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Acoustic wave controlled X-ray diffraction and emission processes in crystals

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Summary. — Possibilities to exert control over electromagnetic processes in a crystal by an acoustic wave are considered. The influence of such a wave on the parametric X-ray radiation and coherent bremsstrahlung from relativistic electrons in a crystal are studied in this paper.

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1. – Introduction

The development of X-ray–based technologies causes an increasing interest in high energy electromagnetic processes in crystals with lattice periodically deformed by the action of internal fields (for example, by an acoustic wave). The possibility to change yields of such processes is one of the main aims of theoretical and experimental studies conducted in this field.

The influence of an acoustic wave on coherent bremsstrahlung has been considered in works [1, 2] (it should be noted that this problem connects strongly with that known as "crystalline undulator", see review [3] and references therein). Some works are devoted to the coherent electron-positron pair production in a crystal, excited by an acoustic wave [4, 5], diffraction of free photons [6, 7], parametric X-rays (PXR) [8-10].

It is good to bear in mind that the studies of the above processes are developing at present, which is why some experimental results have no adequate theoretical explanation up to now (this is especially true in regard to the effect of PXR yield amplification under the action of acoustic wave [10], or to the similar effect of X-ray reflection increasing).

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The aim of our work is to develop a simple kinematical approach to the description of PXR. In addition to this the modification of crystalline undulator scheme is proposed and studied in the work.

The paper is organized as follows. The modified crystalline undulator is considered in sect. **2**. The next section is devoted to the problem of PXR in periodically deformed crystalline lattice.

Relativistic system of units $\hbar = c = 1$ is used in our calculations.

2. – Mechanism of coherent bremsstrahlung enhancement by acoustic wave in conditions of incident electron interaction with a gas of deformed atomic strings

The traditional approach to the realization of the crystalline-undulator–based source under discussion suggests the emission from relativistic positrons channeling in a system of periodically deformed atomic planes of the crystal. The main disadvantage of this scheme consists in a small achievable length of such source caused by dechanneling processes.

Another way to realize gamma-ray source based on the crystalline undulator consists in the use of above-barrier particle emission in the potential of periodically deformed atomic planes in the crystal [2]. Such scheme has no need of an emitting particle channeling and hence one can increase the emission yield substantially using electron beam instead of low-power positron beam. It should be noted in this connection that coherent bremsstrahlung (CB) from relativistic electrons moving in a crystal with periodically deformed crystalline lattice was studied before within the frame of the more general problem of controlling the characteristics of electromagnetic processes in crystals by the influence of external fields, such as acoustic waves, temperature gradient, etc. [1]. Performed studies have shown that the interaction of fast electrons with deformed atomic strings allows one to realize more intensive emission than that with deformed atomic planes [2] and so CB from relativistic electrons on a system of periodically deformed atomic strings is the subject for our study. The aim of the paper is to show the possibility to realize a new scheme of crystalline undulators, permitting one to produce intensive quasi-monochromatic X-rays in the range of about 10 keV and more by electron beams with energies of the order of 100 MeV.

Let us consider bremsstrahlung from relativistic electrons moving in a crystalline target near parallel to one of the main crystallographyc axes. Since the emission of photons with energies ω much less than the energy ε_0 of an emitting electron is of interest in the process being studied, classical electrodynamics may be used for the determination of the emission amplitude $\mathbf{A}$:

$$\mathbf{A} = \frac{i\omega e}{2\pi} \int \mathrm{d}t \left(\mathbf{V}(t) - \frac{\mathbf{n}}{\sqrt{\varepsilon}} \right) \exp\left[i\omega(t - \sqrt{\varepsilon}\mathbf{n} \cdot \mathbf{r}(t)) \right], \tag{1}$$

where $\mathbf{V} = \mathrm{d}\mathbf{r}/\mathrm{d}t$, $\mathbf{r}(t)$ is the trajectory of the emitting electron, ε is the target dielectric permeability, $\mathbf{n}$ is the unit vector to the direction of the emitted photon propagation. Supposing the characteristic angle of electron scattering by atomic string to be less than $\gamma^{-1} = m/\varepsilon_0$ we will use the approximation of rectilinear trajectory $\mathbf{r}(t) = \mathbf{V}t + \mathbf{r}_0$ to calculate the integral (1). The result of calculations can be expressed in terms of the potential of target atoms

