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Polarization bremsstrahlung by fast charge on atomic bound electrons—Analog of nuclear Mössbauer's effect

V. K. Grishin(*) and D. P. Nikitin
Skobel'tsyn Institute of Nuclear Physics, Lomonosov Moscow State University - Moscow, Russia

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Summary. — Features of polarization bremsstrahlung radiation (PB) by a relativistic charge on medium electrons bound in atoms are discussed. PB is considered as a dispersion of virtual photons of an electromagnetic field of a fast charge on atomic bound electrons. In this case atomic electron can get dispersed at a recoil energy in only certain portions, as in the nuclear Mössbauer process. Because of this a spectrum of dispersed photons is degenerated in a series of narrow peaks.

PACS 41.60.-m – Radiation by moving charges.
PACS 03.50.-z – Classical field theories.

1. – Introduction

Recently PB by fast electrons on medium electrons has been actively discussed since PB is rather sensitive to the features of electronic configuration. Consequently it can be used, *e.g.*, for diagnostics of a substance structure. One of the ways of PB characteristics research is an approach, which represents PB as a process of dispersion of virtual photons, "contained" in an electromagnetic field of a fast charge, on atomic electrons in a medium [1-3]. But usually it is proposed that in the process of dispersion the electrons are actually free. Nevertheless, as it follows from the laws of conservation of energy and momentum, the recoil energy transmitted to electrons during interaction with a fast electron field appears as an enough appreciable portion only in the high-frequency area of photons. Therefore, in this case (as opposite to Compton's photon-electron interaction) the dispersion shows interesting features, because probability of transferring a share of pulse and energy (lost by virtual photons during dispersion) to atomic electrons sharply depends on the quantum state of the atom. So, a portion of the energy transmitted to the atomic electron appreciably differs from the energy of bound electron's transition to a new quantum state; the momentum can be transferred during dispersion to the whole atom, *i.e.* actually without transfers of energy due to ruggedness of atom. Probability of

(*) E-mail: vkgris@mail.ru

a similar dispersion is strongly suppressed, but it becomes strong when the specified portions of energy coincide. To sum up, the spectrum of dispersed photons (*i.e.* irradiated photons) results in the peak structure of PB on bound electrons.

Thus, a radiation spectrum is displaced in a X-ray range as it follows from kinematic ratios and also it is divided into two groups, which range poorly and strongly depend on the energy of a fast charge.

Specified PB features on bound electrons are described below on an example of hydrogen-like one-electronic atom.

2. – Analytical description: a stream of virtual waves

So, let us present electromagnetic field of a fast charge (a relativistic electron) as a stream of virtual photons of various frequencies, addressing to the following scheme. Fast electron at a speed $\vec{v}$, with a charge e and relativistic factor $\gamma_e = 1/\sqrt{1 - v^2/c^2}$ distributed in the medium along some directions z, is scattered by a medium atom. As a result of interaction of a projectile with one or several atomic electrons (further the process of scattering of a fast electron field is considered), a photon is emitted under the angle ϑ along the unit vector $\vec{n'}$.

The electric field of a fast electron with spatial density of charge $\rho = e\delta(\vec{r} - \vec{v}t)$ can be submitted as a stream of plane waves

$$E(t, \vec{r}\,) = \int E_{\omega,\vec{k}} \exp\left[i(\vec{k}\vec{r} - \omega t)\right] \mathrm{d}\omega\, \mathrm{d}\vec{k}. \tag{1}$$

However, in this case it becomes useful to describe the axial-symmetric field as a wave stream (a stream of pseudo- or virtual photons; their velocities coincide with the velocity of fast electrons) with indication of coordinate dependence in transverse planes [4]:

$$\vec{E}(t, \vec{r}_\perp, z) = \int \vec{E}_\omega(\vec{r}_\perp) \exp\left[i(k_z z - \omega t)\right] \mathrm{d}\omega\, \mathrm{d}k_z, \tag{2}$$

where $\vec{r} = (\vec{r}_\perp, z)$; $r_\perp$ is the radius-vector transverse to the trajectory plane.

