

Breaking the symmetry. The first steps of a new way of thinking

I. A. SARDELLA(*)

*Seminario di Storia della Scienza, Università degli Studi di Bari - Piazza Umberto I
70121, Bari, Italy*

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Summary. — The concept of Spontaneous Symmetry Breaking (SSB) represents a real breakthrough for the present description of fundamental interactions by means of gauge theories. Although the underlying ideas were ancient, their formalization required a long time, due to epistemological obstacles and technical difficulties. In this paper, the main steps of SSB evolution are briefly outlined, from the introduction of the order parameter in the Thirties to the birth of the many-body theory at the end of the Fifties. In this context, the contribute of the capital L. Landau's works on phase transitions and quantum fluids, as well as of the seminal ideas of F. London, is highlighted, and the phenomenological approach in theoretical physics (whose features are schematically underlined) is showed to be crucial in the rising field of complex systems.

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1. – Introduction

The role of Spontaneous Symmetry Breaking (SSB from now on) in theoretical physics of the last fifty years can be hardly exaggerated. In the Standard Model it represents the key concept to achieve the formalization of the electroweak sector of the theory on the base of gauge field symmetry. Moreover, the Higgs mechanism, accounting for the mass of the intermediate vector bosons, is a next decisive step towards the unification of forces on the ground of symmetry principle [1]. However, the role of SSB cannot be limited to the area of particles physics; because it represents the pivot of the building of the general theory of phase transitions, it is obviously widely used in ferromagnetism and ferroelectricity, as well as in superfluidity, superconductivity, Bose-Einstein condensation

(*) E-mail: eziosardella@libero.it

phenomena [2]. From a philosophical point of view, SSB has sometimes been considered as the theoretical background for the understanding of emergence phenomena, in the context of a non-reductionist conception of the hierarchical structure of theories [3, 4].

Despite their relevance, the first steps of SSB have not yet received, from historical studies, an adequate attention, apart from reminiscences by physicists⁽¹⁾ [5-7] and not many recollections by professional historians [8, 9]. As a matter of fact, scholars are mainly devoted to the so-called revolutionary periods of science, whereas SSB as a formalized concept started after the quantum mechanics paradigm was established (in Kuhnian words: in a *normal* phase of science). Moreover, it has to be evidenced the objective difficulty of retracing a long and tortuous path, only recently completed, which involves the interlacement of different search fields (not yet recognized as autonomous at the time).

In this paper, an essential historical reconstruction is attempted. Keeping in mind that SSB, perhaps more than other theoretical issues, was the result of a collective work, only some of the essential contributors will be mentioned. Quite conventionally, the end of the story can be fixed at the beginning of Sixties. At that time, the SSB idea acquired a general theoretical meaning, since it was introduced in the framework of particle models, coming from the original area of condensed matter physics. After Heisenberg seminal ideas at the end of the Fifties, this *cross fertilization* was mainly due to Y. Nambu and G. Jona-Lasinio papers about the analogy between the BCS theory of superconductivity and the quantum field theory [10]. It is much more difficult to locate the beginning of the story. Maybe, it can be placed at the end of the XVIII century, with the first Euler's considerations about a cylindrical rod which, subjected to an increasing force, flexes in an apparently casual direction [11]; or, in the first half of the XIX century, with Jacobi's work about a mass subjected to self-gravitation which, due to the rapid rotation, changes its initial spherical form into an ellipsoidal one [12]. In these and other similar observations, it can be found a symmetrical situation which is converted to a less symmetrical one when a parameter exceeds a given value [13].

2. – Why such a long road to SSB?

Now, a first question might arise why did SSB take such a long time to emerge as a general theoretical concept? An answer could be given paying attention to: 1) the philosophical conception supporting SSB and epistemological obstacles (in the Bachelard's sense [14]) interposing to its acceptance; 2) the network of mathematical and physical tools involved in a formalized SSB.

