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Stochastic mechanism of a high-energy charged-particle beam deflection by a bent crystal

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Summary. — The efficiency dependence on particles energy for the stochastic mechanism of a high-energy charged-particle beam deflection by a bent crystal was analysed. It was shown that with particles energy decrease the maximal possible angle of beam deflection increases. The influence of incoherent effects on particles scattering was also analysed.

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PACS `29.27.Eg` – Beam handling; beam transport.
PACS `07.05.Tp` – Computer modeling and simulation.

Scattering at collision of a high-energy charged particle with an atomic string in a crystal, when particle falls on the string at small angle with respect to the string axis, occurs mainly along the azimuthal angle in the plane orthogonal to the string axis. As a result of multiple scattering on different crystal atomic strings, the angular redistribution of particles takes place. It leads to the so-called doughnut particles scattering. Special interest represents the case when particles momentum incidence angle with respect to the atomic string axis ψ is less or about the critical axial channeling angle ψ_c, because in this case we deal with fast equilibration in particles azimuthal angles as well as with the possibility of particles axial channeling (their finite motion in the field of one or several atomic strings) [1]. Such a motion may take place in the case when crystal atomic strings are slightly bent, resulting in beam deflection. Such effect is observable both for finite and infinite particles motion with respect to atomic strings. Special interest represents the case of infinite motion, because for positively charged particles potential wells, in which finite motion occurs, are typically not deep. For negatively charged particles the fast dechanneling occurs due to the intensive incoherent scattering.

In the work [2] the condition, under which the beam deflection at multiple particles scattering on bent crystal atomic strings takes place, was revealed. Experimental

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confirmation of such effect for positively and negatively charged particles with energies exceeding 100 GeV was given in works [3, 4]. Obtained results indicate very good efficiency of this deflection mechanism for particles with such energy by the angles significantly exceeding the critical axial channeling angle. With particles energy decrease the beam bend on greater angles can be realized. However, in this case the contribution of incoherent effects in scattering also increases.

The present work analyzes the possibilities for stochastic mechanism of beam deflection by means of a bent crystal at the decrease of particle energy. The value of critical axial channeling angle increases at energy decrease, hence we can expect the increase of maximal possible beam deflection angle under the energy decrease. However, the increase of incoherent effects at scattering may lead to the effect of destruction. That is why studying the energy dependence of the stochastic beam deflection mechanism is of great interest.

In the paper [2] it was shown that for fast charged particles moving in the stochastic regime of multiple scattering on bent crystal atomic strings, the following relation holds between the mean-square deflection angle of particle momentum direction with respect to the current direction of the crystal atomic string $\overline{\psi^2}$, the mean free path of a particle between successive collisions with atomic strings l and the crystal curvature radius R:

$$\frac{\mathrm{d}}{\mathrm{d}z}\overline{\psi^2} = \frac{l}{R^2}\,, \tag{1}$$

where the axis z is curved. If the crystal atomic string potential has the form $\phi(\rho) = \frac{Ze}{d}\frac{\pi R_a}{2\rho}$ (this potential is often used in the theory of channeling [1]), where R_a is the atomic screening radius, then $l = l_0\psi/\psi_c$, where $l_0 = 4/(\pi^2 n d R_a \psi_c)$, n is the concentration of atoms in the crystal, $\psi_c = \sqrt{4Z|qe|/(pvd)}$, e is the electron charge, $Z|e|$ is the crystal atomic charge, q is the particle charge, p and v are its momentum and velocity and d is the distance between neighboring atoms in the atomic string parallel to the selected axis. Using this expression for the described potential, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}z}\overline{\psi^2} = \frac{l_0}{R^2}\frac{\sqrt{\overline{\psi^2}}}{\psi_c}\,, \tag{2}$$

where ψ in the expression for $l(\psi)$ was replaced by $\sqrt{\overline{\psi^2}}$. The solution of this equation has the form

$$\overline{\psi^2} = \frac{1}{4\psi_c^2}\left(\frac{l_0 L}{R^2}\right)^2. \tag{3}$$

