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Angular distributions of bent-crystal deflected protons

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Summary. — Penetration of relativistic protons into bent crystals at small angles with respect to the bent crystallographic planes has been evaluated within continuous potential approximation. Namely, in this paper the numerical solution of the equation of motion for channeled and quasi-channeled relativistic protons is presented. Proton trajectories under the conditions of both channeling and volume reflection were simulated. The angular distributions of outgoing beam protons are calculated with the parameters of recent CERN experiments. The rather good agreement with experimental data is achieved.

PACS 02.60.Cb – Numerical simulations; solution of equations.
PACS 61.85.+p – Channeling phenomena (blocking, energy loss, etc.).

1. – Introduction

The use of bent crystals to manipulate charge particle beams was first suggested by Tsyganov [1]. At present, while this technique became of great importance as one of the accelerator techniques (see, for example, [2-4] and references therein), the theoretical background has been continuously developing [2, 5-9]. The request for modern computer codes has been determined by the increasing studies of the processes of relativistic projectile scattering in crystals of various geometries. Last experiments on relativistic proton channeling in crystals, especially, crystal collimation experiments at CERN and Fermilab, as well as the experimental results on relativistic heavy ion scattering under the channeling conditions have revealed new features to be additionally investigated.

This work aims at developing a new computer code to simulate the deflection of relativistic protons by means of a bent crystal. In our work the simulation results have been obtained following mainly the model described in [5, 6], but with some distinctions. In particular, in our approach there is no necessity for integrating the proton motion

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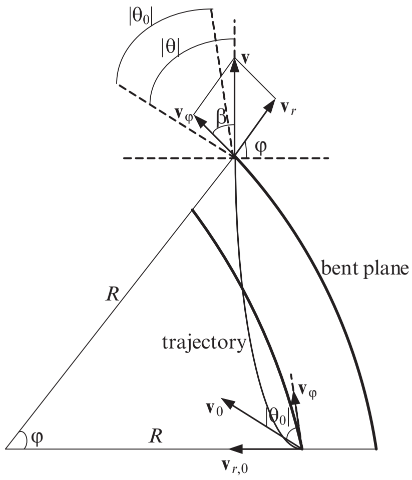

Fig. 1. – The geometrical scheme of proton motion.

equations. We have tested this code to describe the experimental results [10], where the motion of 400 GeV protons through Si crystal bent along (220) planes has been investigated. The angular distributions of protons in the outgoing beam are calculated. The angular divergence for initial proton beam is also taken into account.

2. – Proton motion in a bent crystal planar potential

Let a crystal be bent with the radius R. In this case the continuous planar potential of crystallographic planes is axially symmetric in the transverse cross-section. The motion of a proton, which penetrates into a bent crystal under small angle θ_0 to the crystallographic planes, is governed by an averaged planar potential resulting generally in a rather complicated proton trajectory. In dependence on the initial point of trajectory, the proton can be captured in the planar channeling regime, and, hence, either the projectile follows the bent planes, *i.e.* it channels, or the proton can occur in a quasi-channeling state. In the latter case the proton is not constrained within one planar channel. However, in both cases simple geometrical consideration can be applied to define the angle of deflection from the initial direction (see fig. 1).

Let the proton initial velocity be equal to v_0. The proton azimuthal velocity in the axial field $v_\phi = v_0 \cos\theta_0$ is constant. The evolution of proton radial velocity v_r is defined by the motion equation [5, 6]:

$$\gamma m \frac{\mathrm{d}v_r}{\mathrm{d}t} = -\frac{\mathrm{d}U_{\mathrm{cr}}}{\mathrm{d}r} + F_{\mathrm{C}}\,, \tag{1}$$

where γ is the relativistic factor, and m is the rest mass of the proton; U_{cr} is the crystal planar potential (in our simulations the potential in Molier approximation is used); F_{C} is the centrifugal force. The centrifugal force is determined by the expression

$$F_{\mathrm{C}} = \frac{E_{\mathrm{kin}} + mc^2}{R}\frac{v_0^2}{c^2}\,, \tag{2}$$

where E_{kin} is the proton kinetic energy, c is the speed of light.