The main idea of SSB is clearly expressed by the inventors of the term, Baker and Glashow [15]: it is plausible that the real-world complexity is not reflected in an equally intricate fundamental theory. The idea of SSB tries to satisfy a comprehensible desire of a scientist: the asymmetry (and complexity) of physical phenomena does not imply that we cannot use symmetry (and simplicity) in describing Nature at a fundamental level. In mathematical terms: the complexity is not present in equations; it arises only in solutions. In this conception it can be found a sort of reconciliation between two

⁽¹⁾ Precious recollections are available in the Archives for the History of Quantum Physics (AHQP, Niels Bohr Library, American Institute of Physics, College Park, Maryland). See in particular the interviews with: F. Bloch (1964, 1968, 1981); P. W. Anderson (1987, 1999); W. Heisenberg (1962, 1970); Y. Nambu (2004).

important ways of thinking in Occidental culture [16]: the idea that the laws of Nature are obscure and inaccessible, because of the complexity of physical phenomena and, on the other side, the representation of Nature as an open, intelligible book⁽²⁾. One could think that an epistemological obstacle may arise from the philosophical difficulty to conceive a disagreement between the deep essence of reality (the laws of Nature) and the manifestation of it (the physical phenomena). This can be seen more clearly keeping in mind the reflections of Pierre Curie, the first one at the end of XIX century to stress the importance of asymmetry in physics⁽³⁾. In 1894 he stated the following laws: 1) For a phenomenon to appear, it is necessary that certain elements of symmetry are missing: *c'est la dissymétrie qui crée le phénomène*; 2) the symmetry of the causes must be preserved in effects; 3) the asymmetry of a phenomenon must arise from asymmetrical causes [18].

In the first law Curie made an important step towards SSB. Indeed, he underlined the connection between the occurrence of a phenomenon and the generation of the asymmetry conditions compatible with it⁽⁴⁾. However, in the last two laws, Curie assumed that causes and effects must have the same degree of symmetry, according to Leibniz principle of sufficient reason. That was really a strong paradigm, in scientific community. As an evidence for this, L. Radicati mentions [13] that Curie did not interpret the experiment of his colleague Bénard in 1900 as a falsification of his laws. In Bénard's experiment, a liquid in a beaker was uniformly heated from below. As soon as the vertical temperature gradient reached a critical value (later measured by Rayleigh), a regular pattern of convection cells appeared, breaking the symmetry in horizontal planes [19]. From considerations dating back to Euler [11], this kind of phenomena could be explained through subtle asymmetric sources existing in causes and manifesting in macroscopic evidence in effects; however, it seemed difficult to accept, in opposition to a generally recognized *continuity principle*, that such effects could manifest only above a definite value of a parameter. A mature awareness of the typical consequences of non-linearity occurred only much later.

In 1950, the American mathematician G. Birkhoff dedicated the first chapter of his book *Hydrodynamics* [20] to the so-called *paradoxes* (in which he mentioned Bénard's experiment). He ironically opposed the established beliefs stating that "Nature has desire for symmetry in the same way as it abhors the vacuum" and explained macroscopic manifestations of asymmetries on the ground that "although the symmetric causes should produce symmetrical effects, nearly symmetrical causes not necessarily produce almost symmetrical effects; a symmetric problem does not necessarily have a *stable* symmetric solution"⁽⁵⁾.

⁽²⁾ The first position is well expressed by the famous Heraclito's sentence: "Nature loves to hide"; the second is poetically rendered by the verses of Goethe: "Seize, then, with no delay, the sacred mystery in broad daylight" [17].

⁽³⁾ The deep interest in symmetry questions by Pierre Curie, head of the prestigious French school of crystallography, led him first to the discover of piezoelectric effect and then to an acute analysis of the reflection properties of electromagnetic fields.

⁽⁴⁾ Pierre Curie expressed his first law in terms of group theory. He applied the law to several physical situations, such as the Wiedemann effect, noting that it can give only a necessary condition for the occurrence of a phenomenon.

⁽⁵⁾ Birkhoff imputed to physicists and engineers a lack of mathematical rigor in the interpretation of hydrodynamic phenomena. He recalled the Oseen result of the *limit paradox*: in systems of differential equations, the presence of arbitrary small terms of high order can completely change the aspect of the solutions.

In more recent years, G. Jona-Lasinio recalled the resistance expressed from particle physics community towards the acceptance of SSB concept. After defining SSB in condensed matter as “the situation occurring when the lowest energy state of a system can have a lower symmetry than the forces acting among its constituents and on the system as a whole”, he remarked that “to appreciate the innovative character of this concept in particle physics one should consider the strict dogmas which constituted the foundation of the relativistic quantum field theory in the late Fifties. One of the dogmas stated that the lowest energy state, the vacuum, should not possess observable physical properties, and all the symmetries of the theory, implemented by unitary operators, should leave it invariant” [10].

Apart from the overcoming of epistemological barriers, the elaboration of the SSB idea required the development of a quantity of theoretical tools.

