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Optical transition radiation in fused quartz under external acoustic field

A. R. Mkrtchyan(1)(*), A. H. Mkrtchyan(1), V. R. Kocharyan(1), A. E. Movsisyan(1), A. H. Aslanian(1), Z. G. Amirkhanyan(1) and P. N. Zhukova(2)

(1) *Institute of Applied Problems of Physics - Nersessian Str. 25, 375014 Yerevan, Armenia*
(2) *Belgorod State University - Studencheskaya Str. 14, 308007 Belgorod, Russia*

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Summary. — In the present paper we have investigated the optical transition radiation in a plate excited by a longitudinal acoustic wave. The spectral-angular density of the radiated energy is calculated. The numerical examples are given for a plate of fused quartz. These results show that the acoustic waves allow to control the parameters of the radiation. In particular, new resonance peaks appear in the spectral distribution of the radiation intensity. The height of the peaks can be tuned by choosing the parameters of the acoustic wave.

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1. – Introduction

In recent accelerator technology the problem of the beam diagnostics is very urgent because it is necessary not only to measure the size and the position of the beam, but also to control its profile and angular divergence [1-5]. A common method is based on the detection of optical transition radiation (OTR) from the beam of charged particles.

It is well known that a relativistic electron flying through the stack of plates emits photons. This phenomenon was studied in detail in [6,7] for X-rays. In [8,9], the problem of an electron moving in a medium with dielectric permittivity varying in the space by a harmonic law was discussed. In these papers the possibility of creating an intense source of monochromatic X-rays with the spatial and time control is discussed based on the transition radiation of relativistic electrons in an acoustic superlattice, excited in a plate of amorphous quartz. In [10], OTR from relativistic electrons is investigated in an ultrasonic superlattice excited in a finite thickness plate. In the quasi-classical

(*) E-mail: malpic@sci.am

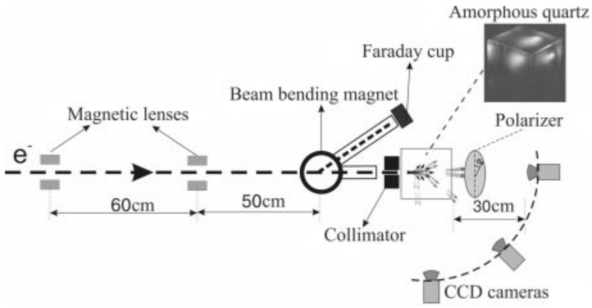


Fig. 1. – The experimental setup for studying OTR.

approximation the formulae are derived for the vector potential of the electromagnetic field and for the spectral-angular distribution of the radiation intensity.

In this paper, for the first time, we have investigated, both experimentally and theoretically, the phenomenon of OTR on the acoustic superlattice excited in a plate of amorphous quartz. The paper is organized as follows. In the next section we describe the experimental apparatus and the results of the measurements. Section **3** is devoted to the theoretical analysis of the problem. The main results are summarized in sect. **4**.

2. – Experiment

The studies were carried out on the microtone of Yerevan Physics Institute. The principal components of the experimental setup are shown in fig. 1. The operating parameters of the electron beam are presented in table I. After the formation by the magnetic lenses, the electron beam passes through the lead collimator (with the diameter 4 mm and the length 5 cm) and then it is scattered on the sample. A rotating magnet is placed before the collimator, which was used to change the direction of the beam propagation. The beam current is measured by using the Faraday cup. The detection of the OTR intensity was carried out by the photomultiplier PMT-85, and the frequency distribution of the radiation was measured by using the filters, which were placed before PMT-85. For the measurements of the polarization of the beam, special lenses were used. The camera had the opportunity to work in the experience mode.

