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Probing correlations of gaseous microwires in lattice potentials via inelastic light scattering

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Summary. — In this work, inelastic light-scattering (Bragg spectroscopy) is used to study strongly correlated phases of ultracold 1D gases in optical lattices. We investigate the crossover from correlated superfluids to Mott insulators. Light-scattering creates in the system elementary excitations with non-zero momentum, and the response of the correlated gases is in the linear regime. This allows for extracting information about the atomic many-body state in terms of its particle-hole excitations, as common in solid-state physics. In particular, we characterize the Mott state both via intra-band and inter-band spectroscopy, the former giving access to the dynamical structure factor $S(q, \omega)$ and the latter to the one-particle spectral function $A(q, \omega)$.

PACS 67.85.Hj – Bose-Einstein condensates in optical potentials.
PACS 67.85.De – Dynamic properties of condensates; excitations, and superfluid flow.

1. – Introduction

Ultracold gases in light-induced periodic potentials (optical lattices) are a fertile ground for studying physics of quantum many-body systems with strong correlations and fluctuations. In particular, they are a valuable tool to realize one-dimensional systems, where the role of interactions and fluctuations is enhanced due to the reduced dimensionality [1]. Those are among the most intriguing physical systems, since no simple picture captures their behaviour. Such problems have many realizations, from material science to chemistry [2, 3], but many important questions have still to be addressed.On this prospect, new possibilities have been opened by the realization of ultracold gases in degenerate quantum states and by the development of techniques to manipulate them in optical lattices [4]. As a matter of fact, gaseous systems can realize interactions-induced

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strongly-correlated systems in an unprecedent way, and present some important advantages compared to alternative realizations of condensed-matter physics. For example, it is possible to switch the potential on and off abruptly, as well as to modulate it. Besides, optical lattices provide an effective tool for tuning the atom-atom interactions, since the depth of the periodic potential can be easily modified by changing frequency and intensity of the laser light producing the lattice. This makes possible to strictly compare experiment and theory. For these reasons, ultracold gases in optical lattices offer the possibility to implement theoretical models in an almost perfect manner. In particular, it is a virtually perfect rendering of the Hubbard-type models [5]. This is in line with the original idea proposed by R. Feynman [6] to realize a quantum simulator of models describing many-particle quantum systems, specialized in this case for simulating condensed-matter models.

In this context, it is crucial to better characterize the strongly correlated quantum phases of gases in the optical lattices. On this purpose, we used mainly inelastic light scattering (Bragg spectroscopy) [7] as a probe technique. As it is commonly performed in solid-state physics, this probe allowed us to gain important information about the atomic many-body state, by investigating its response *in the linear regime* to an excitation *at non-zero momentum*. In this regime, Bragg spectroscopy gives us access to the dynamical structure factor $S(q,\omega)$ and the one-particle spectral function $A(q,\omega)$ of the system. This paper presents our recent experimental work [8-10] which consists in exploring correlated states of ultracold bosons confined in one-dimensional microwires: We measure the excitation spectra of 1D Bose gases in the correlated superfluid regime as well as in the Mott insulator regime, observing novel experimental features.