In order to highlight this point, it is useful to sketch a modern possible definition of SSB. The framework in which this idea can be found is that of a theory expressed by non-linear equations, invariant for transformations by a continuous symmetry group⁽⁶⁾, with a degenerate vacuum structure (*i.e.* the set of physically equivalent states, transforming according to the symmetry group). In the thermodynamic limit (the number of degrees of freedom which tends to infinity), by varying some parameters a bifurcation point can occur in which stable solutions have a lower symmetry than the Lagrangian. The *jump* to the less symmetrical states can be due to fluctuations. The breaking of continuous symmetries are manifested by the existence of collective excitations, whose quanta are the so-called *Goldstone bosons*, massless, corresponding to the group generators in the broken symmetry directions.

As can be seen, SSB is a confluence of sophisticated theoretical concepts coming from linear algebra and group theory, phase transitions, theory of fluctuations and collective excitations. All these theories were laboriously developed over 50 years, on the basis of the pillars of statistical mechanics and quantum mechanics, in a process that led to the birth of the so-called many-body theory.

3. – Measuring the order

Taking into account the above considerations, a possible schematic reconstruction of the nodal points of historical process driving to SSB can be exposed as follows. At the end of the XIX century, capital works by S. Lie and F. Engel on the invariance of equations in symmetry transformations stressed the importance of symmetry considerations in the laws of physics [21]. On the other hand, Pierre Curie underlined the role of asymmetry in physical phenomena. It was just the tension between these two instances which became the natural background for a careful analysis of phenomena, like ferromagnetism, deeply involved in symmetry questions. Ferromagnetism had an essential role inasmuch as it allowed, in a context sufficiently explored in its macroscopic features (among others, by P. Curie himself, whilst microscopic details were to remain mysterious at least until the famous Heisenberg’s work about the quantum exchange interactions in 1928), the easier comprehension of some of the main elements of complex systems: asymmetric ground states, relevance of fluctuations at critical point, collective excitations.

At the beginning of the last century, a very important step was made by the French physicist Pierre Weiss. First, because he focused the attention on ferromagnetism in

⁽⁶⁾ It is possible to have SSB with a discrete symmetry group, such as in the Ising model, but continuous symmetries are mainly involved in gauge field description of fundamental interactions.

crystals (in this way, he was led to emphasize the role of symmetry questions in such a phenomenon⁽⁷⁾); more important, by virtue of his conception of ferromagnetism as a *cooperative phenomenon*. Indeed, Weiss deduced the relationship between magnetic susceptibility and temperature by means of the introduction of a *molecular field*⁽⁸⁾, in order to take into account the collective action of the rest of the system on a given elementary magnetic dipole [23]. The fundamental assumption of such a *mean-field* approach was that of proportionality between the molecular field and the magnetization, that is to say, the degree of asymmetry existing in the system in the ferromagnetic phase. So, Weiss was able to obtain a self-consistent equation to calculate the magnetization and identify a definite temperature at which the transition occurs between the paramagnetic and the ferromagnetic phase⁽⁹⁾. This kind of approach was phenomenological because the molecular field was not really deduced from molecular interactions.

In the first decades of the century, the analysis of ferromagnetic systems intertwined with the issues coming from a still lacking theory of phase transitions. In a pretty obscure paper, in 1914, the Dutch physicists Ornstein and Zernike stressed the importance of density fluctuations in phase transitions deducing the phenomenon of critical opalescence from the existence of density correlations which increase indefinitely in range as the critical point is approached [24]. That was really a crucial step for phase transition understanding, since it allowed to go beyond the statistical mechanics assumption of neglecting fluctuations in favor of the average values.

At the end of Twenties, the mean-field technique adopted in ferromagnetism proved to be very useful in the theoretical approach to the so-called *superlattices*, a new line of research in the field of crystallography. At that time, mainly due to constant advances in resistivity measures and X-ray spectroscopy technology, a precise analysis of the lattice structure of binary alloys had become possible. As a clear experimental result, at low temperature the atoms of two different metals were found to be strictly ordered in a regular pattern (*e.g.*, each atom of one metal surrounded by atoms of the other metal, in a body-centered cubic lattice), whilst at high temperature they appeared to be accidentally distributed among the sites of the lattice⁽¹⁰⁾. Through a series of investigations mainly due to Gorsky in Leningrad, Dehlinger in Stuttgart and the group of Borelius, Linde and Johansson in Stockholm, it became evident that it was involved a transition with no latent heat, occurring in a fairly small temperature range and accompanied by large specific heat and electric resistance [25]. In order to describe a transition within two solid phases, these physicists realized that a new thermodynamic parameter was needed, whose aim was to measure the degree of order in the lattice due to the atomic distribution. In 1934, the British physicists W. L. Bragg and E. J. Williams defined the concept in a profitable way to build a mean-field theory of superlattices⁽¹¹⁾. Under the crucial assumption of