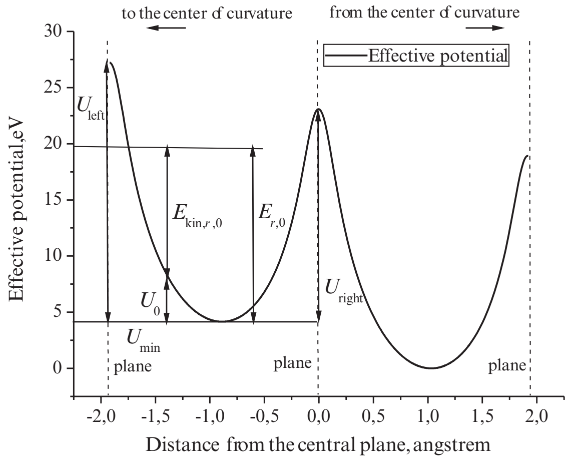

Fig. 2. – The effective radial potential for 400 GeV protons moving through the Si crystal bent along (220) planes with radius 1852 cm (conditions of [10]).

Since the radial coordinate r is counted from the center of curvature, the radial velocity is negative, $v_r < 0$, when proton moves to the center of curvature and positive, $v_r > 0$, in the opposite case. Hence, it is convenient to define $\theta_0 < 0$ if the proton initially moves to the center of curvature, and $\theta_0 > 0$ from the center of curvature. This choice of the sign for angle θ_0 is argued by the sign of angle β between the proton velocity $\mathbf{v}$ and its azimuthal velocity $\mathbf{v}_\phi$ in an arbitrary point of the trajectory (fig. 1)

$$\tan\beta = \frac{v_r}{v_\phi}\,. \tag{3}$$

Finally, defining the proton angular path ϕ, the deflection angle θ from the initial motion direction can be calculated by the formula

$$\theta = \beta - \phi - \theta_0, \tag{4}$$

where the angle β of eq. (3) is derived for the final point of the trajectory and the radial velocity v_r is numerically reduced from the differential equation (1). The suggested method for calculating the deflection angle does not require the integration of the motion equation, and this fact, perhaps, is of great advantage and importance for various simulation applications. The result that $\theta < 0$ means the proton deflection directs to the center of curvature, while the direction changes for the case $\theta > 0$.

Actually, one can see from eq. (1) that the radial proton motion can be presented as a motion in the effective potential (fig. 2) [5, 6]

$$U_{\text{eff}}(r) = U_{\text{cr}}(r) - F_{\text{C}}r + \text{const.} \tag{5}$$

The effective radial potential is characterized by the fact that the left and right barriers of the channel have different heights (see fig. 2). This feature explains the projectile motion in a bent crystal. The character of proton motion is determined by its radial energy at the initial point of the trajectory

$$E_{r,0} = U_0 + E_{\mathrm{kin},r,0}\,, \tag{6}$$

where U_0 is the potential energy at the initial point of the trajectory in a crystal. The second term of the expression $E_{\mathrm{kin},r,0}$ relates to the initial radial kinetic energy

$$E_{\mathrm{kin},r,0} = \frac{1}{2}\left(E_{\mathrm{kin}} + mc^2\right)\frac{v_0^2}{c^2}\sin^2\theta_0. \tag{7}$$

Obviously, the initial radial energy depends on the incident point into the channel. If the proton has the initial radial energy $E_{r,0} < U_{\mathrm{right}}$ (smaller barrier limiting the channel, see fig. 2) the proton is captured in the channeling regime. If the proton has the initial radial energy $E_{r,0} > U_{\mathrm{right}}$, it is quasi-channeled. The deflection angle θ for the channeled proton is close to the bend angle of a crystal α, and, moreover, $\theta < 0$, whereas the deflection angle of the quasi-channeled protons can be either positive or negative. If the angle $\theta_0 < 0$, *i.e.* a proton moves initially to the center of curvature, its radial energy decreases. At some point of the trajectory this energy will equal zero. This point is the turning point. After this moment the proton moves from the center of curvature. And this described feature corresponds to the phenomenon of volume reflection. If angle $\theta_0 > 0$, a proton initially moves from the center of curvature, its radial energy increases. This proton leaves the crystal without reflection.

