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Thermal stability of a strip crystal for channeling of halo particles in the Large Hadron Collider

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Summary. — For possible application of a silicon bent crystal as a beam collimator in the Large Hadron Collider, the thermal stability of a strip-like crystal has been studied. The thermal power generated by the beam inside the crystal has been evaluated through a simulation and used as a source in the heat equation. The equation has been solved numerically with the semi-analytical method of lines to show the thermal stability of crystal-assisted collimation with the parameters currently being considered for its exploitation. The method we presented appears to be useful for calculation of the thermal load concerning any situation involving crystal channeling.

PACS 44.05.+e – Analytical and numerical techniques.
PACS 61.85.+p – Channeling phenomena (blocking, energy loss, etc.).

1. – Introduction

The possibility to perform halo collimation in hadron colliders through channeling of particles in a bent crystal has been investigated since the nineties [1]. Since then, significant progress has been made since the first appearance of the idea [2-6]. Crystal collimation could be a valid scheme for halo collimation [7], allowing the deflection of halo particles with high efficiency thanks to multi-turn extraction mechanism [8], or can be used as a precision diagnostics to aid the traditional collimation systems [9].

In particular, the application of a silicon bent crystal at the Large Hadron Collider could serve to kick most external particles in the particle beam, directing them onto a secondary collimator [10]. Among the many challenges concerned with implementation of a crystal-based collimation scheme, such as alignment with beam, compensation of aberrations in crystal bending, radiation hardness of the crystal, the problem of crystal heating has never been addressed. Yet, although the crystal is intended to intercept halo

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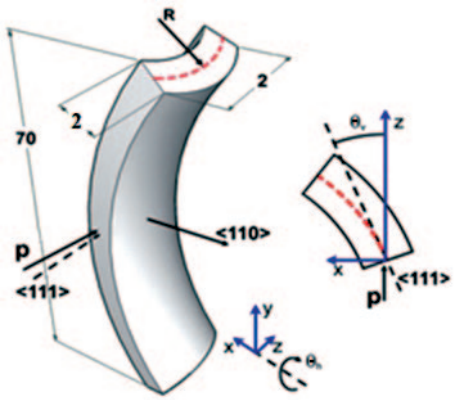

Fig. 1. – Sketch of the channeling crystal, with the details of the curvatures.

particles, *i.e.*, to be lodged at the periphery of the beam, the beam circulating in the LHC will store the largest amount of energy at the highest energy (up to 7 TeV) than any other accelerator ever. Thus the problem of thermal stability of the crystal exposed to the beam is an important issue to be addressed to ensure proper working conditions.

In this work, the behaviour of a strip-like crystal, shown in fig. 1, has been studied from a thermal point of view, during its operation as channeling device for the LHC proton beam. Beam parameters are those reported in ref. [11], the most relevant of which to our purpose are summarized in table I.

The crystal, during its operation, is hit by high-energy protons, up to 7 TeV. Beam particles lose part of their energy during the motion under channeling, acting as a heat source for the crystal, while heat dissipation occurs through radiation and conduction only because convection is prevented due to ultra-high-vacuum condition.

2. – Evaluation of the heat source

The energy lost by a single proton inside a silicon crystal has been evaluated using a GEANT4 framework, with a custom-made geometry description, inserting a silicon layer with a thickness of 1 cm. Since GEANT4 is not equipped with a routine to simulate the dynamics of particle motion, the interaction of particles with the crystal was treated as it were an amorphous material with the same density as for a Si crystal. The dynamics of particles channelled in a crystal is subject to the planar potential, *i.e.*, the particles oscillate between neighbouring crystalline planes while following the global curvature of the crystal. It means that interaction with nuclei is minimal and the density of the electron cloud experienced by channelled particles is always lower than for interaction with an equivalent amorphous medium. Thus, the heat rate evaluated through GEANT4 is to be regarded as an overestimate of the current heating.

TABLE I. – *LHC beam parameters.*

No. of bunches	Protons/bunch	Total intensity	Bunch length	Bunch spacing
2808	1.15×10^{11}	3.23×10^{14}	7.55 cm	25 ns

For the simulation, 10^5 protons were hit onto the crystal at 7 TeV energy, then we determined the energy deposited per unit length with the following strategy: during the reconstruction of the particle's track, we summed up the deposited energy along the path of an individual particle, and we divided it by the length of interaction with the material. In particular, GEANT4 renders a lot of useful information about the energy deposited and the length of the interaction path of the particle with the material.