⁽⁷⁾ With respect to this point, it was very important the availability of high-quality crystals. Weiss managed to seize pyrrhotite crystals from Brazil perfectly homogeneous over long distances [22].

⁽⁸⁾ Suggested to him by the internal pressure parameter in van der Waals equation of real fluids.

⁽⁹⁾ In 1910, the transition temperature was named *Curie point*, on a proposal by P. Weiss and K. Onnes, even if then French physicist localized only an interval of transition.

⁽¹⁰⁾ The first indications for this behavior were due to Tammann in 1919.

⁽¹¹⁾ Bragg and Williams defined the parameter S , which they called *degree of order*, in the following way: $S = \frac{p-r}{1-r}$, where p is the actual probability that a given position in the lattice is occupied by the atom required for complete order; r is the value of p in case of complete disorder. When order is complete, $p = 1$ and $S = 1$; at the transition point, when order disappears, $p = r$

the linear dependence of the potential energy involved in an atomic interchange from the order parameter, they obtained a self-condition equation which gave a sharp transition point [26]. In a series of three papers, they clarified the relation between the order parameter and the presence of long-range order in the lattice. Successively, Bethe [27] and Peierls [28] introduced a new parameter (unlike the previous one, persisting above the transition point) to take account for short-range correlations, neglected by Bragg and Williams, and improve the approximations. In the following debate, interesting issues related to the evaluation of long- and short-range order in mean-field theories were addressed [29].

As a matter of fact, the order parameter was conceived as an instrumental tool for mean-field theories in order-disorder transitions. However, its introduction was also needed to identify continuous transitions even in situations where the presence of “order” was less obvious. In the early Thirties, the Dutch physicists Gorter and Casimir, with a phenomenological approach [30], applied thermodynamics on superconductivity⁽¹²⁾. Adopting Kronig’s hypothesis of superconductive phase as a coexistence of a gas of free electrons with a sort of a lattice of electrons, they considered the transition from superconducting to normal phase as a phenomenon of evaporation of electrons. Defining a so-called “internal parameter” as the fraction of the electrons being in the lattice, they identified the transition temperature as the point in which that parameter vanishes. More important, they used the concept as a way to affirm, in a lively exchange of views with von Laue and Justi, an essential feature of a continuous transition: the transformation of one phase into another without metastable states [31].

4. – The Landau’s theory of phase transitions

In 1937, Landau’s work about the theory of phase transitions [32] responded to the need for clarification of key questions relating mainly to the concept of continuous transition. Suggested by Keesom’s observation of a peak in the specific heat of ^4He at the so-called λ -point (*i.e.* the transition temperature between the superfluid and the normal phase), the Ehrenfest classification of phase transitions based on discontinuity of the derivatives of thermodynamic potentials showed the possibility of a general description but did not clarify crucial questions such as the conditions of their occurrence, the physical mechanism driving to discontinuities, the possibility of metastable states [33]. The fundamental Landau’s belief was that such transitions can be understood in terms of a change in the symmetry properties of the stable states of the system, through a mechanism of spontaneous symmetry breaking. That conception implies that in a continuous transition an order parameter is always involved, and a proper description can only be given in terms of its variations⁽¹³⁾. Landau referred to the principle: *a given symmetry*

and $S = 0$. However, they recognized the birthright of Gorsky in 1928 in identifying a degree of order and applying the mean-field method to the alloys.

⁽¹²⁾ The thermodynamic approach was supported in Netherlands by Ehrenfest and Rutgers even before the fundamental Meissner and Ochsenfeld’s experiment in 1933 that, showing the phenomenon of screening of the magnetic field from the interior of the metal, proved the reversibility of the superconducting state and launched the new paradigm of superconductor as a perfect diamagnetic material.