Using the ratio for the electric field of a fast electron (see for example [4, §18.5]), we can write for the Fourier components:

$$\vec{E}_\omega(r_\perp, z) = \left(\vec{e_\perp} \frac{e\zeta K_1(\zeta)}{\pi\varepsilon(\omega) v r_\perp} + \vec{e}_z \frac{e\zeta K_0(\zeta)}{\pi\varepsilon(\omega) v \gamma_\otimes r_\perp}\right) \exp[ik_z z], \tag{3}$$

where $\zeta = \omega r_\perp / v\gamma_\otimes$.

Here $\vec{e}_\perp, \vec{e}_z$ are the transverse and longitudinal unit vectors, accordingly, $\varepsilon(\omega) = 1 - \chi$ is the dielectric permeability of the medium crossed by the electron, $\chi = \omega_0^2/\omega^2, \omega_0$ is the plasma frequency of the medium ($\omega_0 \simeq 30$–$40\,\mathrm{eV}$ for various substances), $1/\gamma_\otimes^2 = 1/\gamma_e^2 + \chi\beta^2, \beta = v/c$, $K_{0,1}$ are the Macdonald functions. The wave stream is distributed along the axis Z with velocity v and wave vector $k_z = \omega/v$, *i.e.* $\vec{k} = \omega\vec{v}/v^2$.

In the description (3) the sizes of "a central stain" of the virtual stream are precisely traced and are defined by the ratio $\zeta \leq 1$, as at the big values of argument $K_{0,1} \to 0$. The radius of the spot being

$$R_s \simeq \beta\gamma_\otimes c/\omega \to \beta\gamma_\otimes \lambda/2\pi, \tag{4}$$

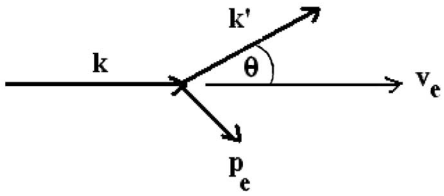

Fig. 1. – Scheme of the virtual photon (wave vector $\vec{k}$) dispersion on the electron. $\vec{k}'$ is the wave vector (real) of the dispersed photon, $\vec{p_e}$ is the recoil momentum transmitted to the electron.

where λ is the wavelength. The same ratio defines the width of virtual frequencies spectrum and its change in the process of radiation separation from electron trajectory.

It is typical that spectra for longitudinal and transverse parts of the fields differ by the frequency dependence as well as by the intensity one. As $1/\gamma_{\otimes} \ll 1$ even for moderately relativistic electrons, the contribution of longitudinal fields in (3) as a whole is small, see for example [5].

3. – Scattering of virtual photons by electron

Scattering of a virtual photon with initial momentum $\vec{k} = \omega\vec{v}/v^2$ and a real photon with momentum $\vec{k}' = \omega'\vec{n}'/c$, where the velocity $\vec{v}$ of the fast electron is proposed to be constant during the dispersion (here $\hbar = 1$; ω and ω' are the frequencies of a photon before and after scattering, accordingly, c is the velocity of light in vacuum), is described by the ratio (see fig. 1)

$$\vec{k} = \vec{k}' + \vec{p_e}, \tag{5}$$

or

$$\frac{\omega\vec{v}}{v^2} = \frac{\omega'\vec{n}'}{c} + \vec{p_e}, \tag{6}$$

where $\vec{p_e}$ is the momentum transmitted to the electron.

As $\omega - \omega' \ll \omega$ (that is quite natural in the range of moderate energies of photons as the latter are more “simple particles” in comparison with electrons), we obtain

$$\omega = \frac{|vp_e \cos\vartheta|}{1 - \beta\cos\vartheta}\,, \tag{7}$$

where $\beta = v/c$.