$$\mathbf{A} = \frac{ie^3}{2\pi^2\varepsilon_0}\frac{1}{1-\sqrt{\varepsilon_0}\mathbf{n}\cdot\mathbf{V}}\int\frac{\mathrm{d}^3k}{k^2}\left(Z-F(k)\right) \times\sum_n \exp\left[i\mathbf{k}(\mathbf{r}_n-\mathbf{r}_0)\right]\mathbf{T}(\mathbf{k})\delta\left(\omega(1-\sqrt{\varepsilon}\mathbf{n}\cdot\mathbf{V})-\mathbf{k}\cdot\mathbf{V}\right), \tag{2}$$

$$\mathbf{T}(\mathbf{k}) = \mathbf{k}-\mathbf{V}\mathbf{k}\cdot\mathbf{V}-\frac{\mathbf{n}-\sqrt{\varepsilon}\mathbf{V}}{1-\sqrt{\varepsilon}\mathbf{n}\cdot\mathbf{V}}\left(\mathbf{n}\cdot\mathbf{k}-\mathbf{n}\cdot\mathbf{V}\mathbf{k}\cdot\mathbf{V}\right).$$

Here Z is the atomic number, $F(k)$ is the form factor of an atom, $\mathbf{r}_n$ is the coordinate of the n-th atom, the summation is over all atoms in the target.

Performing the averaging over $\mathbf{r}_n$ one should take into account the periodical displacement of atoms in a crystalline lattice (for simplicity's sake assume that only one atom is placed in an elementary cell) and the perturbation of the crystalline lattice by acoustic wave, so that the coordinate $\mathbf{r}_n$ may be presented in the form

$$\mathbf{r}_n = \mathbf{R}_n + \mathbf{u}_n + \mathbf{a}\sin(\bar{\xi}\cdot\mathbf{R}_n), \tag{3}$$

where $\mathbf{R}_n$ is the equilibrium position of the n-th atom in the lattice, $\mathbf{u}_n$ is its thermal displacement, $\mathbf{a}$ and $\bar{\xi}$ are the amplitude and wave vector of the acoustic wave, respectively. With account of (3) one can obtain from (2) the following expression for the emission density:

$$\omega\frac{\mathrm{d}N^{\mathrm{coh}}}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}\Omega} = \frac{8\pi Z^2e^6n_0^2}{\varepsilon_0^2}\frac{1}{(1-\sqrt{\varepsilon}\mathbf{n}\cdot\mathbf{V})^2} \times\sum_{\mathbf{g}}\sum_l\frac{\exp\left[-g_l^2u_T^2\right]}{\left(g_l^2+R^{-2}\right)^2}J_l^2(\mathbf{g}_l\cdot\mathbf{a})T^2(\mathbf{g}_l)\delta\left(\omega(1-\sqrt{\varepsilon}\mathbf{n}\cdot\mathbf{V})-\mathbf{g}_l\cdot\mathbf{V}\right), \tag{4}$$

where $\mathbf{g}_l = \mathbf{g} - l\vec{\xi}$, the simplest statistical model of an atom with exponential screening is used in (4), R is the screening radius in the Fermi-Thomas model.

The result (4) is a basic one for further analysis. Obviously, in the limiting case $a \to 0$ only item with index $l = 0$ takes the contribution to the sum (4). As this takes place, the result (4) is reduced to the well-known formula for traditional coherent bremsstrahlung from relativistic electrons moving in a crystal with rectilinear atomic strings.

Let us use the result (4) for studies of the influence of acoustic wave on CB characteristics. It is convenient for further analysis to simplify (4) with account of the property $g \gg l\xi$ allowing one to neglect the difference between $\mathbf{g}$ and $\mathbf{g}_l$ in the argument of Bessel function in (4). The wavelength of acoustic wave exceeds essentially the distance between atoms in the target and hence the above difference is not substantial throughout this formula except the argument of the δ-function.

