IL NUOVO CIMENTO Vol. 34 C, N. 4 Luglio-Agosto 2011
DOI 10.1393/ncc/i2011-10949-y

Colloquia: Channeling 2010

A model of parametric X-ray radiation for application to diagnostic radiology

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(ricevuto il 22 Dicembre 2010; pubblicato online il 29 Agosto 2011)

Summary. — Parametric X-ray Radiation (PXR) is well known as an intense, tunable and quasi-monochromatic X-ray source. From the very first work of Ter-Mikaelian, who proposed the interaction phenomenon for Parametric X-rays many theoretical and experimental studies have investigated the characteristics of such a novel X-ray source. Within the framework of classical electrodynamics, we have thoroughly studied the physical implications of electrons moving through a medium at relativistic speed and then developed an analytical model of X-ray diffraction based on the PXR phenomenon. The model has been used to obtain information on the characteristics of PXR diffracted beam in terms of X-ray intensity, energy spectrum and angular distribution. Several crystals have been studied both in Bragg and Laue geometry and their relative yield has been compared. Preliminary results on the diagnostic potential of PXR have shown that, at a distance from the crystal which produces a size of the X-ray field useful for an imaging application, the photon yield of PXR is higher than that produced by a conventional X-ray tube, provided that a similar electron current is available.

PACS 41.50.+h – X-ray beams and X-ray optics.
PACS 41.60.-m – Radiation by moving charges.

1. – Introduction

The availability of a monochromatic X-ray beam having a tunable energy is a major breakthrough in diagnostic radiology for it allows a better image quality at reduced dose. Synchrotron Radiation has been the first monochromatic X-ray source to be successfully applied to medical imaging but its cost and size are prohibitive for the clinical use. Recently, small-scale synchrotrons [1] and betatrons [2] have been proposed as recycling source of electrons, and when a target is inserted in the electron path intense hard X-rays

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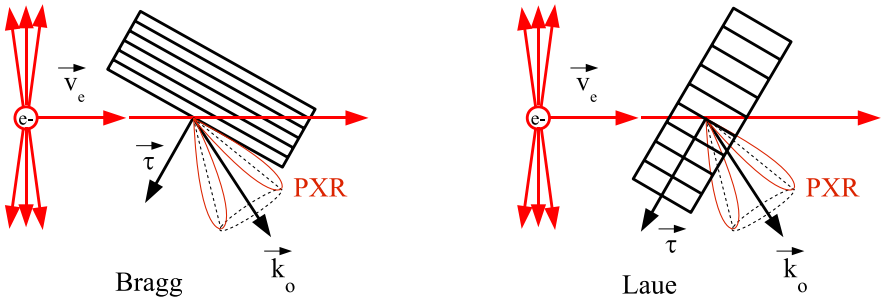

Fig. 1. – Bragg geometry (left) and Laue geometry (right) for PXR emission.

are produced by bremsstrahlung or parametric effect. In order to obtain quantitative characteristics of PXR, it is necessary to include into the theory of PXR the contribution of electron multiple scattering into the crystal, the beam divergence and eventually the crystal mosaicity [3]. Starting from the approach proposed by Asano *et al.* [4], where the process of PXR emission and multiple scattering are independent (incoherent model), and applying the uniform technique presented by Potylitsyn [5], we have developed a stochastic-convolutional model to simulate the conditions of a real PXR experiment, taking into account such phenomena for 20 MeV electron beam of MIRRORCLE-20ST model.

2. – Parametric X-ray radiation

The angular distribution of emitted PXR photons per solid angle and per electron according to the Feranchuk-Ivashin model [6, 7] is

$$\frac{\mathrm{d}^2N}{\mathrm{d}\theta_x\mathrm{d}\theta_y} = \frac{\alpha}{4\pi}\frac{\omega_B}{c}\frac{|\chi_h|^2e^{-2W}}{\sin^2\theta_B}f_{\mathrm{geo}}(\hat{\tau},\hat{k},\hat{v})\Lambda(\theta_x,\theta_y), \tag{1}$$

