IL NUOVO CIMENTO Vol. 34 C, N. 4 Luglio-Agosto 2011
DOI 10.1393/ncc/i2011-10901-3

Colloquia: Channeling 2010

On transition radiation by relativistic electron

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(ricevuto il 22 Dicembre 2010; pubblicato online il 27 Giugno 2011)

Summary. — The problem of relativistic electron transition radiation on a thin ideally conducting plate is considered. The space-time evolution of the electromagnetic field, which arises before and after the electron's traverse of the plate, is analyzed. It is shown that at the moment of time t after the electron's traverse of the plate for distances $r > ct$ the reflected field is different from zero and entirely coincides with the Coulomb field of oppositely charged particle which should move in the direction of reflection. For distances $r < ct$ the total field equals zero. A similar picture of field evolution takes place in the opposite direction. The potential jump at $r = ct$ entirely defines the characteristics of backward transition radiation. The analogy in the development of the space-time picture of the electromagnetic-field evolution in the processes of transition radiation and bremsstrahlung at the momentary scattering of the relativistic electron to a large angle is discussed.

PACS 41.20.-q – Applied classical electromagnetism.
PACS 41.60.-m – Radiation by moving charges.

1. – Introduction

During the motion of a charged particle through a medium with variable permittivity the transition radiation occurs. Beginning from the work of Ginzburg and Frank [1] a great number of papers was devoted to theoretical and experimental investigations of this process owing to the multiplicity of its various realizations (see monographs [2-6], devoted to this topic and references therein). The transition radiation by ultrarelativistic particles is of great interest, because for such particles the radiation is mainly concentrated inside the narrow angular region along the direction of the particle's motion. In the last decades the new turns of works in this area, which are connected to the possibility of the realization of ultralarge transition radiation formation lengths, when not only the target but the whole measuring apparatus can be situated in the pre wave zone, were outlined (see, for example, [7-15]). In this case the analysis of the structure of electromagnetic fields which arise during the transition radiation process turns to be of great importance. This paper is devoted to the analysis of this problem.

In the given work the main attention is paid to the analysis of the structure of the electromagnetic field which takes place during the electron's traverse of the thin ideally conducting plate. In this case wave packets of reflected and passed fields are the packets of free electromagnetic waves that gradually transform into transition radiation field. We show that the structure of these packets is similar to the structure of packets that arise at momentary scattering of an electron to a large angle, which explains the analogy in some characteristics of transition radiation and bremsstrahlung in the considered cases.

2. – Scalar and vector potentials of transition radiation field

Let us consider the problem about the transition radiation that arises during the normal traverse of the thin ideally conducting plate, situated in the plane $z = 0$, by the electron which moves along the z-axis from $z = -\infty$ to $z = +\infty$. Let us investigate the structure of electromagnetic fields that take place before and after the electron's traverse of the plate in vacuum.

The scalar and the vector potentials of the electromagnetic field, which is generated by the electron moving in vacuum, are the solutions of the inhomogeneous Maxwell equations

$$(1)\qquad \Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi e\delta(\mathbf{r}-\mathbf{v}t), \qquad \Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t),$$

where c is the speed of light, e the electron's charge ($e < 0$), $\mathbf{v}$ its velocity, $\mathbf{r} = (\boldsymbol{\rho}, z)$, $\boldsymbol{\rho}$ is orthogonal to the z-axis. The equations set (1) in the considered problem should be supplemented by a boundary condition which corresponds to the fact that on the plate's surface the tangential component of the total electric field equals zero.

The general solution of eq. (1) for the scalar potential $\varphi(\mathbf{r}, t)$ can be represented in the form of the following Fourier expansion:

$$(2)\qquad \varphi(\mathbf{r},t) = \int \frac{\mathrm{d}^3k\mathrm{d}\omega}{(2\pi)^4} e^{i(\mathbf{kr}-\omega t)} \left[\varphi^C_{\mathbf{k},\omega}\delta(\omega - \mathbf{kv}) + \varphi^f_{\mathbf{k},\omega}\delta(k^2-\omega^2)\right].$$

Replacing here φ by $\mathbf{A}$ we obtain the analogous solution for the vector potential.

Here and further the system in which the speed of light equals unit, $c = 1$, is used.