2. – Realizing the Bose-Hubbard model with 1D lattice gases

We start from a three-dimensional gas of ^{87}Rb atoms, driven to the quantum degenerate state of Bose-Einstein condensate (BEC). The gas consists in $N \simeq 1.5\times10^5$ particles and it has chemical potential $\mu_{3D} = h\times 740\,\text{Hz}$, $h = 2\pi\hbar$ being Planck's constant. We confine it in two orthogonal red-detuned optical lattices, each created by a laser standing wave with wavelength $\lambda_L = 2\pi/k_L = 830\,\text{nm}$. These transverse lattices have the same, large amplitude $s_\perp = V_\perp/E_R = 35$, where $E_R = h^2/(2m\lambda_L^2)$, m being the atomic mass. This configuration produces a two-dimensional array of one-dimensional gaseous microwires, as sketched in fig. 1. The 1D gases can be considered as independent since the tunneling rate of the atoms through the potential barriers which separate different 1D gases is $\sim 0.75\,\text{s}^{-1}$, much smaller than the inverse timescale of the experiment (lasting a few tens of ms). A crucial quantity for describing these systems is the ratio of interaction to kinetic energy $\gamma = mg_{1\text{D}}/(\hbar^2 n_{1\text{D}})$ [11], $n_{1\text{D}}$ being the 1D density and g_{1D} the interatomic coupling in the 1D gas. As a remarkable difference from the usual physics in three dimensions, the lower the density the stronger are interactions despite to kinetic energy. For $\gamma \ll 1$, the interactions are well described by a mean-field picture, while for $\gamma \gg 1$ the 1D gas enters the Tonks-Girardeau regime [12]. In our case $\gamma \simeq 0.6$, thus the correlations are stronger than in the mean-field regime.

Along the axial direction of the 1D gases ($\hat{y}$), we superimpose an additional optical lattice with the same wavelength of the transverse ones, but variable amplitude $s_y = V(y)/E_R$ (see fig. 1).

These gases immersed in the axial lattice realize the Bose-Hubbard model [5], that is the extension for bosons of the Hubbard model describing fermions in a periodic potential (such as electrons in a crystal lattice). In this framework, the system is described by the

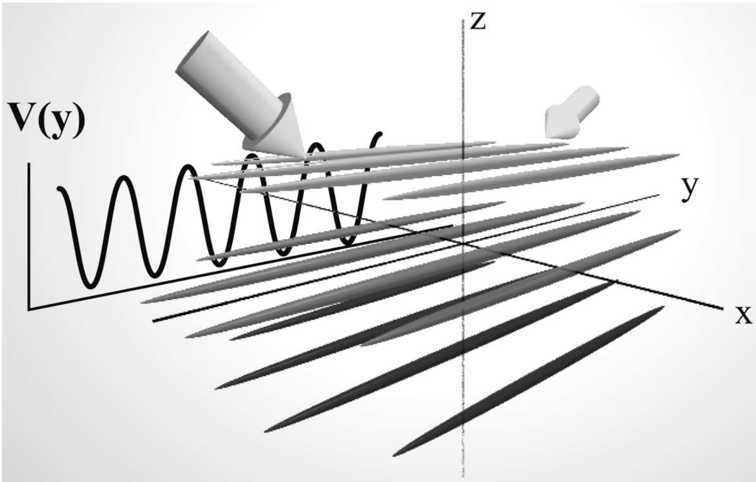

Fig. 1. – Sketch of the experimental system. An array of 1D tubes is produced trapping a three-dimensional Bose-condensed gas in a two-dimensional optical lattice (the latter being not shown for simplicity); along the axis of the atomic microwires ($\hat{y}$ in the figure) a third optical lattice with amplitude $V(y)$ is superimposed (dark curve), for driving the gases across the quantum phase transition from a superfluid state to a Mott insulator state. The system is investigated by using two simultaneous laser pulses (depicted by the arrows) with slightly different frequency (ν_1 and ν_2) and different momentum ($\hbar\mathbf{q}_1, \hbar\mathbf{q}_2$) inducing an excitations with frequency $\nu_1 - \nu_2$ and momentum $\hbar\mathbf{q}_1 - \hbar\mathbf{q}_2$.