The acoustic vibrations in amorphous quartz were excited by using single crystals of quartz (see fig. 2). Initially, on the amorphous quartz was attached a copper ring with

TABLE I. – *Electron beam parameters.*

Electron energy	7.5 MeV
Average current of microtrone	$I \approx 0.05$–$0.1\,\mu$A
Beam diameter	4 mm
Angular divergence	≈ 0.02 radian
Duration of macropulse	$2 \cdot 10^{-6}$ radian
Pulse repetition frequency	50 Hz
Number of micropulses in macropulse	6000

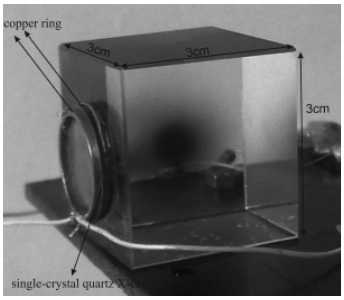


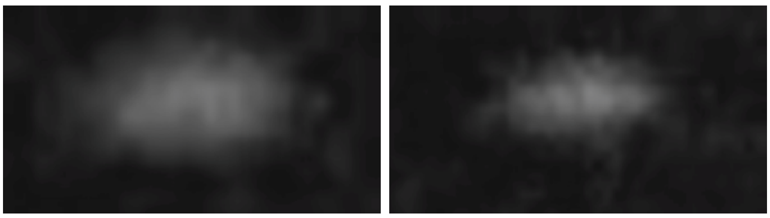
Fig. 2. – The general view of the operation cell for the investigation of the OTR.

the outer and inner diameters 19 mm and 18 mm, respectively, and with the thickness 1 mm. On the free side of the copper ring, a single-crystal quartz plate was attached with the diameter 20 mm and with the thickness in the range 120–700 μm. For the excitation of acoustic oscillations, silver contacts were implanted on the surface of the single crystal. The contact of the silver-copper-ring was homogenous with an error $\leq \lambda_{us}/8$, where λ_{us} is the acoustic wavelength. On the other side of the single-crystal quartz, the second copper ring was attached identical to the first one. The volume between the amorphous quartz and the single crystal of quartz (the volume of the inner cylinder of the washer) was filled with Vaseline in order to achieve even transmission of acoustic oscillations in the amorphous sample. On the copper washers a microwave field was applied.

The results of the experimental investigation of OTR in the amorphous quartz from relativistic electrons with the energy 7.5 MeV, in the presence and in the absence of the acoustic field with the frequency of 5 MHz are shown in fig. 3, a, b, respectively. From the obtained preliminary results, it follows that as a result of the modulation of the dielectric permittivity by an external acoustic field, the center of the spectral distribution is shifted from the blue radiation range to the infrared one.

3. – Theory

Let an electron, moving with constant velocity $\mathbf{v} = \mathrm{v}\mathbf{n}_z$ ($\mathbf{n}_z$ is the normal to the boundary), enter the plate perpendicular to the plane (fig. 4). We consider the case when the longitudinal acoustic vibrations are excited along the axis Z, forming a superlattice



(a) (b)

Fig. 3. – OTR from relativistic electrons in the amorphous quartz a) without of the acoustic field, b) with the acoustic field.

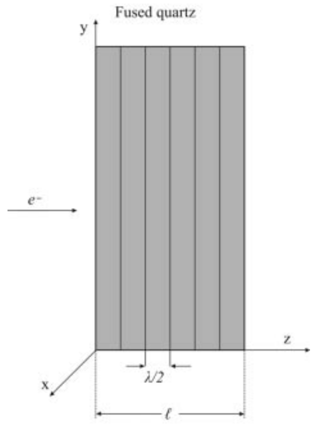


Fig. 4. – The schematic view of superlattice in the amorphous quartz.

with the permittivity

$$\varepsilon(\omega, z) = \begin{cases} 1, & z > l, z < 0, \\ \varepsilon_0(\omega) + \varepsilon_1(\omega, z) & 0 < z < l, \end{cases}$$

where $\varepsilon_0(\omega)$ is the dielectric permittivity of the medium without acoustic waves (see table II), $\varepsilon_1(\omega, z)$ is the change of the dielectric permittivity induced by the acoustic vibrations

(1) $$\varepsilon_1(\omega, z) = \Delta\varepsilon \cos(k_0 z + \varphi_1),$$

where $\Delta\varepsilon = \frac{\Delta n}{n_0}(\varepsilon_0(\omega) - 1)$, n_0 is the number density of electrons, ω is the radiation frequency, k_0 is the wave number of the ultrasound.