3. – Angular distributions of deflected protons

Simulated deflection angles of 400 GeV protons passing through the (220) Si bent crystal [10] ($R = 1852\,\mathrm{cm}$, the bend angle of a crystal $\alpha = 162\,\mu\mathrm{rad}$, the critical channeling angle $\theta_L = 10.2\,\mu\mathrm{rad}$), the effective planar potential of which has been demonstrated in fig. 2, are presented in fig. 3. The angular path in eq. (4) is evaluated as $\phi = \alpha$. In figs. 3, a, b, c at the angles $|\theta_0| \le \theta_L$ we deal with channeled protons (that corresponds to the case $E_{r,0} < U_{\mathrm{right}}$ in fig. 2); these protons are deflected by the angles $\theta \approx -\alpha$. In fig. 3, a we have $\theta_0 > 0$, and in the outgoing proton beam the quasi-channeled non-reflected fraction appears, corresponding to the case $E_{r,0} > U_{\mathrm{right}}$ in fig. 2. These protons are deflected slightly by the angles $\theta > 0$. In figs. 3, b, c at the angles $-\theta_L \le \theta_0 \le 0$ the reflected fraction (the case $E_{r,0} > U_{\mathrm{right}}$ in fig. 2) can be revealed together with the channeled protons. In fig. 3, d at the angle $|\theta_0| > \theta_L$ the initial radial energy for all protons is so high that the channeled fraction is absent, and all protons are reflected.

It is important to underline that protons having the close initial radial coordinates (and, therefore, close initial radial energies) form the groups, namely, the inclined spaces at both right and left sides and the horizontal space at channel center, which reflected by the same bent plane in the infinite periodical structure (see fig. 3, d).

The deflection angle θ is not the best parameter for the correct comparison with experimental data. Indeed, the angle θ is counted from the incident proton direction, but there is a large amount of the incident directions, when the beam has the initial angular divergence, whereas the angle between proton motion direction in the outgoing beam and fixed direction can be experimentally obtained.

Let us suppose that the initial beam is characterized by both positions of the beam center θ_C and initial angular divergence $\Delta\theta$ of a beam, and the incident angles θ_0 of protons are uniformly distributed within the interval $(\theta_C - \Delta\theta/2; \theta_C + \Delta\theta/2)$. Let us