⁽¹³⁾ In comparison with previous concepts, the order parameter assumed with Landau a more crucial role, because it became a general measure of the breaking of symmetry in stable states. This meaning was the key to its extension to more abstract fields than crystallography.

element is either present or absent, to explain the transition as a process of *abrupt* reduction of symmetry⁽¹⁴⁾ in which, nevertheless, below the critical temperature the order parameter starts to increase *continuously* from zero to non-zero values⁽¹⁵⁾. Therefore, L. Landau assumed that the free energy Φ could be expanded in series of the order parameter η , near the critical point: $\Phi = \Phi_0 + A\eta^2 + B\eta^4 + \dots$ (where only the even powers of η are considered for symmetry reasons and the coefficients A, B, \dots are supposed to be functions of the other thermodynamic variables of the system⁽¹⁶⁾). The value of η , at a given temperature, can be obtained through the minimization of Φ . Under a general assumption about the signs of the coefficients, the variation of the order parameter allows to reproduce the phase transition, that is the discontinuity in derivatives of Φ .

The real Landau's breakthrough was the identification of a general mechanism for SSB, in principle regardless of the particular area of application. Moreover, in the framework of the theory, it was easy to show a very important feature of complex systems near the critical point: the *universal behavior*, due to the presence of long wavelength fluctuations which cause the lack of relevance of microscopic details.

Landau's theory had a relevant impact to Soviet scholars [34], but in Western Countries in the early times it was somewhat underestimated, for several reasons. Primarily, due to problems of communication during the War; second, for mathematical doubts about the convergence of the free energy series [35]. Further, it should be recalled that the real possibility to reproduce the singular behavior of thermodynamic potentials in a critical point by means of analytic functions was still an open general question [36].

In 1944, after the Onsager's exact solution of two dimensional Ising model [37], it became clear that phase transitions could be captured by statistical mechanics methods, but also, that Landau's theory suffered from the limitation of any mean-field approach in reproducing accurately the behavior of the system just near the transition. As a result, its physical content was in some extent obscured. In more recent years, the theory was widely put into great consideration: in 1984 Anderson defined Landau's statement about the impossibility to change symmetry gradually as the First Theorem of solid-state physics [38].

5. – Collective excitations

Goldstone theorem (1961) asserts that SSB of a continuous symmetry group implies the existence of massless bosons due to collective excitation in the direction of the breaking of symmetry [39]. Since Debye's work in 1912 in which wave excitations in crystals

⁽¹⁴⁾ In algebraic terms, the symmetry group of one phase (in general, the *disordered* or symmetric phase) is required to admit as a subgroup the one of the other phase (*ordered*, or asymmetric phase).

⁽¹⁵⁾ In the Thirties, a debated question was the possibility of a continuous transition between the solid and the liquid phase. Y. Frenkel expressed the conviction, based on supposed structural similarities between the two phases, that elements of disorder can gradually appear in the crystal lattice driving to a continuous transition to the liquid phase. L. Landau strongly objected to this idea on the base of his principle.

⁽¹⁶⁾ In [32] the order parameter η is first deduced from the expression of the distribution probability of the atoms of a crystal written in terms of the irreducible representations of the its symmetry group; however, in Landau-Lifshitz volume dedicated to Statistical Mechanics the free energy expansion precedes the discussion on the distribution probability, which indicates the intention to give it a more general meaning.

were considered in continuous limit (in that case, involving SSB in rotational and translational symmetry), Goldstone's fundamental issue was (unconsciously) anticipated from several condensed matter results. Undoubtedly, an important step was Bloch's paper in 1930 on ferromagnetism [40]. He studied the behavior of a ferromagnetic material at very low temperature, showing that, in the same way as Debye's propagation of elastic perturbations in crystals, the deviations from perfect order (*i.e.* complete alignment of spins) occur through propagation of spin waves. Of these collective excitations, Bloch obtained the continuous energetic spectrum at low frequency. Afterward, Soviet physicists applied the Dirac's method of II quantization to elastic and magnetic oscillation fields by introducing *phonons* (Tamm, 1930) and *magnons* (Pomeranchuk, 1944).

As a matter of fact, the collective behavior had a very important role in the Soviet approach to condensed matter physics [41]. In the Twenties, in the lively debate between the proponents of the opposite models of conductivity based on completely free and quite bounded electrons, Y. Frenkel⁽¹⁷⁾ proposed an intermediate model based on collective sharing of electrons by the nuclei of the metal and, later, introduced a quantum of collective excitation called *exciton*. Because of the lack of a satisfactory formalization of his theory and of some redundancy of political metaphors [41], Frenkel's ideas were not widely considered in the Western scientific community. However, that line of research was prosecuted, on one hand, by the works of Bohm and Pines on plasma in the Fifties [42] and, on the other hand, by the fundamental Landau's work on superfluidity in 1941.