Hamiltonian

$$H = -J \sum_{<i,j>} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i n_i(n_i - 1) + \sum_i \epsilon_i n_i. \tag{1}$$

Here $a_i^\dagger$, a_i are the creation and annihilation operator of one boson at site i and $n_i = a_i^\dagger a_i$ is the particle number operator. The parameters U and J describe the energy scales which govern the behaviour of the system: U is the repulsive interaction energy between two atoms in the same lattice site, J is the the hopping energy between next-neighbours sites. Both U and J depend on the axial-lattice depth s_y, thus they can be easily tuned by changing the intensity of the laser standing wave that creates this lattice [13]. ϵ_i is the energy offset experienced by an atom on site i due to the presence of an overall harmonic confining potential. When $U/J \ll 1$, the system experiences a superfluid state in which the particles are delocalized over the whole lattice. In the opposite regime, $U/J \gg 1$, the energy cost for putting two particles in the same lattice site overcomes the kinetic hopping energy and particles tend to localize in the lattice sites: This realizes an interaction-induced insulator, the Mott insulator [5]. However, due to the presence of the external trap, one expects to have several insulating regions with different average occupation, separated by superfluid lobes [14].

3. – The spectroscopic method

The lattice gas is investigated via Bragg spectroscopy. In practice, two simultaneous off-resonant laser pulses (Bragg beams) induce a two-photon transition between two different momentum states of the same internal atomic ground-state. The Bragg beams

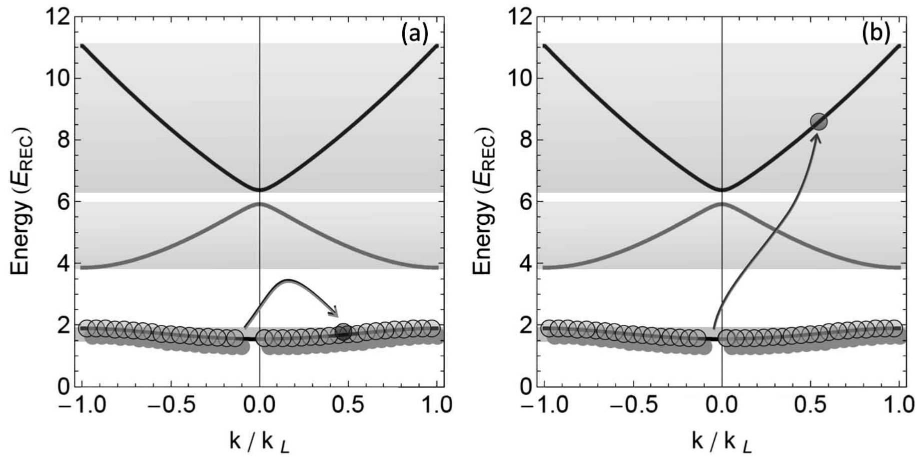

Fig. 2. – (a) Excitation *within* the lowest-energy band (lowest curve) of the system in a periodic potential, *i.e.* a particle-hole pair living in the ground state. (b) If the energy of the perturbation exceeds the band gap of the system, a particle is removed from the many-body ground state, populating a single-particle state in a high-energy band (upper curve) and leaving a hole in the lowest band.

have a tunable frequency difference $\delta\nu = \nu_1 - \nu_2$ and propagate with a relative angle θ (see arrows in fig. 1). This corresponds to creating an excitation with energy $h\delta\nu$ and momentum $\hbar\mathbf{q}_B$, the modulus of the latter depending on θ. In the experiment, the geometry of the Bragg beams is chosen to align $\hbar\mathbf{q}_B$ to the axis of the 1D gases. With the appropriate configuration of the Bragg beams, the momentum imparted to the atoms is $7.3(2)\,\mu\text{m}^{-1} \times \hbar = 0.96\hbar k_L$, close to the edge of the Brillouin zone defined by the axial lattice. Here, the response of the Mott insulator is expected to be the strongest [15]. After the excitation, all the lattice amplitudes are turned down in 15 ms to a low value ($s_\perp = s_y = 5$) where the different tubes are no longer independent allowing the system to re-thermalize via atom-atom collisions. After 5 ms more, both optical and magnetic trap are simultaneously switched off and the system is observed after a time of flight (TOF) $t_{TOF} = 21$ ms. Expanding from a phase-coherent state in a 3D optical lattice, the atomic distribution exhibits an interference pattern which is the analogous of the diffraction pattern of light from a grating [16]. From this interference pattern we extract the size of the central peak of the atomic cloud, that increases when the excitation is on resonance. This provide a measurement of the energy ΔE transferred to the system, as we verified in [9].