where χ_h is the Fourier expansion of the dielectric susceptibility and can be viewed as the crystal's diffraction efficiency. The *geometrical factor* f_{geo}, which accounts for the photon absorption along the escape path out of the crystal, is given by

$$f_{\mathrm{geo}}(\hat{\tau},\hat{k},\hat{v}) = L_a\left|\frac{\hat{\tau}\cdot\hat{k}}{\hat{\tau}\cdot\hat{v}}\right|\left(1-e^{-\frac{L}{L_a|\hat{\tau}\cdot\hat{k}|}}\right) \tag{2}$$

where $\hat{\tau}$ denotes the crystal surface unit vector, $\hat{k}$ the direction of the emitted photon and $\hat{v}$ the direction of the incident electron as shown in fig. 1. Here L and L_a are, respectively, the thickness and the absorption length of the target crystal.

The *angular factor* $\Lambda(\theta_x,\theta_y)$ is

$$\Lambda(\theta_x,\theta_y) = \frac{\theta_x^2\cdot\cos^2 2\theta_B+\theta_y^2}{[\theta_x^2+\theta_y^2+\theta_{\mathrm{ph}}^2]^2}, \tag{3}$$

where θ_x and θ_y refer to the deviation from the Bragg direction in and perpendicular to the scattering plane given by $\hat{v}$ and $\hat{k}$. The characteristic angle θ_{ph}, which defines

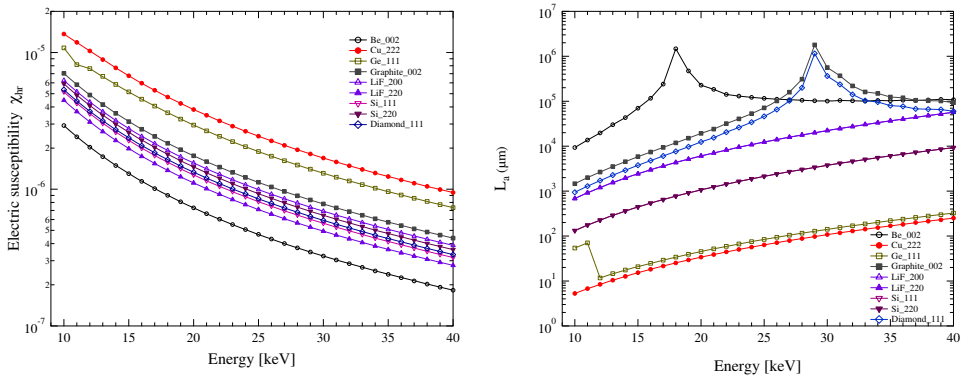

Fig. 2. – The real component of electric susceptivity χ_{hr} (left), the geometrical factor $f_{\rm geo}$ (right) as a function of X-ray energy for different target crystals.

the spread of the cone of PXR photons, is given by $\theta_{\rm ph}^2 = 1/\gamma^2 + (\omega_p/\omega)^2$ and it is the only term related to the electron energy. It differs from the corresponding term in Feranchuk-Ivashin equation because the coherent scattering term θ_s^2 has been omitted in accord with various experimental observations [4, 8]. Both L_a and χ_h,obtained by X0h program [9, 10], depend on PXR energy as shown in fig. 2, but in opposing ways: L_a increases with photon energy and χ_h decreases with photon energy.

The analysis of these two competing factors could be used to determine what is the target (crystal) suited for X-ray production of a given energy by the PXR effect. Then, it is useful to define the multiplicative term of the angular function $\Lambda(\theta_x, \theta_y)$ in eq. (1) as the *yield factor*, that represents the PXR quality factor for a crystal. In fig. 3 we show the calculated *yield factor* for Bragg and Laue geometry for one L_a crystal thickness, and the results suggest low-Z crystals as the best choice for PXR.