Note that under the condition $\omega_s \ell/\omega \ll 1$ during the transition time of the electron, the dielectric permittivity in the superlattice is not notably changed. For the relativistic electrons and for the plate thickness $\ell \leq 1$ cm this leads to the constraint $\omega_s \ll 10^{11}$ Hz.

In the following calculations, in the expression for the permittivity the cosine function is replaced by the step function. To solve the corresponding problem we have developed a matrix method for solving the Maxwell equations, by taking into account the radiation absorption in the medium. This method allows us to speed up the computation significantly. In the case when the dielectric permittivity of the medium is constant, the solutions for the transition radiation fields are well known. For the spectral components of the self-field displacement vector, $\mathbf{D}_n^e$ and for the radiation field displacement vector, $\mathbf{D}_n^r$ we have

(2) $$\mathbf{D}_n^e = \left(\frac{iev}{\pi\omega c}\right) \iint \frac{\left(\left(k^2 - \varepsilon_n k_\omega^2\right)\mathbf{k} + k^2\mathbf{q}\right)}{k_\omega(k^2 + q^2 - \varepsilon_n k_\omega^2)} \exp[-i\mathbf{r}(\mathbf{k} + \mathbf{q})]\mathrm{d}\mathbf{q},$$

$$\mathbf{D}_n^r = \left(\frac{iev}{\pi\omega c}\right) \iint \left(\mathbf{d}_n^+ \exp[i\mathbf{r}\mathbf{p}_n] + \mathbf{d}_n^- \exp[-i\mathbf{r}\mathbf{p}_n]\right) \exp[-i\mathbf{r}\mathbf{q}]\mathrm{d}\mathbf{q},$$

TABLE II. – *The value of the fused quartz refraction index as a function of the wavelength.*

Wavelengths microns	Index of refraction	Wavelengths microns	Index of refraction
0.2	1.551	1.0	1.450
0.22	1.528	1.1	1.450
0.25	1.507	1.2	1.448
0.3	1.488	1.3	1.447
0.32	1.483	1.5	1.445
0.36	1.475	1.6	1.443
0.4	1.470	1.7	1.442
0.45	1.466	1.8	1.441
0.5	1.462	1.9	1.440
0.55	1.460	2.0	1.438
0.60	1.458	2.2	1.435
0.65	1.457	2.4	1.431
0.7	1.455	2.6	1.428
0.75	1.454	2.8	1.424
0.8	1.453	3.0	1.419
0.85	1.452	3.2	1.414
0.9	1.452	3.37	1.410

where $\mathbf{k}$ is the radiation wave vector, $\mathbf{q}$ is the component of the vector $\mathbf{k}$, perpendicular to the axis Z, and $k = \omega/c$.

In order to obtain the radiation field in such a medium, we have developed a new algorithm on the basis of the matrix representation. The solution of the Maxwell equations is presented in the matrix form:

$$\left(\frac{ie\omega}{\pi c^2}\beta\right)^{-1}\begin{pmatrix}\frac{\boldsymbol{\kappa}\mathbf{D}(z)}{\kappa}\\ \frac{\mathbf{n}\mathbf{D}(z)}{\varepsilon_n}\end{pmatrix} = \frac{\exp[-i\kappa t]}{\left(\gamma_n^{-2}+\sin^2\theta\,\beta^2\right)}\begin{pmatrix}\gamma_n^{-2}\\ \frac{\sin\theta}{\varepsilon_n}\end{pmatrix} +$$