The regime of the stochastic particles deflection by a bent crystal is realized until $\overline{\psi^2} \lesssim \psi_c^2$. For the mentioned atomic string potential it means that beam deflects up to the crystal thickness

$$L_{\mathrm{cr}} = 2R^2\psi_c^2/l_0, \tag{4}$$

and the particle beam bend in the regime of stochastic deflection is possible on the angle

$$\alpha_{\mathrm{cr}} = \frac{L_{\mathrm{cr}}}{R} = \frac{2R\psi_c^2}{l_0}\,. \tag{5}$$

TABLE I. – *The dependence of* L_{cr} *and* α_{cr} *on the particle energy.*

E (GeV)	1		10		100	
Particle	proton	π^--meson	proton	π^--meson	proton	π^--meson
L_{cr} (cm)	22.0	33.5	1.1	1.2	0.04	0.04
α_{cr} (mrad)	146.9	223.3	7.4	8.2	0.26	0.26

It should be noted that in formula (1) deducing incoherent effects in particles scattering on crystal atomic string are not taken into account. As shown below, these effects for negatively charged particles lead to a significant change in α_{cr} at energies $E \lesssim 10\,\mathrm{GeV}$ (E corresponds to the particle kinetic energy).

The problem of high-energy charged particles motion in a bent crystal near one of crystal axes is rather complicated, because in this case, there is the possibility of the existence of dynamic chaos effect [5], and incoherent scattering can have a significant impact on the process of motion. That is why in this process analysis methods of computer simulation of charged particles passing through the crystal play an important role. In this work, the analysis of the particles passage through the bent crystal was made on the basis of the method, developed in [6], of numerical simulation of high-energy particles passage through straight and bent crystals near one of the crystal axes. This simulation allows to analyse the influence of various factors on the particle motion in a crystal, such as the dynamical chaos phenomenon, the influence of crystal atomic planes in the particle motion in a crystal, the role of incoherent effects in scattering, etc. In this method, to find the particle trajectory in a bent crystal, we divide the crystal into a large number of straight parts, each of which is slightly rotated relative to the previous one. The trajectory of particles in each part of the crystal is determined by solving the motion equation in the continuous potential of bent atomic strings $U_c(\vec{\rho})$ [7]

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{\rho} = -\frac{1}{E}\frac{\partial}{\partial\vec{\rho}}U_c(\vec{\rho}), \tag{6}$$

where $\vec{\rho}(t)$ is the particle trajectory in the plane orthogonal to the current direction of the crystal axis. The crystal curvature and incoherent scattering effects in this program are discounted at the end of each straight part of the crystal.

Let us consider the simulation results of fast charged particles motion in a bent silicon crystal with length of 1.5 mm and radius of curvature $R = 1.5\,\mathrm{m}$ near to the $\langle 110\rangle$ crystal axis. The crystal bend angle was $\alpha = L/R = 1\,\mathrm{mrad}$. Figure 1 shows the angular distribution of protons with (a) $E = 1\,\mathrm{GeV}$, (b) $E = 10\,\mathrm{GeV}$ and (c) $E = 100\,\mathrm{GeV}$, after the passage of the described crystal and the horizontal profiles of these distributions ($\eta = N(\theta_x)/N$, where $N(\theta_x)$ is the number of particles, deflected in the range of angles $(\theta_x; \theta_x + \Delta\theta)$, $\Delta\theta$ is the histogram step, N is the total number of particles). For these energies and for the given crystal values of L_{cr} and α_{cr}, calculated from formulas (4) and (5), are presented in table I. The simulation was carried for the beam entering the crystal parallel to the $\langle 110\rangle$ crystal axis. Beam divergence was $10\,\mu\mathrm{rad}$.