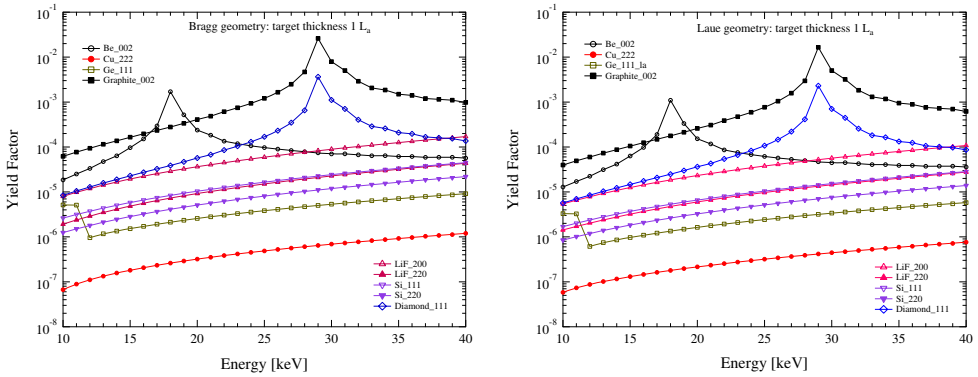

Fig. 3. – The yield factor for Bragg geometry (right) and Laue geometry (left) as a function of X-ray energy for different target crystals.

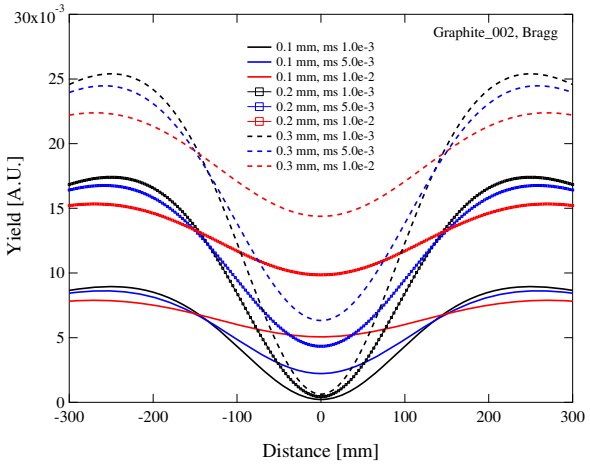

Fig. 4. – Calculation of PXR intensity profiles for different values of multiple scattering and crystal thickness.

3. – Stochastic-convolutional model

However, it is well known that real experimental conditions broaden theoretical PXR photon distribution. To include such effects in eq. (1), the approach proposed by Potylitsyn [5] has been considered. Convolution with a Gaussian distribution that accounts for multiple scattering, divergence of electron beam and crystal mosaicity gives

$$(4)\quad \mathrm{d}N = \frac{\alpha}{4\pi}\frac{\omega_B}{c} f_{\mathrm{geo}}(\hat{\tau},\hat{k},\hat{v})\frac{\chi_h^2 e^{-2W}}{\sin^2\theta_B}\left\{\int_{\theta_x}\int_{\theta_y}\mathrm{d}\theta_x\mathrm{d}\theta_y \right.$$
$$\left.\times\left[\int_{\Delta_x}\int_{\Delta_y}\mathrm{d}\Delta_x\mathrm{d}\Delta_y\frac{(\theta_x+\Delta_x)^2\cdot\cos^2 2\theta_B+(\theta_y-\Delta_y)^2}{[(\theta_x+\Delta_x)^2+(\theta_y-\Delta_y)^2+\theta_{\mathrm{ph}}^2]^2}\frac{e^{-\left(\frac{\Delta_x^2}{2\sigma_{mmsdx}^2}+\frac{\Delta_y^2}{2\sigma_{mmsdy}^2}\right)}}{2\pi\sigma_{mmsdx}\sigma_{mmsdy}}\right]\right\},$$

where $\sigma_{mmsdx}^2=\sigma_{msx}^2+\sigma_{dx}^2+4\sigma_m^2$ and $\sigma_{mmsdy}^2=\sigma_{msy}^2+\sigma_{dy}^2+4\sin^2\theta_B\sigma_m^2$.