$$+\frac{1}{\left(\gamma_n^{-2}+\sin^2\theta\beta^2\right)}\begin{pmatrix}\beta^2\sin^2\theta\exp[i\pi_n\kappa t], & \beta^2\sin^2\theta\exp[-i\pi_n\kappa t]\\ \frac{\pi_n\sin\theta}{\varepsilon_n}\exp[i\pi_n\kappa t], & -\frac{\pi_n\sin\theta}{\varepsilon_n}\exp[-i\pi_n\kappa t]\end{pmatrix}\begin{pmatrix}s_n^+\\ s_n^-\end{pmatrix},$$

where $\beta = \mathbf{v}/c$, $\mathbf{k} = k\,\boldsymbol{\kappa}$, $\mathbf{q} = k\,\mathbf{n}\,\sin\theta$, with $\mathbf{n}$ being the normal vector, and

$$\gamma_n = \frac{1}{\sqrt{1-\varepsilon_n\beta^2}}\,, \qquad \boldsymbol{\kappa} = \frac{c}{v}\frac{\mathbf{v}}{v}\,.$$

Other notations are as follows: $\mathbf{p}_n = k\,\pi_n\boldsymbol{\kappa}$, $\pi_n = \beta\sqrt{\varepsilon_n - \sin^2\theta}$, $d = t_n - t_{n-1} = \lambda/2n$ and ε_n are the thickness and the dielectric permittivity of the n-th layer.

On the base of the suggested method, the general solution for the radiation fields in the forward and backward directions can be presented as follows:

$$\mathbf{D}^r_{\leftarrow} = \left(\frac{ie\omega}{\pi c^2}\beta\right) \iint D^+_{\leftarrow} \left(\beta^2 \sin\theta\, \boldsymbol{\kappa} + \pi_0 \mathbf{n}\right) \exp\left[-i \sin\theta\, \mathbf{n}\boldsymbol{\rho}\right] \mathrm{d}\mathbf{q}, \tag{3}$$

$$\mathbf{D}^r_{\rightarrow} = \left(\frac{ie\omega}{\pi c^2}\beta\right) \iint D^-_{\rightarrow} \left(\beta^2 \sin\theta\, \boldsymbol{\kappa} - \pi_0 \mathbf{n}\right) \exp\left[-i\, \sin\theta\, \mathbf{n}\boldsymbol{\rho}\right] \mathrm{d}\mathbf{q},$$

where $\mathbf{D}^r_{\leftarrow}$ and $\mathbf{D}^r_{\rightarrow}$ are the corresponding displacement vectors,

$$D^+_{\leftarrow} = -\frac{\pi_0 Q^{\uparrow}_M + \beta^2\, \sin\theta\, Q^{\downarrow}_M}{\pi_0 \left(\beta^2 \sin\theta\, \Omega^M_{11} + \pi_0 \Omega^M_{12}\right) + \beta^2 \sin\theta \left(\beta^2 \sin\theta\, \Omega^M_{21} + \pi_0 \Omega^M_{22}\right)},$$

$$D^-_{\rightarrow} = \frac{\left(\beta^2 \sin\theta\, \Omega^M_{21} + \pi_0 \Omega^M_{22}\right) Q^{\uparrow}_M - \left(\beta^2 \sin\theta\, \Omega^M_{11} + \pi_0 \Omega^M_{12}\right) Q^{\downarrow}_M}{\pi_0 \left(\beta^2 \sin\theta\, \Omega^M_{11} + \pi_0 \Omega^M_{12}\right) + \beta^2 \sin\theta \left(\beta^2 \sin\theta\, \Omega^M_{21} + \pi_0 \Omega^M_{22}\right)}.$$

Here

$$\mathbf{Q}_M = \exp\left[-i\kappa t_0\right] \cdot \left(\Omega^M - \exp[-i\kappa M d]\right) \left(\mathbf{B}_{1,0} + \frac{\exp[-i\kappa d]\mathbf{Q}_0}{\left(\Omega - \exp[-i\kappa d]\right)}\right). \tag{4}$$