The measure of the energy transferred to the system gives us access to its polarizability $\chi''(\omega)$, *i.e.* its response-function to an external excitation, thanks to the relation [17]

$$\Delta E \propto V_B^2 \omega \chi''(\omega) \Delta t_B, \tag{2}$$

where V_B and Δt_B are the amplitude and the time duration of the perturbing potential.

In the presence of a periodic potential, the energy spectrum shows a band structure, and the imparted momentum is defined *modulus* the momentum $\hbar k_L$ associated to the standing wave producing the optical lattice. Thus, the two-photon Bragg transition imparting a given momentum along the direction of the optical lattice can couple atomic states in the same energy band of the lattice or in different bands (see [18]). These excitations consist in removing a particle with momentum k from the many-body ground state (where a hole is left) and creating an excited particle with momentum $\hbar(k + q_B)$. If the particle-hole pair is created *within* the lowest energy band, as in fig. 2 (a), the response of the system can be expressed in terms of the dynamical structure factor of the system $S(\omega, q)$, that is the Fourier transform of the two-particle correlation function, and is given by the sum of the all the possible excitations of the system [19]. Instead, when the perturbation carries high energy compared to the band-gap of the system, a hole is created in the lowest band and the particle jumps to a higher-energy band (see fig. 2 (b)). If the excited single-particle does not interact with the ground-state of the system, the probability of creating a hole in the many-body ground-state is described by the single-particle spectral function of the system $A(q, \omega)$ [20].

4. – Dynamical structure factor of the many-body atomic state

Figure 3 displays the excitation spectra of the system in the lowest-energy band for six different amplitudes s_y of the axial lattice. A single broad resonance is observed for low values of the lattice depth, with a width diminishing as s_y increases. For $s_y = 0$ the gases are superfluids, that we demonstrate by measuring the frequencies of the collective modes([1]). Although, due to thermal effects and correlations a large distribution of quasi-momenta is populated also at low values of s_y, and the full width of the resonance is comparable with the bandwidth of the lowest energy band ($\sim 5\,\mathrm{kHz}$). Increasing s_y, and thus diminishing J, the band flattens (the bandwidth being $\sim 4J$) and this can explain the reduction of the response width. When s_y further increases, the 1D gases are driven into an inhomogeneous Mott insulating state, where Mott lobes with integer site-filling alternate superfluid regions. As a response of the Mott state, new resonances are expected at energy above the lowest-energy band of the optical lattice. As long as the peaks of the superfluid and insulating regions are not resolved (*e.g.*, $s_y = 7$ in fig. 3 (c)), we can still define the width of the envelope, which suddenly increases again when the first Mott lobe appears. This analysis allows us to locate the appearance of the first Mott region in the range $s_y = 5 - 6$ ($U/J \sim 8 - 10$) according to recent theoretical predictions [14].

For $s_y \geq 10$ (fig. 2 (d-f)) the complex structure of the Mott response is well resolved. The distinctive signature of the dynamical structure factor of the Mott insulator consists in a resonance corresponding to the particle-hole excitation energy $\Delta_{\mathrm{ph}} \sim U$. It appears in the spectra, as pointed out by the vertical dashed line in fig. 3 (e) at $\sim 2\,\mathrm{kHz}$. We also observe a peak at the double frequency $\sim 4\,\mathrm{kHz}$. This can be attributed to the inhomogeneity of the experimental system, coming from the trapping potential and to a loading in the optical lattice which might not be fully adiabatic([2]).