The standard deviations σ_{ms}, σ_d and σ_m correspond to electron multiple scattering, beam divergence and crystal mosaicity, respectively. The multiple scattering contribution could be evaluated by using formula (5):

$$(5)\qquad \sigma_{ms,x,y}\cong\frac{14\,\mathrm{MeV}}{\sqrt{T^2+2Tmc^2}\sqrt{1-\gamma^{-2}}}\cdot\sqrt{\frac{\delta L_{\mathrm{path}}}{L_{\mathrm{rad}}}}\left[1+\frac{1}{9}\log_{10}\left(\frac{\delta L_{\mathrm{path}}}{L_{\mathrm{rad}}}\right)\right].$$

Including these contributions the characteristic two-lobe intensity profile of PXR is reduced and the beam is more suitable for X-ray imaging as shown in fig. 4.

By using recycling electron source like betatron where the target is mounted inside the accelerator, the crystal thickness could be chosen to limit both the multiple scattering and the energy loss by electrons. To accomplish this task, we have used the following criteria:

TABLE I. – *The calculated crystal thickness for Laue and Bragg geometry by using the upper criteria. The electron beam energy is* 20 MeV *and the X-ray energy is* 20 keV*. The thickness is in meter.*

	Si 111	Be 002	LiF 220	Diamond 111	Si 220	Graphite 002	Li 200
Laue	2.7E-04	5.8E-04	3.6E-04	2.6E-04	2.7E-04	5.4E-04	3.7E-04
Bragg	2.7E-05	1.0E-04	8.1E-05	4.0E-05	4.3E-05	5.1E-05	5.7E-05

1) select one of the acceptable values for the multiple scattering: from eq. (5) we obtain δL_{path} and by using the stopping power equation (6) we evaluate if the electron energy loss $\mathrm{d}T$ is acceptable;

2) select one of the acceptable values for electron energy loss $\mathrm{d}T$, from the stopping power equation (6) we obtain δL_{path} and by using eq. (5) we evaluate if the electron multiple scattering $\sigma_{ms,x,y}$ is acceptable.

$$S(T) = -\frac{1}{\rho}\frac{\mathrm{d}T}{\mathrm{d}L}\,. \tag{6}$$

In this work we studied the 20 keV PXR production by a 20 MeV electron beam fixing the acceptable value for the multiple scattering to $\frac{1}{\gamma}$ and for the energy loss to 1% of electron starting kinetic energy. The calculated thickness, by using the upper criteria, for typical PXR target is shown in table I.

The 2D distribution of photon yield and energy spectrum are shown in fig. 5 for a Graphite 002 crystal. The energy spectrum image shows a large gradient on θ_x direction which reduces the useful X-ray field to $\approx \pm 10$ mrad for 2 keV energy band. The photon yield shows instead a more uniform spatial distribution. At a distance of 3 meters the dimension of the useful X-ray field is 6 cm × 18 cm.

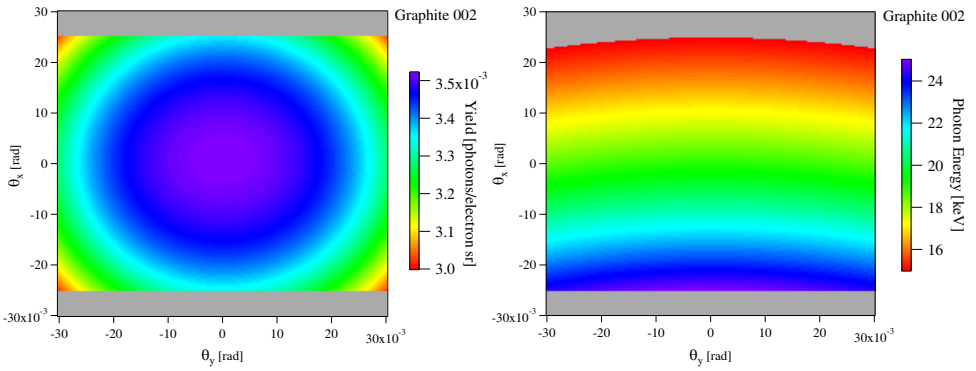

Fig. 5. – The 2D distribution of photon yield (left), the 2D distribution of energy spectrum (right) for Graphite 002 in Bragg geometry. The crystal thickness has been chosen following the previous criteria and it is 51 μm.

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Research supported by European Commission under FP7 program-LABSYNC project.

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