The first item in (2) is the Fourier expansion of electron's Coulomb field, for which

$$(3)\qquad \varphi^C_{\mathbf{k},\omega} = -\frac{8\pi^2 e}{\omega^2 - k^2}, \qquad \mathbf{A}^C_{\mathbf{k},\omega} = \frac{\mathbf{v}}{c}\varphi^C_{\mathbf{k},\omega}.$$

The second item in (2) is the Fourier expansion of the field of induced surface currents on the plate (free field), for which in vacuum $|\mathbf{k}| = \omega$. At large distances from the region in which the transformation of the surrounding electron field takes place, this item forms the radiation field. It can be derived from the boundary condition for the total electric field $\mathbf{E} = \mathbf{E}^C + \mathbf{E}^f$ on the surface of the plate:

$$(4)\qquad \mathbf{E}^C_\perp(\boldsymbol{\rho}, z=0, t) + \mathbf{E}^f_\perp(\boldsymbol{\rho}, z=0, t) = 0.$$

Here $\mathbf{E}^C$ is the electron's Coulomb field and $\mathbf{E}^f$ is the field of induced surface currents on the plate.

The scalar and the vector potentials define the electric and magnetic fields

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi, \qquad \mathbf{H} = \nabla\times\mathbf{A}. \tag{5}$$

Due to the symmetry of the problem, the vector potential is directed along the particle velocity $\mathbf{v}$ [2,3] and the relation $|\mathbf{A}^C(\mathbf{r},t)| = v|\varphi^C(\mathbf{r},t)|$ for the particle's field in vacuum is valid.

Using Fourier expansion (2) it is possible to derive the potential $\varphi(\mathbf{r},t)$ from the condition (4). For this let us perform the integration over the component k_z in (2). Thus we obtain the following expression for the transversal component of the electric field:

$$\mathbf{E}_\perp(\mathbf{r},t) = -\frac{\partial}{\partial\boldsymbol{\rho}}\int\frac{\mathrm{d}^3k_\perp\mathrm{d}\omega}{(2\pi)^4}e^{i(\mathbf{k}_\perp\boldsymbol{\rho}-\omega t)}\Bigg[\varphi^C_{\mathbf{k},\omega}e^{ik_zz}|_{k_z=\frac{\omega}{v}} + \frac{1}{2\left|\sqrt{\omega^2-k_\perp^2}\right|}\left(\varphi^f_{\mathbf{k},\omega}e^{ik_zz}|_{k_z=\sqrt{\omega^2-k_\perp^2}} + \varphi^f_{\mathbf{k},\omega}e^{ik_zz}|_{k_z=-\sqrt{\omega^2-k_\perp^2}}\right)\Bigg]. \tag{6}$$

For $z=0$, according to (4) this component should equal zero. From this we find that

$$-\varphi^C_{\mathbf{k},\omega}|_{k_z=\frac{\omega}{v}} = \frac{1}{2\left|\sqrt{\omega^2-k_\perp^2}\right|}\left(\varphi^f_{\mathbf{k},\omega}|_{k_z=\sqrt{\omega^2-k_\perp^2}} + \varphi^f_{\mathbf{k},\omega}|_{k_z=-\sqrt{\omega^2-k_\perp^2}}\right). \tag{7}$$

The values $k_z = \pm\sqrt{\omega^2-k_\perp^2}$ satisfy the dispersion relation $\omega^2 = k_z^2 + k_\perp^2$, which is defined by the respective δ-function in (2). The sign before the square root $\sqrt{\omega^2-k_\perp^2}$ determines the direction of propagation of plane waves (Fourier components) with the given values of ω and $|k_\perp|$ [2,3]. Indeed, the equation of a plane wave constant phase along the z-axis is $k_zz-\omega t = \text{const}$. The plane waves, which the free field produced by the plate consists of, should propagate away from the plate. Hence for $\omega>0$ on the right of the plate (which means $z>0$) it is necessary to take into account only the Fourier components with positive sign before the root $\sqrt{\omega^2-k_\perp^2}$ in (7), while for $\omega<0$ and $z>0$ only the Fourier components with negative sign before this root in (7). In the region $z<0$ for $\omega>0$ and $\omega<0$ in (7) we should take into account the items with opposite signs before the root $\sqrt{\omega^2-k_\perp^2}$ relatively to the case for $z>0$ [2,3]. The value of the square root itself is considered either positive or belonging to the upper complex half plane.

Thus, taking into account all requirements mentioned above we can write the scalar potential of the free field in the following form:

$$\varphi^f(\mathbf{r},t) = -\frac{e}{2\pi^2v}\int\mathrm{d}^2k_\perp\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{k_\perp^2+\omega^2/p^2}e^{i(z\omega\sqrt{1-k_\perp^2/\omega^2}-\omega t+\mathbf{k}_\perp\boldsymbol{\rho})}, \tag{8}$$

where $p = v\gamma$ (γ is the electron's Lorentz factor). It is a packet of free electromagnetic waves which gradually turns into the field of transition radiation in such a way that each harmonic with frequency ω reconstructs into a diverging spherical wave at the distance $z = l_C \approx 2\gamma^2/\omega$ which is the formation length of the radiation process [16].