A striking fact is the observation of a response from the system at very low frequencies, up to 1.5 kHz, not previously revealed by using other spectroscopic techniques such as lattice modulation [22]. The superfluid part of the whole gas is expected to contribute to the response of the system with a signal stretched out in the energy-range $4J$–$12J$ [23], corresponding to $200-500$ Hz. However, a response has been revealed also at in the energy

([1]) For a superfluid, the ratio of frequencies of the breathing mode to the dipole mode is $\sqrt{3}$, whereas it is 2 for a classical ensemble [21].
([2]) We exclude non-linear processes since the response to the light scattering lies in the linear regime as demonstrated in [9].

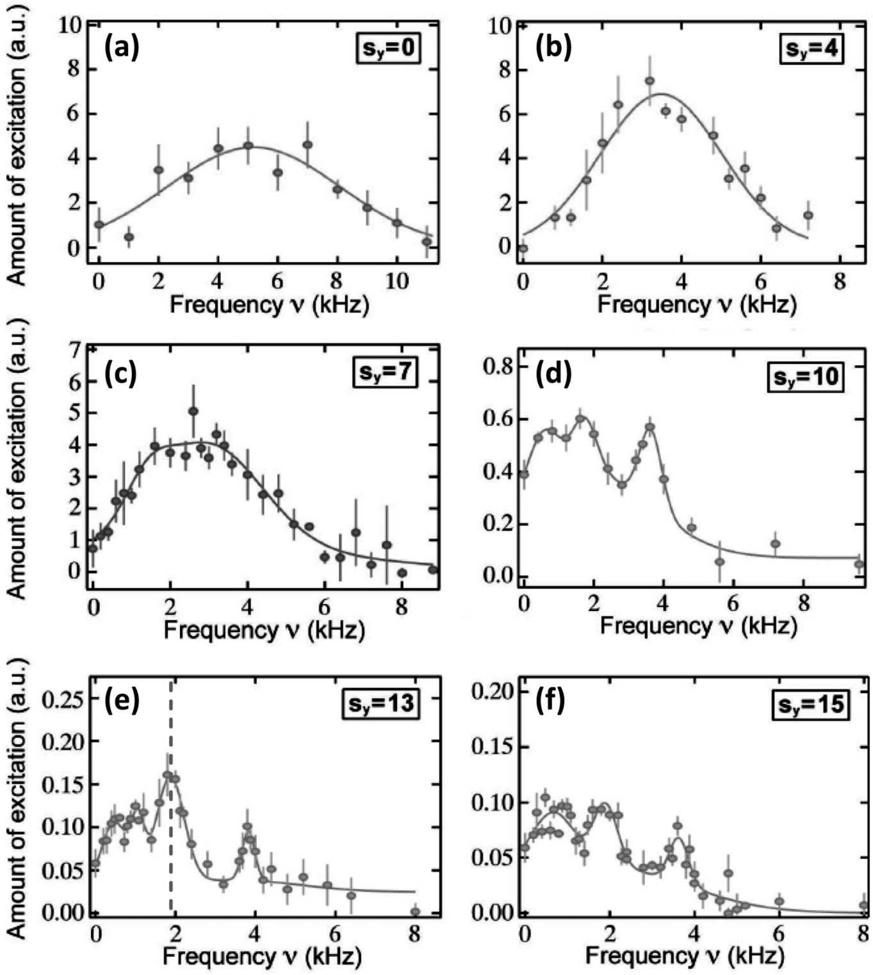

Fig. 3. – Spectra in the lowest-energy band measured across the transition from superfluid to Mott insulator. Solid lines are guides to the eye. The vertical dashed line in (e) marks the particle-hole excitation energy $\Delta_{\mathrm{ph}}(q_B)$ (see text). Note the drop in the amplitude of the response (vertical scale).

range 0.5–1.5 kHz, which cannot be attributed to the superfluid domains. This could be explained by the thermal population of several particle-hole excitations. These modes are separated to each other by a fraction of the bandwidth, namely by an energy of the order of J, thus they can be simply activated for effect of finite temperature (J/k_B typically amounts to few nanokelvin, in a Mott insulating state). Yet, how to extract a quantitative temperature from the position and/or width of these low-energy excitations is not clear.