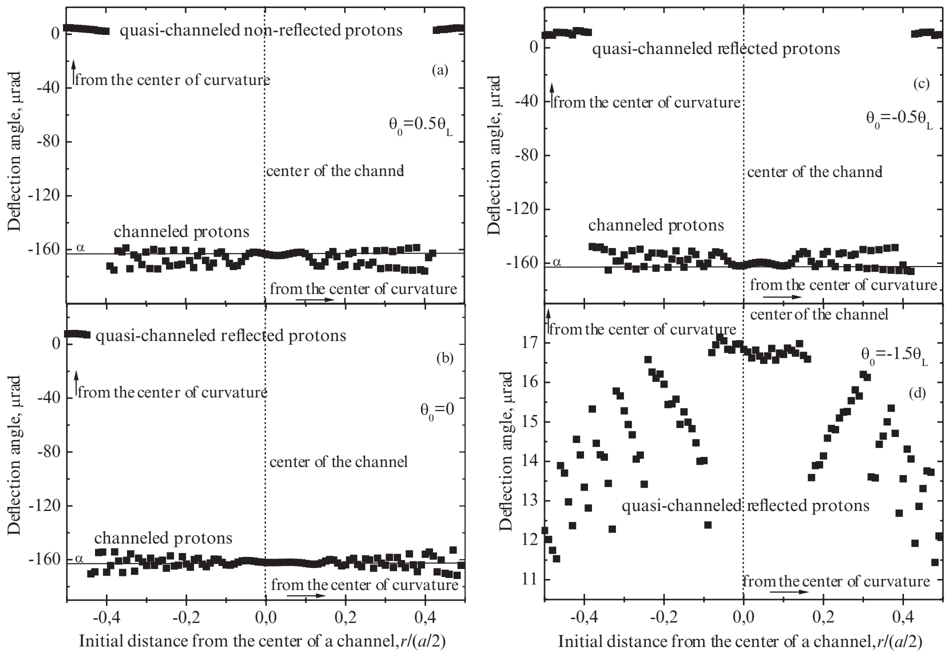

Fig. 3. – Deflection angles of 400 GeV protons in bent (220) Si crystal at the different angles θ_0 as the function of incidence coordinate into the channel; the radius $R = 1852\,\text{cm}$, crystal bend angle $\alpha = 162\,\mu\text{rad}$, a is the interplanar distance, the critical channeling angle $\theta_L = 10.2\,\mu\text{rad}$.

introduce the angle $\delta = \theta + \theta_0 - \theta_C$, which is the angle between the proton motion direction in the outgoing beam and the direction corresponding to the center of the incident beam.

Within this consideration let us describe the experimental data presented in [10], where the initial angular divergence of proton beam was $\Delta\theta = 8\,\mu\text{rad}$. The simulations were carried out at the angles θ_0 within the interval $(\theta_C - \Delta\theta/2; \theta_C + \Delta\theta/2)$ with an angular step of $0.5\,\mu\text{rad}$. For each chosen incident angle θ_0 the 300 trajectories with initial points uniformly distributed across the channel were simulated, and for each trajectory the angle δ was obtained. The space of the angles δ was divided onto sections at $0.2\,\mu\text{rad}$ width. For each section the number of trajectories, for which the angle δ "hits this section", was defined. This number of trajectories is pointed out along the Y-axis, whereas corresponding sections are pointed out along the X-axis. Hence, the obtained histogram presents the angular distributions of protons at a given direction of the center of proton beam, *i.e.* at the angle θ_C. Angular distributions of protons in dependence on angles δ are shown in fig. 4.

The distributions in figs. 4a-c are divided onto two parts: channeled and quasi-channeled protons. The channeled protons are deflected along the channels at angles $\theta \approx \delta \approx -\alpha$. When the absolute value of angle θ_C increases (from fig. 4, a to fig. 4, c), the number of quasi-channeled protons increases due to the increase of initial radial energy. The distribution of quasi-channeled protons shifts to the side of the large angles δ, the width of quasi-channeled protons distribution increases. It should be underlined that in fig. 4, a ($\theta_C = 0$) the quasi-channeled fraction includes not only reflected protons