Let us note that the value of $|k_\perp|$ in (8) is arbitrary. Therefore, it is necessary to perform the integration in (8) not only over the travelling waves $k_\perp^2 < \omega^2$, but over the surface ones $k_\perp^2 > \omega^2$ as well.

Equations (1) are presented in the Lorentz gauge $\mathrm{div}\mathbf{A} + \partial\varphi/\partial t = 0$.

If we know $\varphi(\mathbf{r}, t)$, we can derive the vector potential $A^f = |\mathbf{A}^f|$ from this equation:

$$(9) \quad A^f(\mathbf{r}, t) = -\frac{e}{2\pi^2 v}\int \mathrm{d}^2 k_\perp \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{k_\perp^2 + \omega^2/p^2}\frac{1}{\sqrt{1 - k_\perp^2/\omega^2}} e^{i(z\omega\sqrt{1-k_\perp^2/\omega^2} - \omega t + \mathbf{k}_\perp \boldsymbol{\rho})}.$$

Making in (8) the substitution $|k_\perp| = |\omega| x$ and separating the contributions to the potential by the free field of travelling and surface waves, we can write the potential of this field in the following form:

$$(10) \qquad \varphi^f(\mathbf{r}, t) = \Phi_1(\mathbf{r}, t) + \Phi_2(\mathbf{r}, t),$$

where

$$(11) \qquad \Phi_1(\mathbf{r}, t) = -\frac{2e}{\pi v}\int_0^1 \frac{x\mathrm{d}x}{x^2 + p^{-2}} \int_0^\infty \mathrm{d}\omega J_0(\omega x \rho) \cos\left[\omega\left(|z|\sqrt{1 - x^2} - t\right)\right],$$

$$(12) \qquad \Phi_2(\mathbf{r}, t) = -\frac{2e}{\pi v}\int_1^\infty \frac{x\mathrm{d}x}{x^2 + p^{-2}} \int_0^\infty \mathrm{d}\omega J_0(\omega x \rho) \cos(\omega t)\, e^{-|z|\omega\sqrt{x^2-1}}.$$

Deriving (10) we performed in (8) the integration over the azimuth angle between $\mathbf{k}_\perp$ and $\boldsymbol{\rho}$ and proceeded from integration over ω along the interval $+\infty > \omega > -\infty$ to the integration over the only positive values of this variable. The corresponding expressions for vector potential $A^f = A_1 + A_2$, according to (9), differ from (11) and (12) only by the additional factor $1/\sqrt{1 - x^2}$ in the integrands.

3. – The structure of the transition radiation field

Let us discuss the structure of the fields that arise during the electron's traverse of a thin ideally conducting plate. The integrals in (11) and (12) can be analytically calculated. After rather long calculations we finally obtain the following expression for $\varphi^f(\mathbf{r}, t)$:

$$(13) \qquad \varphi^f(\mathbf{r}, t) = -\frac{e}{\sqrt{\rho^2\gamma^{-2} + (|z| - vt)^2}}\theta(r - t) - \frac{e}{\sqrt{\rho^2\gamma^{-2} + (|z| + vt)^2}}\theta(t - r),$$

where $\theta(x)$ is the step function which is equal to unit for $x > 0$ and to zero for $x < 0$.

The total field, produced by the electron and the plate, can be obtained by adding to (13) the expression for the electron's own Coulomb field $\varphi^C(\mathbf{r}, t) = e/\sqrt{\rho^2\gamma^{-2} + (z - vt)^2}$.

Thus, if $t < 0$ the total field in the left half-space is equal to the sum of the electron's Coulomb field in this half-space and the field of electron's image:

$$(14) \qquad \varphi(\mathbf{r}, t) = \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z - vt)^2}} - \frac{e}{\sqrt{\rho^2\gamma^{-2} + (|z| - vt)^2}}.$$

In the right half-space the total field equals zero for $t < 0$.