In eq. (4), the following notations are introduced:

$$\boldsymbol{\Omega} = \prod_{p=0}^{K-1} \boldsymbol{\alpha}_{K-p} = \prod_{p=1}^{K} \boldsymbol{\alpha}_p,$$

$$\mathbf{Q}_0 = \left(\mathbf{B}_{1,K} + \sum_{l=1}^{K-1} \left(\exp\left[i\kappa\left(t_K - t_{K-l}\right)\right] \cdot \left(\prod_{p=0}^{l-1} \boldsymbol{\alpha}_{K-p}\right) \cdot \mathbf{B}_{K-l+1,K-l}\right)\right),$$

$$\boldsymbol{\alpha}_n = \begin{pmatrix} \cos(\pi_n \kappa d_n), & i\frac{\beta^2 \varepsilon_n}{\pi_n \sin\theta} \sin(\pi_n \kappa d_n) \\ i\frac{\pi_n \sin\theta}{\beta^2 \varepsilon_n} \sin(\pi_n \kappa d_n), & \cos(\pi_n \kappa d_n) \end{pmatrix},$$

$$\mathbf{B}_{n+1,n} = \frac{\sin\theta \left(\varepsilon_{n+1} - \varepsilon_n\right)}{\left(\gamma_n^{-2} + \sin^2\theta\, \beta^2\right)\left(\gamma_{n+1}^{-2} + \sin^2\theta \beta^2\right)} \begin{pmatrix} \sin\theta\, \beta^4 \\ \frac{1-\beta^2(\varepsilon_{n+1}+\varepsilon_n)+\sin^2\theta\beta^2}{\varepsilon_n \varepsilon_{n+1}} \end{pmatrix},$$

with $\mathbf{B}_{n+1,n} = -\mathbf{B}_{n,n+1}$

In the relations given above M is the pile number, *i.e.* how many periods we have due to the acoustic field excitation, and θ is the angle between the wave vector of the photon and $\mathbf{n}_z$. The radiation intensity in the frequency range $(\omega, \omega + d\omega)$ and along the angles $(\theta, \theta + \mathrm{d}\theta)$ is given by the expression

$$I(\omega, \theta)\mathrm{d}\omega\mathrm{d}\theta = (2\pi)^3 \frac{\omega^2}{c} \sin^3\theta \cos^2\theta \left|\mathbf{D}^r_{\leftarrow}(\theta, z \gg \ell, \omega)\right|^2 \mathrm{d}\omega\mathrm{d}\theta. \tag{5}$$

For the corresponding spectral density one has

$$W(\omega) = \int I(\omega, \theta)\mathrm{d}\theta. \tag{6}$$

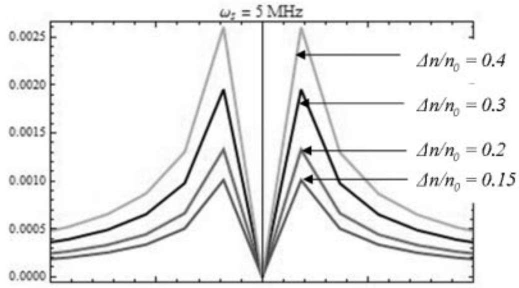


Fig. 5. – The angular distribution of OTR for electrons with energy 7.5 MeV in amorphous quartz on an ultrasonic superlattice. The frequency of the radiation is $3.6 \cdot 10^{14}$ Hz and the frequency of the acoustic field is 5 MHz.

For the numerical calculation of the transition radiation intensity on the base of formulas given above we have developed programs in C++ and Mathematica. Numerical evaluations were carried out for amorphous quartz with the density $2.2\,\mathrm{g/cm^3}$. The velocity of the corresponding longitudinal ultrasonic vibrations is $\omega_0/k_0 \approx 5.6\times10^5$ cm/s.

We have evaluated the spectral-angular distribution of OTR in the forward direction for different values of the parameters for the acoustic field (frequency, amplitude). The results show that in the case $\Delta n/n_0 \ll 1$, the ultrasonic vibrations do not essentially change the intensity of the transition radiation. In the case $\Delta n/n_0 \geq 0.05$, the intensity of the OTR increases with the increase of the acoustic wave amplitude due to the interaction of the electrons with the induced superlattice. Below we present the results of the numerical evaluations for the electron energy 7.5 MeV, and for the frequencies of the ultrasonic field $\omega_s = 5$ MHz, 10 MHz.