The thermal power transferred to the material can be calculated by taking into account beam parameters, in particular the number of protons per bunch and the length of the bunch, both reported in table I. Beam intensity, *i.e.* the number of protons per unit area and time, decreases with the radial coordinate r (the distance from the central axis), and could be written as $F(r) = \rho(r)v$, where $\rho(r)$ is the volumetric density of protons and v is the velocity of the particles ($v \cong c$). Since the radial profile of the concentration of protons can be modelled with a Gaussian function with r.m.s. $\sigma = 0.1\,\mathrm{mm}$, beam intensity can be written as

$$F(r) = \rho_0 c\, e^{-r^2/2\sigma^2},$$

where ρ_0 is the density on the centre of the beam. By imposing that the integral of ρ over the whole bunch must be equal to N, it readily holds

$$\rho_0 = \frac{N}{2\pi\sigma^2 L}\,.$$

If a single proton looses an energy per unit length equal to $\mathrm{d}E/\mathrm{d}x$, the power transferred to the material per unit volume as a function of r is $S(r) = \mathrm{d}E/\mathrm{d}x F(r)$, therefore

$$S(r) = S_0\, e^{-r^2/2\sigma^2}, \qquad S_0 = \frac{\mathrm{d}E}{\mathrm{d}x}\frac{Nc}{2\pi\sigma^2 L}\,. \tag{1}$$

Numerically, with a beam energy of 7 TeV, it results $S_0 = 1.33 \times 10^{17}\,\mathrm{W/m^3}$.

3. – Heat equation: steady state

In order to solve the thermal problem, the two-dimensional heat equation with the Gaussian-like heat source of eq. (1) has been applied to the strip-like crystal. In this section we will approach the problem with the thermal power source of eq. (1) as it was due to interaction with a coasting beam with the same density as that in the bunch for the real case; then, interaction with a bunched beam will be worked out in sect. **4**.

3˙1. *Geometry of the problem.* – The geometry of the problem is sketched in fig. 2, which is a cross-section orthogonal to the nominal axis of the beam. The length of the longer side is $L = 70\,\mathrm{mm}$, whereas the shorter one is $l = 2\,\mathrm{mm}$. We choose a Cartesian coordinate system, with the origin in the left edge on the lower side of the crystal. For symmetry reasons, we can reduce the problem to a half crystal, within the domain $[0, L/2] \times [0, l]$.

Once the coordinate system and its origin have been fixed, heat equation reduces to the Poisson equation:

$$\lambda\nabla^2 u + S_0\, e^{-r^2/2\sigma^2} = 0, \tag{2}$$

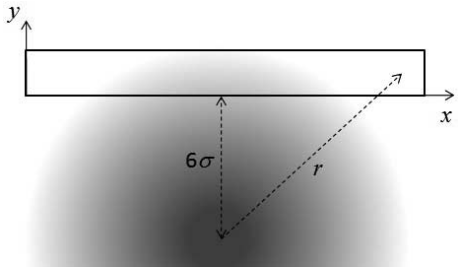

Fig. 2. – Sketch of the geometry of the problem. The origin of the coordinate system coincides with the left edge on the lower side of the crystal. Therefore, the centre of the beam is a point of coordinates $x = L/2$, $y = -6\sigma$.

where $u = u(x, y)$ is the local temperature of the crystal and $\lambda = 148\,\mathrm{W/mK}$ is the conduction coefficient for Si. This equation has been numerically solved with the semi-analytical method of lines [12].

3˙2. *Solution with the semi-analytical method of lines.* – This numerical technique is very powerful to solve elliptic partial differential equations in 2 variables, because it requires a much smaller number of divisions of the domain than finite differences methods. It consists in the discretization of only one variable, leading to a system of coupled ordinary differential equations in the other variable; thus, the solution is given as a set of functions. Equation (2) in Cartesian coordinates is written as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -f(x, y), \tag{3}$$

$$f(x, y) = \frac{S_0}{\lambda} \exp\left[-\frac{(x - L/2)^2 + (y + 6\sigma)^2}{2\sigma^2}\right].$$

As a first step, we consider the thermal dissipation through radiation to be negligible, therefore the crystal transfers thermal power only to the metallic support, which provides to hold and bend it. Since heat capacity of the holder is much larger than that of the crystal, the holder can be regarded as a heat reservoir, *i.e.*, it is kept at constant room temperature T_0. With this statement, the conditions for the differential equation (3) at the boarder of the domain are the following:

- $x = 0$: a Dirichlet condition which fixes the value of the temperature of the left side;
- $y = 0$ and $y = l$ Neumann conditions which fix the heat transfer along y;
- $x = L/2$: a Neumann condition which equals to zero the partial derivative of u with respect to x, for symmetry reasons.

Therefore, the boundary conditions for eq. (3) are

$$\begin{array}{ll} u(0, y) = T_0 & u_y(x, l) = 0 \\ u_y(x, 0) = 0 & u_x(L/2, y) = 0, \end{array} \tag{4}$$

where the subscript x and y indicate the partial derivatives with respect to x and y, respectively.