Thus, based on the criterion (5) in cases (b) and (c) beam as a whole should be deflected by the crystal at 1 mrad. Simulation results shown in fig. 1 (a,b), are in agreement with this criterion. In the case (c) $\alpha_{\mathrm{cr}} \ll \alpha$, so most of the beam particles leave the stochastic deflection regime at small crystal thicknesses, but continue to deviate, falling into the channels formed by bent $(1\bar{1}1)$ and $(1\bar{1}\bar{1})$ crystal planes.

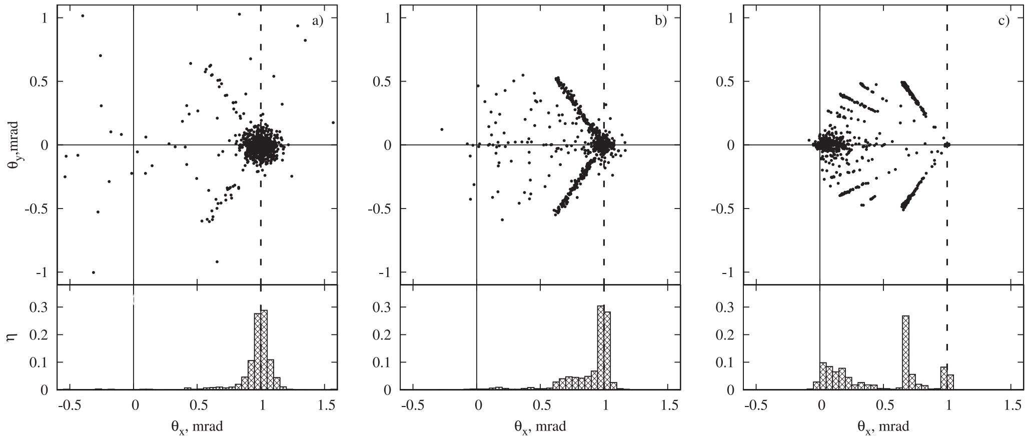

Fig. 1. – Angular distributions of protons with energies (a) $E = 1\,\mathrm{GeV}$, (b) $E = 10\,\mathrm{GeV}$, (c) $E = 100\,\mathrm{GeV}$ after passing 1.5 mm of bent Si crystal and horizontal profiles of these distributions.

For negatively charged particles at selected energies the scattering pattern substantially differs from the case of positively charged particles. When negatively charged particles move in a crystal, the potential energy of their interaction with crystal atoms is negative, so the region of their motion is much closer to the crystal atoms than in the case of positively charged particles. It means that scattering on thermal oscillations is more intensive for negative than for positively charged particles. Since the root mean square angle of multiple scattering on thermal atomic oscillations in a crystal is inversely proportional to the square of the particle energy [8], then with energy decrease the intensity of such scattering rapidly increases. This leads to the fact that particles deflect from the current direction of the crystal axis, near which they are moving, at an angle exceeding the critical axial channeling angle, long before the thickness stated in eq. (4). In confirmation of this fig. 2 shows the angular distribution of π^--mesons with