Figure 5 shows the results of the calculations for the spectral-angular density of the radiation intensity, $I(\omega,\theta)/h$, as a function of the radiation direction for different values of the relative amplitude $\Delta n/n_0$. As is seen, $I(\omega,\theta)/h$ increases with increasing of $\Delta n/n_0$.

Figure 6 shows the results of the spectral distribution of the radiation intensity for different values of the amplitude of the acoustic field for the frequency 5 MHz. For the values of the parameters corresponding to figs. 6 a, the influence of the acoustic field can be neglected. With further increase of the amplitude of the acoustic field new intense peaks appear in fig. 6 b, c. Figure 7 shows the results corresponding to the acoustic field with the frequency 10 MHz. It is seen that with the increase of the frequency of the acoustic field the intensity of the OTR increases and the center of the spectral distribution is shifted toward lower frequencies.

4. – Conclusion

In the present paper we have investigated the optical transition radiation in a plate excited by a longitudinal acoustic wave. The spectral-angular density of the radiated energy is given by eq. (5). The numerical examples are given for a plate of fused quartz. These results show that the acoustic waves allow us to control the parameters of the radiation. In particular, new resonance peaks appear in the spectral distribution of the radiation intensity. The height of the peaks can be tuned by choosing the parameters of the acoustic wave.

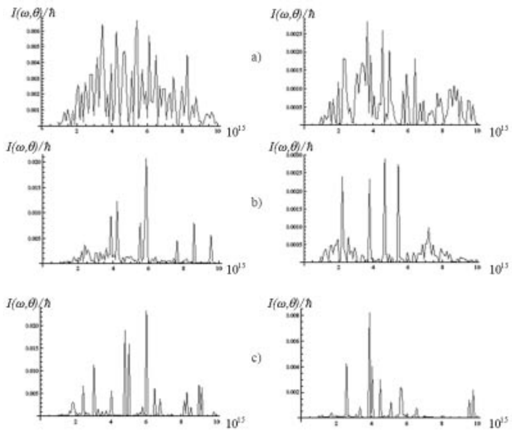


Fig. 6. – The spectral distribution of OTR for $\theta = 0.617$ (left) and $\theta = 0.071$ (right) radian, for different values of the amplitude: a) $\Delta n/n_0 = 0.1$, b) $\Delta n/n_0 = 0.2$, c) $\Delta n/n_0 = 0.3$. The frequency of the acoustic field is $\omega_s = 5\,\text{MHz}$.

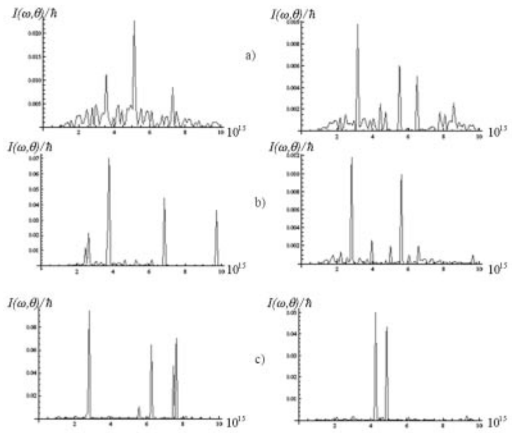


Fig. 7. – The spectral distribution of OTR for $\theta = 0.617$ (left) and $\theta = 0.071$ (right) radian, for different values of the amplitude: a) $\Delta n/n_0 = 0.1$, b) $\Delta n/n_0 = 0.2$, c) $\Delta n/n_0 = 0.3$. The frequency of the acoustic field is $\omega_s = 10\,\text{MHz}$.

The experimental results are in good agreement with the theoretical calculations. In particular, when the acoustic field amplitude increases the spectrum of OTR is shifted towards lower frequencies and the intensities for certain frequencies increase dramatically.

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