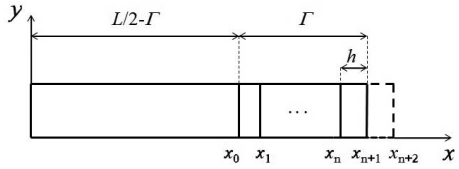

Fig. 3. – Discretization of the horizontal variable x.

The discretization of the horizontal direction must be applied so that the convergence of the solution is achieved with a relatively small number of divisions. Being the characteristic length, σ, of the heat source much smaller than the total length to discretize $L/2$, a convenient choice is to discretize only a zone near the centre of the crystal, where the source $f(x, y)$ of eq. (3) attains its maximum (then vanishes rapidly due to their Gaussian character). Thereby, we chose to discretize only a zone of length Γ starting from the centre of the crystal (see fig. 3). The advantage of this choice is that the intensity of heat source in the left zone can be considered zero, so that the solution is linear, as indicated by the Green's function of eq. (3). As a consequence, the first of the boundary conditions of eqs. (4) must be changed in a condition set at $x = L/2 - \Gamma$: a Robin condition that states that the x-derivative must be equal to the slope of the solution in the first zone,

$$u'(L/2 - \Gamma, y) = \frac{u(L/2 - \Gamma, y) - T_0}{L/2 - \Gamma}\,.$$

Now we introduce $n+1$ points $x_0, \ldots, x_{n+1}$ (where $x_0 = L/2-\Gamma$ and $x_{n+1} = L/2$), which generate $n + 1$ intervals of width $h = \Gamma/(n + 1)$, as shown in fig. 3 (the additional point x_{n+2} is a "virtual point" useful for a better statement of the condition on the partial derivative of u with respect to x in the point x_{n+1}).

Approximating the partial derivative of u with respect to x with the finite differences, eq. (3), with the proper boundary conditions discussed above, becomes a system of $n+1$ ordinary differential equations in the variable y which can be written in the matrix notation:

$$\frac{\mathrm{d}^2 U}{\mathrm{d}y^2} = \frac{1}{h^2} PU - F,$$

with

$$U = \begin{pmatrix} u_1 \\ \vdots \\ u_{n+1} \end{pmatrix} \qquad P = \begin{pmatrix} \frac{L/2-\Gamma+2h}{L/2-\Gamma+h} & -1 & 0 & & & \\ -1 & 2 & -1 & & 0 & \\ & \ddots & \ddots & & & \\ 0 & & & & & \\ & & & -1 & 2 & -1 \\ & 0 & & 0 & -2 & 2 \end{pmatrix}$$

$$F = \begin{pmatrix} \frac{T_0}{h(L/2-\Gamma+h)} + f_1 \\ f_2 \\ \vdots \\ f_{n+1} \end{pmatrix}, \tag{5}$$

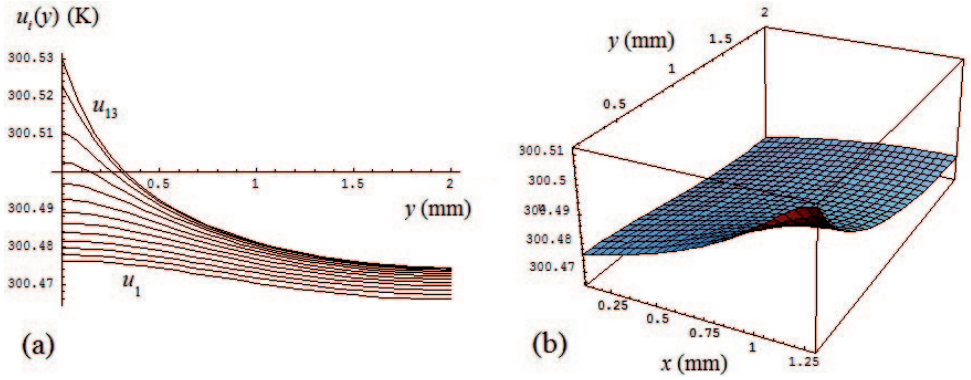

Fig. 4. – (a) Plot of all 13 solutions as a function of y (u_1 is for $x = L/2 - \Gamma$, u_{13} is for $x = L/2$); (b) 3D interpolation of all the $u_i(y)$.

where $u_i(y) = u(x_i, y)$ and $f_i(y) = f(x_i, y)$. The solutions of eqs. (5) can be easily obtained with a change of base that diagonalizes matrix P. Indeed, after diagonalization, we have $n + 1$ independent ordinary differential equations, which can be easily solved numerically. Then, the vector U can be obtained returning to the old base.