Since the coherent contribution of string atoms to the formation of bremsstrahlung yield provides the basis for the emission mechanism in question, one should reserve in the sum $\sum_{\mathbf{n}}$ in (4) summation over transversal relative to string axis $\mathbf{e}_x$ components of reciprocal lattice vectors $\mathbf{g}_\perp$ only. Introducing the angular variables $\vec{\theta}$ and $\vec{\psi}$ by the expressions

$$\mathbf{V} = \mathbf{e}\left(1 - \frac{1}{2}\gamma^{-2}\right), \quad \mathbf{n} = \mathbf{e}\left(1 - \frac{1}{2}\theta^{2}\right) + \vec{\theta}, \quad \mathbf{e}\cdot\vec{\theta} = 0,$$
$$\mathbf{e} = \mathbf{e}_x\left(1 - \frac{1}{2}\psi^{2}\right) + \vec{\psi}, \quad \mathbf{e}_x\cdot\vec{\psi} = 0, \tag{5}$$

one can bring (4) into the form

$$\omega\frac{\mathrm{d}N^{\mathrm{coh}}}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}^2\Omega} = \frac{32\pi Z^2 e^6 n_0^2}{m^2\gamma^2(\gamma_*^{-2}+\theta^2)^2}\sum_{\mathbf{g}_\perp}\sum_{l}\frac{\exp\left[-g_\perp^2 u_T^2\right]}{(g_\perp^2+R^{-2})^2}J_l^2(\mathbf{g}_\perp\cdot\mathbf{a}_\perp)$$
$$\times\left(g_\perp^2 - \frac{4\gamma_*^{-2}(\vec{\theta}\cdot\mathbf{g}_\perp)^2}{(\gamma_*^{-2}+\theta^2)^2}\right)\delta\left(\frac{\omega}{2}(\gamma_*^{-2}+\theta^2) + l\xi_x - \mathbf{g}_\perp\cdot\vec{\psi}\right), \tag{6}$$

where $\gamma_*^{-2} = \gamma^{-2} + \omega_0^2/\omega^2$, ω_0 is the plasma frequency of the target, the ordinary approximation for dielectric permeability is used $\varepsilon = 1 - \omega_0^2/\omega^2$.

Obviously, the emission intensity (6) increases with decreasing orientation angle ψ between string axis and the velocity direction of the emitting electron. On the other hand, there is a strong azimuth scattering of the projectile on the string potential in the range of small orientation angles $\psi \approx \psi_{\mathrm{ch}}$ (ψ_{ch} is the channeling angle), which is why the correlations between consecutive collisions of a fast electron with atomic strings are destroyed and hence such collisions will become accidental. Summation over $\mathbf{g}_\perp$ in (6) can be replaced by integration in the conditions under consideration ($\sum_{\mathbf{g}_\perp} \to (d_\perp/2\pi)^2\int \mathrm{d}^2 g_\perp$, $d_\perp$ is the average distance between atomic strings). The result of integration may be presented as

$$\omega\frac{\mathrm{d}N^{\mathrm{coh}}}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}\theta} = \frac{8\pi Z^2 e^6 n_0^2}{m^2\gamma^2}\frac{R}{d_x\psi}\frac{\gamma_*^{-4}+\theta^4}{(\gamma_*^{-2}+\theta^2)^4}\sum_l J_l^2(\eta_l)\frac{1+2\eta_l^2(R/a_\perp)^2}{(1+\eta_l^2(R/a_\perp)^2)^{3/2}},$$
$$\eta_l = \frac{\omega a_\perp}{2\psi}(\gamma_*^{-2}+\theta^2) + l\frac{\xi_x a_\perp}{\psi}. \tag{7}$$

