\chi_0|^{1/2}}$ is the angle ρ in units of the PXR reflection angular size $\sqrt{|\chi_0|}$. Note that relation (22) is independent of the incident electron energy because the DTR yield Y_{DTR} (13) is independent of the E_e under conditions $\rho_\chi > 1$ and (5). Besides, one can roughly estimate the relation of average spectral densities in the spectral peaks of the DTR and PXR under conditions $\rho_\chi > 1$ and (5):

$$\frac{Y_{\mathrm{DTR}}\cdot\Delta\omega_{\mathrm{PXR}}}{Y_{\mathrm{PXR}}\cdot\Delta\omega_{\mathrm{DTR}}} = \frac{2(\rho_\chi^2+1)^2}{\pi\rho_\chi^4}\left(\frac{|\chi_0''|}{|\chi_{\vec{g}}|}\right)^2 = \frac{2(\rho_\chi^2+1)^2}{\pi\rho_\chi^4}\left(\frac{L_{\mathrm{ext}}}{T_e}\right)^2, \tag{23}$$

that again is a function of the relation $\frac{L_{\mathrm{ext}}}{T_e}$. The relation (23) is less than 0.05 for reflections from the planes (333), (444), (555) of a Si crystal at $\rho_\chi > 1$ in the above considered example. This means that the PXR spectral density is much higher than the DTR one. Therefore the interference can be neglected in these reflections. But relation (23) reaches values 0.89 and 0.35 for the (111) reflection at $\rho_\chi = 1$ and $\rho_\chi = 2$, respectively, because of the small value of T_e in the vicinity if the absorption K-edge of Si. In this case the PXR and DTR cannot be separated and the interference has to be taken into account [10] for more exact calculations of the radiation.

Note that the DTR differently from the PXR cannot produce the radiation in the backward direction due to the plane effect.

7. – Results and discussion

Three kinds of X-ray radiation—CXR, PXR, DTR—are emitted and can be observed in the backward direction from a crystal-radiator of positrons. The observation of radiation emitted at the backward direction is convenient for the reduction of the gamma-ray background that is going mainly to the forward hemisphere. The control of parameters of the DTR and PXR spectral peaks of different frequencies can be applied for the control of the temperature and degradation of the crystal-radiator near surface layers of different thicknesses. In particular, the yields in the PXR and DTR spectral peaks are proportional to values $|\chi_{\vec{g}}|^2$ and $|\chi_{\vec{g}}|$, respectively, and both of them depend on the crystal temperature and degradation. The frequencies of the PXR and DTR spectral peaks can be used for the control of the crystal lattice alignment, *e.g.*, at heating of the crystal at production of positrons. The CXR frequency and yield should be independent of the crystal temperature and degradation and can be used as a reference value for the measurements of the PXR and DTR. Such experimental information can be useful for the control and optimization of the positron production. Others PXR and DTR reflections emitted in the backward hemisphere at angles different from π can be used for the control of the crystal state too.

Besides, the same backward-emitted CXR, PXR, DTR can be used for the control of the crystal-radiator state during production of coherent bremsstrahlung and in experiments on the steering of the particle trajectories by bent crystals. Also, the backward-emitted PXR can be used for measurements of nano-crystals size at moderate incident electron or proton energies [15]. Let us turn the attention to high yield and spectral density of the DTR that increase as the square of the incident particle energy. The quasi-monochromatic DTR of the Bragg frequency ω_B may be considered as a new source of X-ray beam excited by high-energy particle beam. The direction and frequency of the DTR X-ray beam can be smoothly tuned due to rotation of the crystal. Anyway, experimental studies on X-rays emitted in the backward hemisphere are desirable to clear up the real possibility for such applications.

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