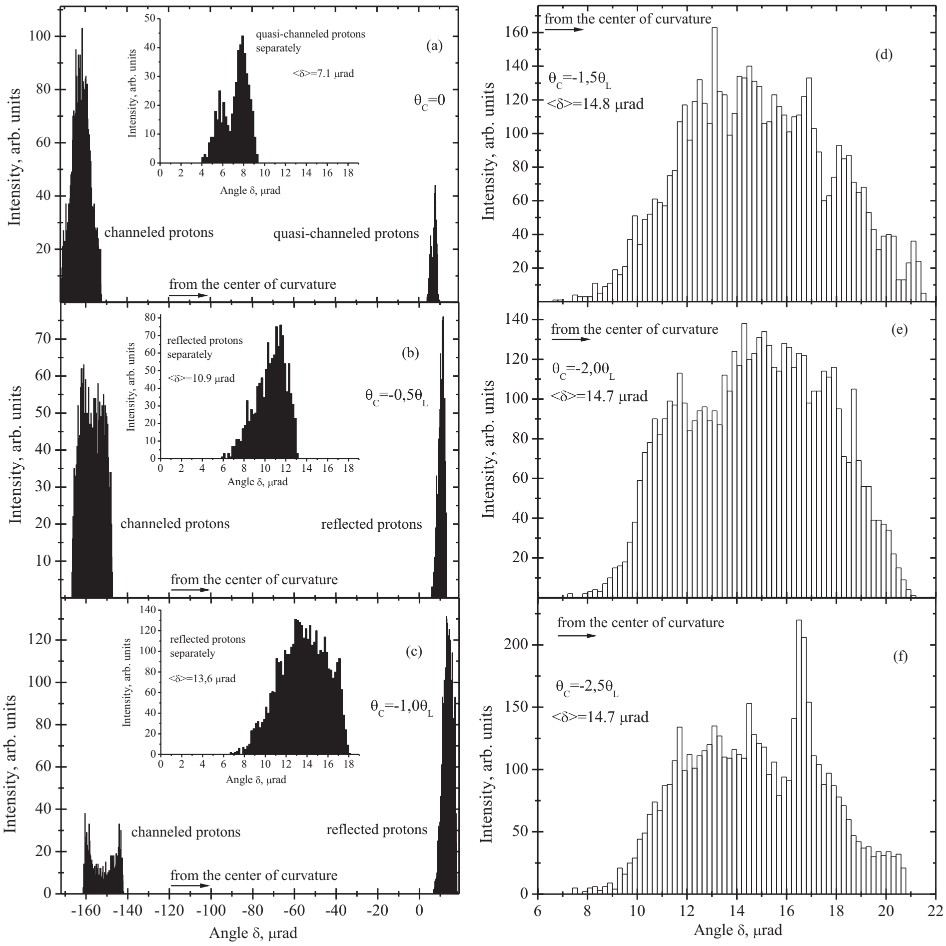

Fig. 4. – Angular distributions in dependence on angles δ of 400 GeV protons passing through bent (220) Si crystal at different angles θ_C; the radius of curvature $R = 1852\,\text{cm}$, the crystal bend angle $\alpha = 162\,\mu\text{rad}$, the critical channeling angle $\theta_L = 10.2\,\mu\text{rad}$, the initial angular divergence $\Delta\theta = 8\,\mu\text{rad}$.

but also the quasi-channeled non-reflected protons characterized by the incident angles $\theta_0 > 0$. In figs. 4, d-f at angles $\theta_C \le -\theta_L$ only reflected protons are observable. When the absolute value of angle θ_C increases (from fig. 4, d to fig. 4, f) both width and position of angular distribution remain almost constant. It should be mentioned that from the data presented in fig. 4, the averaged angle δ at $\theta_C \le -\theta_L$ is approximately doubled in comparison with the averaged angle δ at $\theta_C = 0$ in agreement with the conclusion [5].

4. – Conclusion

The simulations presented above were carried out for the parameters of experiments [10]. The comparison of our calculation results and experimental data shows rather good agreement. In particular, the averaged angle δ of reflected protons (in figs. 4d-f) at angles $\theta_C < -\theta_L$ is obtained $14.7\,\mu$rad, whereas in [10] for this angle the value $13.9 \pm 1.7\,\mu$rad was found. The width of distributions for reflected protons at $\theta_C < -\theta_L$ is obtained to be about $13\,\mu$rad and is in agreement with experimental data also. Rather good agreement for both positions and widths of calculated and experimental channeled fractions are also achieved.

While the proton moves into the crystal, its radial energy might be transferred from the particle to the crystal and from the crystal to the particle. As a result, two effects are taking place: dechanneling, when energy is transferred from crystal to projectile, and volume capture, when energy is transferred from projectile to crystal. Moreover, some accompanying effects might appear. Dechanneling as well as volume capture lead to the transitions of protons from the channeling regime to the quasi-channeling one, and vice versa. The influence of these effects on a proton beam is observed in experiments [4, 10]. In future we will take into account both volume capture and dechanneling effects to obtain a more realistic picture of the process in the proton-bent crystal interaction.

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