5. – One-particle spectral function in a Mott insulator state

Now, we turn on to discussing the response of inhomogeneous Mott insulating states to inter-band excitations. Coupling the many-body ground state to high-energy single-particle states is analogous to angle-resolved photo-emission spectroscopy

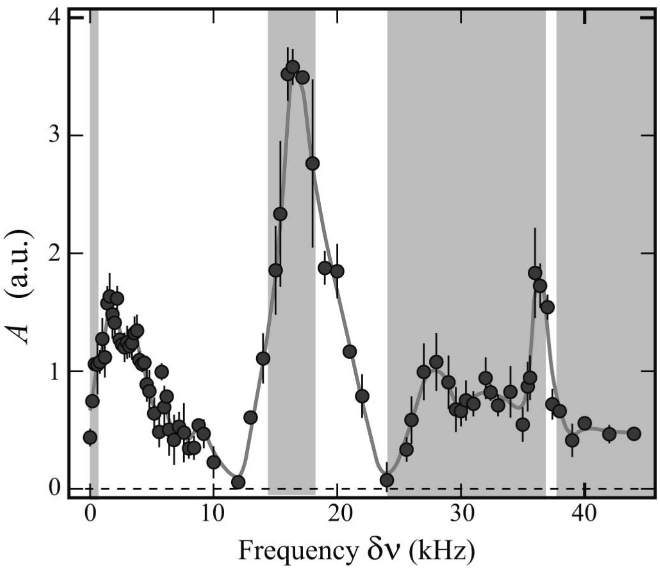

Fig. 4. – Bragg spectrum of an array of 1D BECs ($s_\perp = 35$) loaded in a 1D optical lattice of amplitude $s_y = 9$ (full dots), *i.e.* the 1D gases being in the Mott-insulating state, and for a momentum transfer $0.96(3)\hbar k_L$. The straight line is a guide to the eye. The grey areas cover the energy distribution of each band.

(ARPES) in condensed-matter physics: The excited atoms in the high-energy bands are the equivalent to the removed electrons from the initial many-body state. Thus, the response signal carries out information on the one-particle spectral function of the hole left in the many-body ground state.

Figure 4 depicts the spectrum of the system for $s_\perp = 35, s_y = 9$ on a large energy-scale. On this scale, the response of a Mott insulator exhibits several large resonances that can be identified with transitions towards different energy bands of the optical lattice. Identifying the final momentum state of atoms giving contribution for the different parts of the excitation spectrum can allow us to determine to which band the excitations belong to. On this purpose, we implement a band mapping as in [16]. In practice, once the lattice gas has been perturbed by the Bragg excitation, the optical lattices are turned off on a timescale of $\sim 2\,\text{ms}$. This time is long enough to avoid recombination between different bands, but short with respect the characteristic time associated to the atomic motion in the lattice site, namely, the inverse of the bandwidth of the lowest-energy band. Thereby, the band population is preserved and the quasimomentum states of the lattice gas are projected onto momentum states of the free particles released from the trap. After a time-of-flight of the atomic cloud, the atomic density distribution reflects the band population in the momentum space. As in fig. 5, the band mapping image shows a large squared peak, corresponding to a flat quasimomentum population in the first Brillouin zone. This proves that in a Mott insulating phase atoms cover homogeneously all the states of the lowest-energy band. In addition, a small lateral peak appears corresponding to excited atoms. By tuning the frequency of the Bragg excitation, the position of this peak (*i.e.*, the quasimomentum of the excited atoms) changes: This allows us to reconstruct the dispersion relation of the Mott state in the higher bands (see fig. 6).