After the electron's traverse of the plate, which means for $t > 0$, the total field is defined by the formula

$$\varphi(\mathbf{r},t) = \left[\frac{e}{\sqrt{\rho^2\gamma^{-2} + (z-vt)^2}} - \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z+vt)^2}}\right] \times \left[\theta(r-t)\theta(-z) + \theta(t-r)\theta(z)\right]. \tag{15}$$

Thus for $t > 0$ the picture of the total field which is created by the electron plate system is the following. In the left half-space in the coordinate region $r > t$ it is a sum of the electron's Coulomb field of the opposite sign reflected from the plate and the own field of electron which is situated on the right of the plate. The reflected field in this case moves with velocity $\mathbf{v}$ in the direction opposite to the direction of electron's motion. In the coordinate region $r < t$, in which the signal about the electron's traverse of the plate at $t = 0$ has already reached, the total field equals zero. In the right half-space for $t > 0$ and $r < t$ the total field equals the sum of the fields of the electron and its "image" on the left of the plate. For $r > t$ the total field equals zero. By the field of the electron's image we assume the field which is created by an imaginary particle with a charge of the opposite sign, which is situated on the opposite side of the plate and moves symmetrically to the electron relatively to the plate. The analogous expressions can be obtained for the vector potential as well. Namely, for $t > 0$ the total field vector potential (the sum of particle's Coulomb field and radiation potentials) has the following form:

$$\mathbf{A}(\mathbf{r},t) = \mathbf{v}\left[\frac{e}{\sqrt{\rho^2\gamma^{-2} + (z-vt)^2}} + \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z+vt)^2}}\right] \times \left[\theta(r-t)\theta(-z) + \theta(t-r)\theta(z)\right]. \tag{16}$$

The expression in square brackets in (16) differs from the same expression for the scalar potential (15) only by the sign of the second item. The reason of this can be understood from the following reasoning. For $z < 0$ in the region $r > t$ the field (15) is the difference between two Coulomb fields, the sources of which are the electron and its image. The vector potential of the Coulomb field is related to its scalar potential by $\mathbf{A} = \mathbf{v}\varphi$, where $\mathbf{v}$ is the velocity of the field source. As electron moves with velocity $\mathbf{v}$ and its image with velocity $\mathbf{v}$, their vector potentials, respectively, equal $\mathbf{A}_e = \mathbf{v}\varphi_e$ and $\mathbf{A}_i = \mathbf{v}\varphi_i$. It is the presence of the "minus" sign in the expression for $\mathbf{A}_i$ that causes the discussed difference in the signs of the items in square brackets in (15) and (16).

Let us note that all integrations in (11) and (12) significantly simplify for $\rho = 0$, while all the main regularities of free-field formation in the considered problem remain intact.

The obtained results are valid for arbitrary electron velocities. The case of an ultrarelativistic particle is of special interest because for such particles the reconstruction of the total field, created by the plate and the electron after its traverse of the plate, into the field of radiation occurs at large distances. The results obtained in this case are illustrated in fig. 1. Here the equipotential surfaces of the field reflected to the left half-space and the field around the electron on the right of the plate are presented.

Let us note that our picture of electromagnetic fields, which take place after the electron's traverse of the thin ideally conducting plate, differs from the picture of the fields which is discussed in paper [17]. Namely, in particular in [17] it is stated that

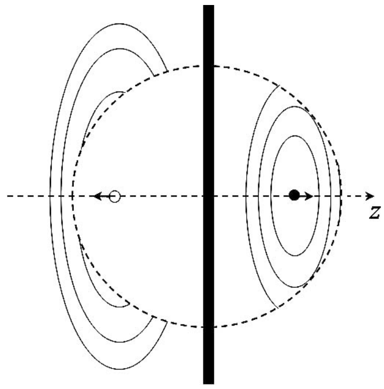

Fig. 1. – The total field in the ultrarelativistic case for $t > 0$; the bold dot is the electron's position after its traverse of the plate, the empty dot is the electron's "image".

"... after the incidence of the charge on the border with the ideally conducting medium the field in the spatial domain $x > 0$ turns out to be different from zero only on the sphere of radius $r = ct$". The given spatial domain ($x > 0$) corresponds to the domain $z < 0$ in our problem statement about transition radiation. The results, obtained by us, (see formula (15)) show that after the electron's traverse of the ideally conducting plate the electromagnetic field equals zero in the region $r < ct$ in the left half-space, while it is different from zero for $r > ct$ in this half-space! This field is caused by the reflection of the particle's field from the plate after its traverse by the electron and cannot instantly disappear in the region $r > ct$. In this case, according to (15), the transition radiation field is defined by the jump of the electromagnetic-field potential on the sphere $r = ct$. Let us note that on this sphere the sharp bend of field lines takes place. In the case of infinite plate, for example, in the region $z < 0$ each field line of the total field (10), which originates on a surface charge of the plate somewhere in the area $\rho > t$ and stretches through the space region $r > t$ to the sphere, should be refracted and stretch further along the surface of the sphere, ending on another surface charge of the plate at $\rho = t$.