Further using the relation $(k')^2 = k^2 + p_e^2 - 2(\vec{p_e}\vec{k})$ we can derive the ratio for electron transmitted energy $E_{\text{rec}} = p_e^2/2m_e$:

$$E_{\text{rec}} = \frac{\omega^2}{2m_e\beta^2c^2}(1 - 2\beta\cos\vartheta + \beta^2). \tag{8}$$

At $\gamma_e^2 \gg 1$ from the ratio (7) it follows that the maximal ($\vartheta \to 0$) and minimal ($\vartheta \to \pi$) values of photon energy are equal according to (see also [6])

$$\omega_{\max} \simeq 2|(\vec{v}\vec{p}_e)|\gamma_e^2, \tag{9}$$

$$\omega_{\min} \simeq \frac{p_e c}{2}\,. \tag{10}$$

Thus the dispersion of virtual photons appreciably differs in frequency-angular dependence on the classical Compton dispersion of real photons. First of all, we specify relativistic effects that was in particular marked on an example of the parametrical radiation in [6] (see also [7]). We remind that energy transmitted to electron (recoil energy) remains appreciably smaller than energies of virtual and irradiated photons.

4. – PB due to bound atomic electron. Predictions

As underlined above, one of the traditional ways to estimate the PB intensity is based on the classical representation of this process as a dispersion of a stream of virtual photons of fast charge on atomic electrons. Appropriate analytical PB description follows, for example, from the procedure stated in [8]. As a result, PB spectral-angular intensity at charged particle scattering by the atom with Z electrons is equal to

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{e^4}{8\pi m^2 c^3}\left|\sum_{s=1}^{\infty}\left[\vec{n'}\vec{E}_{\omega,s}\right]\exp\left[i\vec{p}_e\vec{r'}_s\right]\right|^2, \tag{11}$$

where $\mathrm{d}\Omega$ is the element of a solid angle, $\vec{r}_s$ are the electron radius-vectors within the atom frame. Vector product $[\mathbf{n'E_{\boldsymbol{\omega},s}}]$ is calculated in conformity with the ratio (1) (therefore, atomic electrons have various values of transverse radii (*i.e.* impact parameters), $r_{\perp,s}$; in the chosen system of coordinates $r^2_{\perp,s} = x_s^2 + y_s^2$).

Summing in (10) is distributed over all atomic electrons which are "covered" by the virtual stream. Therefore, the ratio (10), being divided into a stream intensity of virtual photons, represents the effective section of PB process.

However, this classical description does not take into account that atomic electrons are in bound states. And though, the atomic bound weakly affects the accuracy of the ratios (7) and (8), the recoil energy E_{rec} can be transferred effectively to atomic electrons only by certain portions, appropriate for transition of particles to a new quantum state (obviously, it is valid if E_{rec} does not exceed the energy of electrons ionization). In intermediate state the transfer of energy results actually in a virtual excitation of atom. And here it becomes interesting to compare discussed problem and situation when nuclear Mössbauer effect [9] is observed.

The effective section (*i.e.* probability) of a similar process in the elementary nuclear system-atom of hydrogen or deeply "cleared" (bared) more complex atom, *i.e.* one-electron ion, is described following [9, 10] by the ratio (here the most important resonant factor is only underlined)

$$P \propto \frac{\Gamma_s}{(E_{1s} - E_{\text{rec}})^2 + \Gamma_s^2/4}\,, \tag{12}$$

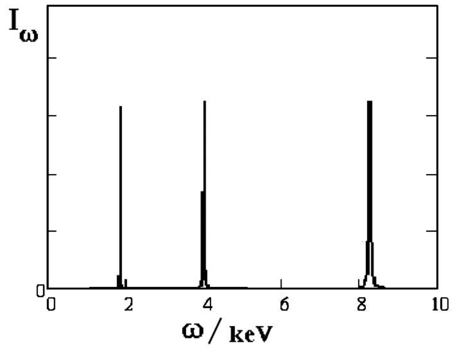

Fig. 2. – Intensity I_ω and PB peak locations at $\gamma = 20$ with different angles of radiation $\theta = \pi$, $\pi/4$, $\pi/8$ (from left to right).

where E_{1s} is the energy difference of electronic states 1 and s, Γ_s is the full width of resonance, directly connected to a constant of disintegration of the excited state (lifetime of this state). Nevertheless, a rather important specification appears in (11) because here a recoil energy transmitted to electron appears directly, see ratio (11) in [10].