Thermal vibrations of crystal atoms are neglected in formula (7) describing CB from relativistic electrons moving in a gas of deformed atomic strings (the characteristic coefficient $R/d_x\psi$ shows the number of string atoms making coherent contribution to the formation of CB yield on a single string, d_x is the distance between neighboring atoms in the string). Formula (7) allows one to describe the influence of acoustic waves on CB spectral-angular properties. Let us consider the orientation dependence of strongly collimated CB. In conditions $\theta^2 \ll \gamma_*^{-2}$ under consideration formula (7) can be presented in the form

$$\omega\frac{\mathrm{d}N^{\mathrm{coh}}}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}\theta} = \frac{8\pi Z^2 e^6 n_0^2}{m^2}\gamma^2\frac{R}{d_x\psi_*}F\left(x,\beta,\tau,l\frac{\psi_*}{\psi}\right), \tag{8}$$

$$F = \left(\frac{x}{x+x^{-1}}\right)^2\frac{\psi_*}{\psi}\sum_l J_l^2\left(\beta(x+x^{-1}) + l\frac{\psi_*}{\psi}\right)\frac{1+2\tau^2\left(\beta(x+x^{-1})+l\frac{\psi_*}{\psi}\right)^2}{\left(1+\tau^2\left(\beta(x+x^{-1})+l\frac{\psi_*}{\psi}\right)^2\right)^{3/2}}$$

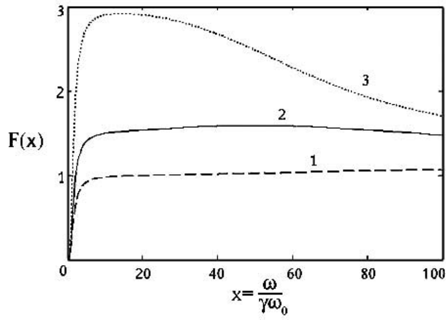

Fig. 1. – Spectrum of strongly collimated CB in a crystal with periodically deformed strings *vs.* the orientation angle $\psi > \psi_*$. The curve 1 corresponds to ordinary CB spectrum described by the function $F_0 = (\frac{x}{x+x^{-1}})^2 \frac{1+2\tau^2(\beta(x+x^{-1}))^2}{(1+\tau^2(\beta(x+x^{-1}))^2)^{3/2}}$. The curves 2 and 3 have been calculated for fixed values of the parameters $\beta = 0.05$, $\tau = 0.1$ and different values of the parameter $\psi_*/\psi = 0.4$ (curve 2) and $\psi_*/\psi = 0.95$ (curve 3).

much convenient for numerical calculations, here $x = \omega/\gamma\omega_0$, $\beta = \omega_0 a_\perp/2\gamma\psi$, $\psi_* = \xi_x a_\perp$, $\tau = R/a_\perp$. The universal function $F(x)$ describing the form of CB spectrum has been calculated for fixed values of the parameters β and τ and different values of the parameter $(\psi_*/\psi) < 1$. Results of calculations are demonstrated in fig. 1. The curve 1 corresponds to an ordinary CB in the potential of rectilinear strings. Curves 2 and 3 have been calculated for growing values of the ratio ψ_*/ψ (amplitude of acoustic wave $a_\perp$ was chosen to be of the order of the distance between atoms, the orientation angle ψ used in calculations was about γ^{-1} and $\gamma \sim 100$). The presented curves 2 and 3 demonstrate the growth of collimated CB yield for $\psi \to \psi_*$. To elucidate this phenomenon let us consider the same orientation dependence but in the range $\psi < \psi_*$. Corresponding curves are presented in fig. 2. According to the this figure the yield peaks in the vicinity of $\psi = \psi_*$ as well, but the achieved enhancement coefficient is greater than that in conditions $\psi > \psi_*$ by a factor of 2. In addition to this, the strong oscillations appear in CB spectrum under the conditions $\psi < \psi_*$ being considered.