4. – The analogy in bremsstrahlung

The fields that are analogous by their structure to (15) can arise in other physical situations. Such fields, for example, occur at the momentary scattering of a fast electron to a large angle, when at the moment of time $t = 0$ in the point $z = 0$ the electron's velocity changes its direction from $\mathbf{v}$ to $\mathbf{v}'$ (fig. 2). In this case the retarded solution for the scalar potential has the following form [16, 18, 19]:

$$\varphi(\mathbf{r}, t) = \theta(r - t)\varphi_{\mathbf{v}}(\mathbf{r}, t) + \theta(t - r)\varphi_{\mathbf{v}'}(\mathbf{r}, t), \tag{17}$$

where $\varphi_{\mathbf{v}}$ and $\varphi_{\mathbf{v}'}$ are the Coulomb potentials of the electrons that are moving uniformly straightforwardly, respectively, along the axes z and z' (see fig. 2). We can see that in the ultrarelativistic case, after the electron's traverse of the plate, the structure of the field (15) on the left of the plate (backward transition radiation field) is analogous, but not identical, to the structure of the field that "tears" off the scattering electron (first item in (17), forward bremsstrahlung). At the same time the structure of the field (15)

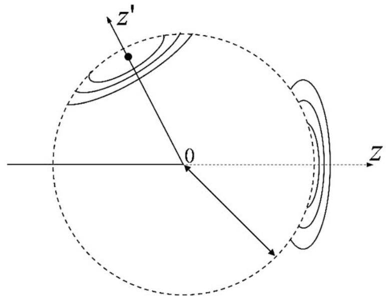

Fig. 2. – The field surrounding the electron after its momentary scattering to a large angle.

around the electron that had traversed the plate is analogous to the field structure around the scattered electron (second item in (17)).

Such structural similarity of the considered fields causes analogous effects in the backward transition radiation and in the forward bremsstrahlung in the discussed processes as well as in the transition radiation by the "half-bare" scattered electron and the electron that had traversed the ideally conducting plate. Namely, in the case when characteristic transversal distances that are responsible for radiation process have macroscopic size, the results of measurements of the characteristics of both backward transition radiation and forward bremsstrahlung should substantially depend on the size of the used detector and its position [20]. Particularly, analogous effects of broadening of the radiation angular distribution in the case of measurements by point detector in the pre wave zone take place for both these types of radiation.

The transition radiation by scattered "half-bare" electron on a thin metallic plate placed in the direction of the scattering is strongly suppressed if the distance between the plate and the scattering point is smaller than the radiation formation length ($l_C \approx 2\gamma^2/\omega$, ω is the measured radiation frequency) [20]. It is analogous to the effect of radiation suppression during further scatterings of the electron which cause the phenomena of radiation suppression in substance (Landau-Pomeranchuk-Migdal effect [21], TSF-effect [22-24]).

A similar effect of radiation suppression is observed for the transition radiation by an electron that had traversed or passed the thin metallic plate [15].

5. – Conclusion

The process of transition radiation which occurs during the fast electron's normal traverse of a thin ideally conducting plate is considered. The space-time structure of electromagnetic-field potentials before and after the electron's traverse of the plate is investigated. It is shown that before the electron's traverse of the plate the field in the left half-space, where the electron is situated, is a sum of the moving electron's own Coulomb field and the field of its image. In the right half-space, the field is absent in this case.

After the electron's traverse of the plate the electromagnetic-field structure substantially changes. In the left half-space the field reflected from the plate arises. This field is a packet of free waves that move oppositely to the direction of the electron's motion. This wave packet gradually transforms into the packet of radiation waves which diverge from the plate. During this process at distances $r > t$ from the traverse point the total field is a sum of the field of electron's image inside the plate, which continues moving in the direction opposite to the direction of the electron's motion, and the electron's own

Coulomb field in this half-space. At distances $r < t$, which the signal about the electron's traverse of the plate has already reached, the electromagnetic field is absent.

In the right half-space, where the moving electron is situated, the picture of fields distribution is opposite to the one of the reflected field. Namely, in the region $r > t$ the electromagnetic field equals zero, while in the region $r < t$ it is a sum of the moving electron's own Coulomb field and the field of its image.

The jump of the field on the sphere $r = t$ defines all the characteristics of transition radiation process.

In the paper it is shown that the structure of the electromagnetic fields, which arise after the electron's traverse of a thin ideally conducting plate is similar to the structure of the fields which take place at the electron's momentary scattering to a large angle. It leads to the fact that there should be analogous effects in radiation in both of these processes.

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