Being limited to similar representations, it is necessary to admit that the basic PB features here are defined by the product of a stream of virtual photons representing the relativistic electron field and the effective section (11). As the lifetime of excited states is small enough (for hydrogen about 10^{-8} s), PB spectrum should degenerate actually in a set of narrow peaks corresponding to various excited states.

For numerical calculations we have to be limited to the approached ratio. So, for electrons in the basic state of the hydrogen atom, the average size of a field E_ω in a virtual stream is of the order (as follows from the ratios (3) and (4))

$$\langle E_\omega \rangle \simeq \frac{2e(1 - \exp[-\Omega_0 \gamma_e/\omega])}{\pi c a_0}\,, \tag{13}$$

where $\Omega_0 = c/a_0 \simeq 4\,\mathrm{keV}$, a_0 is Bohr's radius. Further it is possible to take advantage of the ratio (11).

Thus, there is an additional interesting effect: due to the return of transition-excited atomic electron, characteristic radiation (CR) can be emitted. Paradoxical situations: the frequencies of peaks PB and CR essentially differ, though it would seem both peaks are defined by the same energy transition of the atomic electron. But in reality, for PB peak position is defined by the ratio (8) and only for CR by the energy E_{1s}. Thus, total radiation at the fixed angle of radiation contains two groups of peaks, namely, soft (CR) and hard (PB). Some results of the obtained estimations are presented in fig. 2.

5. – Conclusion

Thus, consideration of polarization bremsstrahlung radiation on bound electrons enriches in a certain extent the traditional picture of the description of the radiating interaction of relativistic electrons with substance. Hence, the analog between considered PB processes and Mössbauer's effect becomes evident.

Of course, the analysis carried out is open for the discussion. It certainly requires specification and wider scope of the object of research. So one of the obvious moments is searching in the considered scheme the conditions for coherent radiations in multi-electron atoms and other collective effects.

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REFERENCES

[1] FERMI E., *Z. Phys.*, **29** (1924) 315.
[2] TER-MIKAELIAN M. L., *High Energy Electromagnetic Processes in Condensed Media* (Wiley, New York) 1972.
[3] AMUS'IA M., BUIMISTROV V., ZON B. *et al.*, *Polarization Bremsstrahlung of Particles and Atoms* (Plenum Press, New York) 1992.
[4] PANOFSKY W. K. H. and PHILLIPS M., *Classical Electricity and Magnetism* (Addison-Wesley Publishing Company, Inc., Cambridge) 1962, §18.5.
[5] GRISHIN V. K., ISHKHANOV B. S., SHVEDUNOV V. I. and NASONOV N. N., *Coherent radiative diagnostic of nanostructure parameter by electron accelerators//vestnik MGU*, Vol. **3**, *Phys. Astron.* (2007) No. 6, pp. 29-33.
[6] BARYSHTVSKY V. G., FERANCHUK I. D. and ULYEANENKOV F. P., *Parametric X-ray Radiation in Crystals*, ISBN-10 (Springer Berlin, Heidelberg, New York) 2005.
[7] BASYLEV V. A. and ZHEVAGO H. K., *Izluchenie bystrykh chastits v veshchestve i vo vneshnikh polyakh* (Nauka, Moscow) 1987.
[8] LANDAU K. D. and LIFSHITS E. M., *Field Theory* (Nauka, Moscow) 1960, §80.
[9] WERTHEIM G. K., *Mössbauer Effect. Principles and Application* (Academic Press, New York) 1964.
[10] AKHIESER A. I. and BERESTETSKI V. B., *Quantum Electrodynamic* (Nauka, Moscow) 1969, §27.2.