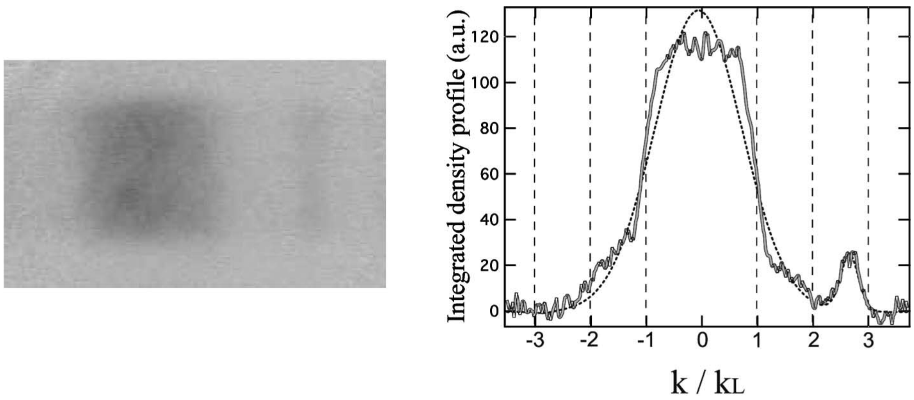

Fig. 5. – (a) Band population of an array of 1D BECs loaded in a 3D optical lattices of amplitudes ($s_\perp = 35, s_y = 9$) after a Bragg excitation with frequency $\delta\nu = 34$ kHz (false colours). (b) Quasi-momentum distribution profile derived from the previous image by integrating along the $\hat{z}$ direction. The horizontal scale is normalized to the momentum $\hbar k_L$ of the longitudinal optical lattice.

It can be convenient to focus on the interesting feature shown by the third band, where the signal amplitude presents a modulation (see fig. 4 in the frequency range from 25 to 36 kHz). The excited particle and the hole have very little spatial overlap and therefore we can approximatively consider it as not interacting with each other. Hence, the high energy particle is essentially free and we expect the signal to carry out information on the hole in the ground state. The spectroscopy in the third band presents an advantage: The spectral properties of the lowest-energy band are mapped onto a relatively large energy range, therefore improving the spectral resolution. The theoretical interpretation of the observed signal is currently under investigation [10].

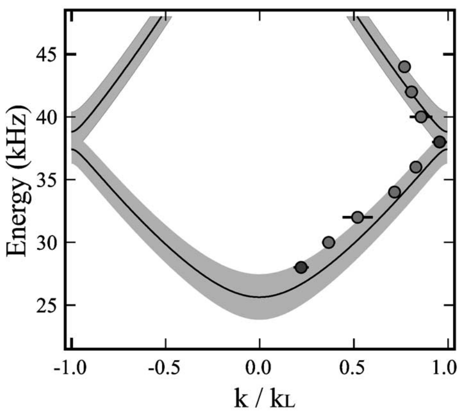

Fig. 6. – Dispersion relation of a Mott state derived from band mapping after a Bragg excitation: Experimental measurements (dots) and single-particle band dispersion relation (straight lines) at $s_y = 10$ with 10% of uncertainty (shaded area) due to the lattice calibration.

6. – Conclusion

In conclusion, we used Bragg spectroscopy to characterize an array of strongly correlated quantum gases in optical lattices. The complexity of such systems appears in the dynamical structure factor we measured via intra-band spectroscopy. From that, we identified the threshold value of the interaction strength above which the first Mott insulating lobe appears, surrounded by a superfluid. In addition, we characterized the novel feature presented by the Mott phase above this quantum critical point. Finally, we implemented inter-band spectroscopy of the Mott state, combined with a band mapping technique which allows us to identify the different bands. We expect that the experimental observations can be described in terms the one-particle spectral function of the hole-like excitation created in the many-body ground-state, that is currently under investigation [10].

* * *

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