The indicated properties are caused by the mechanism of emission enhancement. The greatest contribution to electron CB yield is formed at the parts of its trajectory,

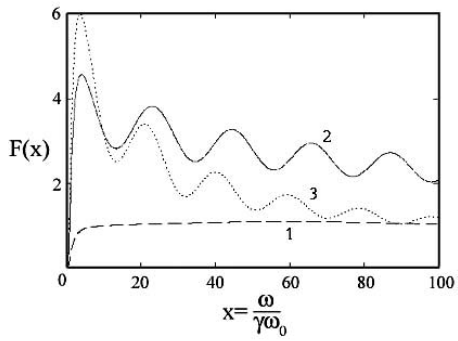

Fig. 2. – The same but for $\psi < \psi_*$, fixed parameters $\beta = 0.24, \tau = 0.05$ and different values of the parameter $\psi_*/\psi = 4$ (curve 2) and $\psi_*/\psi = 1.1$ (curve 3).

parallel to the local direction of bended string axis. Such interaction is not possible under conditions of CB in the potential of rectilinear strings (CB cross-section increases without limits when $\psi \to 0$) and hence it is this case of periodically deformed strings that offers the realization of necessary conditions. It is clear that the length of the above parts of an emitting electron trajectory peaks in the vicinity of $\psi = \psi_*$ (obviously, the angle ψ_* is the maximum angle of a string axis bending). That is the reason why the maximum of CB yield is achieved when $\psi = \psi_*$. One can readily see that an emitting electron has only one intersection with a single string in the range $\psi > \psi_*$, which is why the corresponding emission spectrum has only one maximum (maximum is formed due to the influence of Ter-Mikaelian effect of dielectric suppression). On the other hand, there are several above intersections in the range $\psi < \psi_*$. As a consequence, the interference between elementary waves emitted from different parts of electron trajectory corresponding to different intersections is responsible for oscillations in the emission spectrum.

3. – On the influence of an acoustic wave on parametric X-rays

Diffraction of virtual photons associated with a fast charged particle Coulomb field (PXR process) is considered in this section.

Let us consider PXR from relativistic electrons moving with a constant velocity $\mathbf{V}$ in a crystal excited by an acoustic wave. The initial equation includes the item in the left side describing the induced current density of target electrons, calculated in the high energy limit

$$(k^2 - \omega^2)\mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}(\mathbf{k}\cdot\mathbf{E}_{\omega\mathbf{k}}) + \int \mathrm{d}^3k' G(\mathbf{k'}-\mathbf{k})\mathbf{E}_{\omega\mathbf{k'}} = \tag{9}$$

$$= \frac{i\omega e}{2\pi^2}\mathbf{V}\exp\left[-i\mathbf{k}\cdot\mathbf{r}_0\right]\delta(\omega - \mathbf{k}\cdot\mathbf{V}),$$

$$G(\mathbf{k'}-\mathbf{k}) = \frac{Ze^2}{2\pi^2 m}\frac{1}{1+(\mathbf{k'}-\mathbf{k})^2R^2}\sum_l \exp[i(\mathbf{k'}-\mathbf{k})\mathbf{r}_l].$$

The mathematical treatments analogous to that in PXR theory allow one to find the following expression for the emission spectral-angular distribution:

$$\omega\frac{\mathrm{d}N_g}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}\Omega} \approx \frac{e^2\omega^2\omega_g^4}{2\pi}\sum_p J_p^2(\mathbf{g}_p\cdot\mathbf{a})\frac{(\mathbf{V}-\mathbf{g}_p/\omega\varepsilon)^2 - (\mathbf{n}\cdot\mathbf{V}-\mathbf{n}\cdot\mathbf{g}_p/\omega\varepsilon)^2}{\left(g_p^2 + 2\omega\sqrt{\varepsilon}\mathbf{n}\cdot\mathbf{g}_p\right)^2} \tag{10}$$

$$\times\delta\left(\omega\left(1-\sqrt{\varepsilon}\mathbf{n}\cdot\mathbf{V}\right) - \mathbf{g}_p\cdot\mathbf{V}\right).$$

Here $\mathbf{g}_p = \mathbf{g} - p\vec{\xi}$, $\omega_g^2 = \omega_p^2\exp[-g^2u^2/2](1+g^2R^2)$. It is convenient for further analysis to introduce the variables $\theta_{||}$, $\theta_\perp$ describing the angular distribution of emitted quanta, and the orientation angle θ' as well.