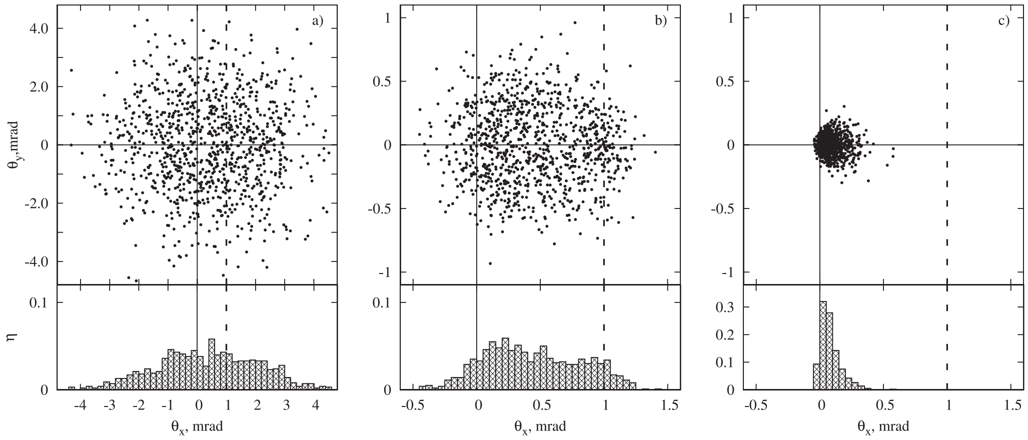

Fig. 2. – Angular distributions of π^--mesons with energies (a) $E = 1\,\mathrm{GeV}$, (b) $E = 10\,\mathrm{GeV}$, (c) $E = 100\,\mathrm{GeV}$ after passing 1.5 mm of bent Si crystal and horizontal profiles of these distributions.

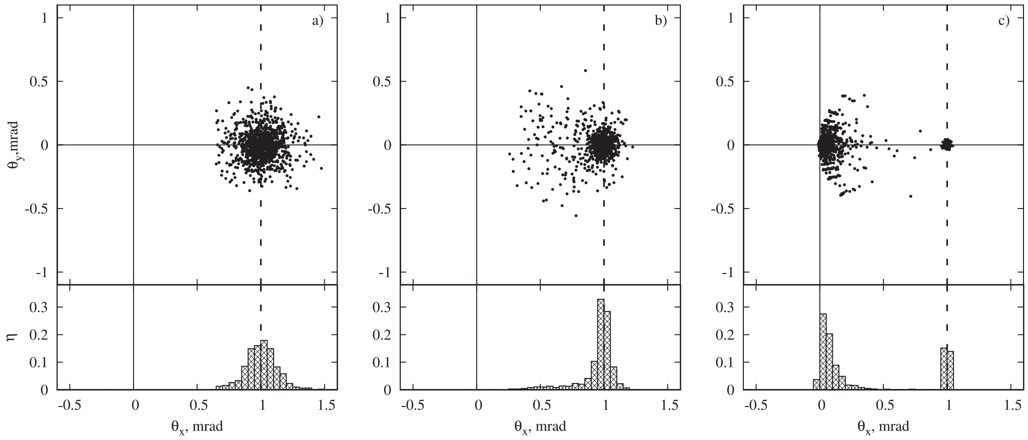

Fig. 3. – The same as fig. 2 without taking into account incoherent scattering.

$E = 1\,\mathrm{GeV}$ (a), $E = 10\,\mathrm{GeV}$ (b) and $E = 100\,\mathrm{GeV}$ (c), after the passage of the bent crystal and horizontal profiles of these distributions. From these figures it is evident that with energy increase, the influence of incoherent effects on the overall scattering picture decreases (and hence the size of the beam at the exit of the crystal decreases), but the angle of beam deflection decreases too.

Figure 3 shows the angular distribution of π^--mesons with energy $E = 1\,\mathrm{GeV}$ (a), $E = 10\,\mathrm{GeV}$ (b) and $E = 100\,\mathrm{GeV}$ (c), after passing through the bent crystal without incoherent effects account and the horizontal profiles of these distributions. Note that the results, as well as in the case of positively charged particles are in good agreement with the criterion (5). In fig. 3(c) we can see that some part of the beam deflects on the angle, exceeding α_{cr}. It happens because these particles are not moving in the stochastic multiple scattering regime. This is the part of π^--mesons that are captured in the axial channeling regime. Remaining in the finite motion state they are deflected by bent atomic strings. The account of incoherent effects in scattering leads to rapid dechanneling of these particles (see fig. 2(c)).

The obtained results allow us to conclude that the process of a high-energy charged-particle beam passage through the bent crystal significantly depends on particles energy. With energy increase, the maximum possible angle of beam deflection due to stochastic multiple scattering on bent crystal atomic strings decreases. However, the beam deflection in this case can be realized by means of the crystals with greater curvature radius defined by relation (5).

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