3˙3. *Results of the calculation.* – Convergence to the second decimal in the solution has been achieved with a number of divisions $n = 12$, and with a discretization length $\Gamma = 1.3\,\text{mm}$. The results of the calculation are shown in fig. 4. In fig. 4a all $u_i(y)$ functions are shown, while fig. 4b illustrates a 3D interpolation of the $u_i(y)$, which clearly highlights a Gaussian-like shape. It is apparent that the increase in temperature of the crystal during operation is negligible inasmuch as the hottest point exhibits an increase of only 0.5 K with respect to room temperature.

The correction due to the thermal power dissipated through radiation is therefore negligible, because the achieved temperatures are too low. This has been confirmed by iterating the solution as follows: each iteration is obtained imposing, in the boundary conditions for the derivatives in $y = 0$ and $y = l$, the Stefan-Boltzmann radiation law with the temperature values obtained in the previous iteration (the first iteration is the solution without radiation). The process converges immediately, indeed every iteration yields the same solution, identical to the one obtained in the absence of radiated power.

We considered the case with beam suffering trajectory perturbation or erroneous insertion of the crystal too deep into the core of the beam. Therefore, the same calculation has been carried out with various distances of the centre of the beam. The results for the hottest point of the crystal, ($x = L/2$, $y = 0$), are reported in table II. The crystal never reaches high temperatures, even in the case of insertion into the beam by $\sigma/2$.

TABLE II. – *Maximum temperature of the crystal as a function of the distance from the beam centre.*

Distance (σ)	6.0	5.9	5.8	5.7	5.6	5.5
$u_{13}(0)$ (K)	300.5	301.0	301.8	303.2	305.8	310.2

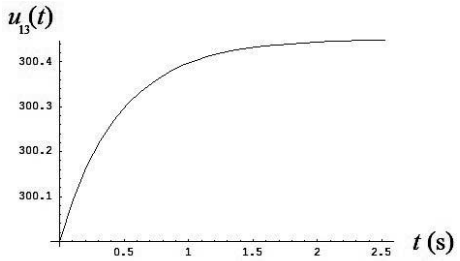

Fig. 5. – Time-dependent solution $u_{13}(t)$ if the time dependence of the heat source were a Heaviside function. The time to reach a steady state is about 2.5 s.

4. – Heat equation: time dependence

In the previous section we worked out with the heat source (1) as it were continuous in time. A more realistic condition is to account for the bunched structure of the beam in the LHC. Each bunch is 250 ps long, with a time spacing between consecutive bunches of 25 ns. In terms of channeling time, a proton spends 0.25 ns every 25 ns inside the crystal, therefore the heat source for the time-dependent heat equation is multiplied by a periodic step function which is 0 for a time 2 orders of magnitude longer than the time in which is 1.

In order to solve also the time-dependent equation with the semi-analytical method of lines, we considered the crystal as it were one-dimensional, a reasonable approximation since the x-length is 35 times greater than the y-length. The equation is written as follows:

$$(6) \qquad \rho C_V \frac{\partial u}{\partial t} = \lambda \frac{\partial^2 u}{\partial x^2} + f(x,t); \qquad f(x,t) = g(t)\, S_0 \, \exp\left[-\frac{(x - L/2)^2 + (6\sigma)^2}{2\sigma^2} \right],$$

where $g(t)$ is the periodic step function discussed above. Applying the discretization on the spatial variable, we obtain

$$\rho C_V \frac{\mathrm{d}u_i}{\mathrm{d}t} = \lambda \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + f_i(t) \quad i = 1, \ldots, n+1,$$

where $\rho = 2330\,\mathrm{kg/m^3}$ is the density of silicon, $C_V = 710\,\mathrm{J/kg\,K}$ the specific heat capacity, and $f_i(t) = f(x_i, t)$. It is a system of first-order ordinary differential equations, which is easily solvable after a diagonalization of the matrix of coefficients. The numerical solution is the constant temperature T_0 of the metallic support with which is in contact. Indeed, the thermal inertia of the crystal is such that the time during which the heat source is "on" is not high enough to increase the temperature of the crystal. A confirmation of that can be found if one evaluates the time constant τ of the thermal behaviour of the system. It can be easily obtained by setting $g(t)$ in eq. (6) equal to the Heaviside function and plot the solution of the equations system as a function of time; the result for τ is of the order of 1 s, as shown in fig. 5.

5. – Conclusions

A model to address the problem of a strip-like crystal heating under channeling condition has been addressed in full generality. The method relies on the numerical solution of heat equation, obtained with the semi-analytical method of lines, which implies the discretization of one variable, in order to obtain a system of ordinary differential equations. In particular, the case of thermal behaviour of a silicon bent crystal during channeling of LHC protons was studied. Particular attention was paid to the spatial domain, since the heat source is negligible within a large part of the crystal. It has been found that crystal heating is never a problem for usage of the crystal as a collimator under channeling condition.

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