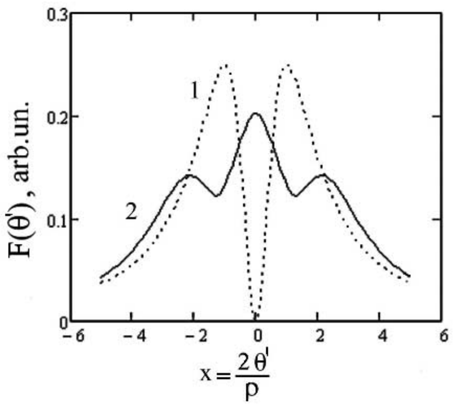

Fig. 3. – Influence of an acoustic wave on PXR angular density. 1–PXR angular dependence in a non-deformed crystal, 2–PXR angular dependence at the present acoustic wave.

Formula (10) has in terms of new variables the form

$$\omega\frac{\mathrm{d}N_g}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}\Omega} \approx \frac{e^2\omega^2\omega_g^4}{\pi g^2}\sum_p J_p^2(\mathbf{g}\cdot\mathbf{a})\frac{\theta_\perp^2+\left(\theta_{||}+2\theta'-2p\frac{\xi_{||}}{g}\right)^2\cos^2\varphi}{\left(\rho^2+\theta_\perp^2+\left(\theta_{||}+2\theta'-2p\frac{\xi_{||}}{g}\right)^2\right)^2}$$
$$\times\delta\left(\omega-\omega_B\left(1+\left(\theta_{||}+\theta'-p\frac{\xi_{||}}{g\cos\varphi/2}\right)\mathrm{ctg}\left(\frac{\varphi}{2}\right)\right)\right), \tag{11}$$

where $\xi_v = \xi_{||}\cos\frac{\varphi}{2} - \xi_\perp\sin\frac{\varphi}{2}$ is the component of acoustic wave vector parallel to the electron velocity, $\rho^2 = \gamma^{-2}+\omega_\rho^2/\omega^2$, γ is the Lorentz factor.

Let us analyze the obtained result. It should be noted that formula (11) is reduced to ordinary distribution of PXR intensity from the non-excited crystal in both limiting cases $a \to 0$, meaning the absence of perturbations and $\xi \to 0$, corresponding to static shift of the crystalline lattice.

Expression (11) shows clearly that the effect appears at once by the bending oscillations of the reflecting plane (only the component of the acoustic wave amplitude being perpendicular to this plane appears in (11)). It is reasonable that the component of $\vec{\xi}$ parallel to reflecting plane gives rise to the angular modulation of emitted photons, because the component $\xi_\perp$ does not result in a bend of the reflecting plane. On the other hand, frequency modulation of PXR is caused by the ξ component parallel to the emitting electron velocity, because it is this component that is responsible for a periodical changing of the distance between consecutive collisions of a fast electron with atoms placed along its trajectory.

In accordance with (11) the physical picture appears as following: the periodical bend of a system of parallel atomic planes determined by the reciprocal lattice vector $\mathbf{g}$ can be represented by changing of a single crystallographic plane to a series of planes with less density turned around each other. As this takes place, the final PXR angular distribution consists of a number of reflexes with the same structure and different amplitudes. These reflexes are shifted relative to each other due to the action of an acoustic wave. It is clear that such quantities as PXR angular density and PXR yield to a finite collimator are free to change under the action of an acoustic wave, but the total yield integrated over observation angles coincides with that for an unperturbable crystal (this statement follows immediately from the formula $\sum_p J_p^2(\mathbf{g}\cdot\mathbf{a}) = 1$).

As an illustration, fig. 3 shows the possibility of radical changes in PXR angular density under the action of an acoustic wave. The orientation dependence of strongly collimated PXR ($\theta_{||}, \theta_\perp < \rho$) is presented by the curves in fig. 3 calculated by the simple formula

$$\frac{\mathrm{d}N_g}{\mathrm{d}t\mathrm{d}\Omega} \approx F(\theta') \sim \sum_p J_p^2(y)\frac{(x-pz)^2}{\left(1+(x-pz)^2\right)^2} \tag{12}$$

following from (11). Here $x = 2\theta'/\rho$, $y = \mathbf{g}\cdot\mathbf{a}$, $z = 2\xi_{||}/g\rho$.

The obtained result allows to present a possible explanation of experiment [10].

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