During the war years, much effort of the scientific community was put in the great puzzles of *quantum fluids*. The interest was stimulated by the very unusual experimental behavior of superconductors and superfluids that could be fraught with technological implications, as well as by more theoretical interest in suspected macroscopic quantum aspects and variety of possible applications, such as atomic nuclei, neutron stars, plasma. However, it soon became clear that a relevant shift was needed in theoretical approach: from the single-particle to the many-particle models in which, in principle, the interactions could not be neglected.

A very bold approach to superfluidity was attempted in 1938 by the German physicist F. London, and prosecuted by the Hungarian L. Tisza, student of Landau in Kharkov. Starting from the alleged knowledge that the atoms of ^4He are bosons, he observed that the experimental measures of the λ -point were not so far from Einstein's estimation of the transition temperature in his debated paper in 1925 about the *Bose-Einstein condensation* (BEC). Therefore, he proposed BEC as a microscopic mechanism for superfluidity [43]. Assuming that at temperatures below the λ -point a finite fraction of atoms can be found in the ground state of zero momentum (*Bose-Einstein condensate*), London could derive the zero viscosity of the superfluid part of helium. London's idea was that it is possible to attribute a single macroscopic wave function to all the particles in the condensate, which therefore moves coherently without friction. The proposal was marked by controversial points: BEC was in doubt for reasons of mathematical rigor⁽¹⁸⁾ and, above all, it referred to an ideal gas and it was not known in advance what the effect of the interactions could be. However, London's view was going to open a fundamental search line [44]. From the point of view of SSB history, it can be considered an essential

⁽¹⁷⁾ Y. Frenkel was the head of the theoretical division in the Physico-Technical Institute in Leningrad. L. Landau worked there, before establishing his famous school in Kharkov.

⁽¹⁸⁾ Among others, Uhlenbeck was not sure, until 1937, of the correctness of Einstein result. That negatively influenced the acceptance of theory.

step, as it was put the focus on different, more abstract kinds of symmetry changes in a phase transition. Indeed, Bose-Einstein condensation involved an order in the space of momenta, rather than in the configurational space as usual⁽¹⁹⁾.

The theory of London proposed a mechanism but was not able to provide quantitative predictions. New technical, as well as conceptual tools were required. Landau rejected London's approach of BEC: in Soviet physics vision, the liquids were considered much more similar to solids than to ideal gases [41]. In that conception, collective excitations were destined to assume an increasing relevant role.

As a matter of fact, Landau's paper in 1941 [45] was really decisive to introduce a new paradigm in condensed-matter physics. It can be stated in the following way: *For any system with strongly interacting particles there must exist a phenomenological description in terms of an ideal gas of fictitious particles (or quasiparticles), describing the collective excitations.* In this framework, Landau gave a description of the very unusual behavior of helium in the superfluid phase. In a fluid flowing through a capillary, the presence of viscosity, due to the kinetic energy loss for frictions with the walls of the capillary and inside the liquid itself, is manifested by the appearance of internal motions in the form of elementary excitations. These can be seen as longitudinal density waves (*phonons*) and vortex excitations (*rotons*). Therefore, at finite temperature below the λ -point, helium liquid was considered by Landau to consist of a gas of phonons and rotons in a superfluid background. The quantitative character of the theory was assured by the assumption of a particular phenomenological form of the energy spectrum of the elementary excitations, in which some parameters had to be fixed according to experimental results. In this way, Landau predicted a critical velocity of the helium liquid (above which superfluidity disappears owing to the excitations of phonons and rotons) and the famous *second sound*, a collective excitation consisting of temperature oscillations.

6. – Complex is better than real

The main difference between Landau and London's approaches to superfluidity can be traced back in the relevance assigned, for the first theory, to the properties of the low-energy spectrum of excitations; for the second one to the effect on the ground state of the symmetry properties of wave functions. The reconciliation between these two aspects came in 1947 from a pretty unacknowledged paper⁽²⁰⁾ by the Soviet physicist N. Bogolubov [47]. Adopting the field-theory method of II quantization he succeeded in demonstrating that BEC is not much altered by the presence of weak interactions and that the presence in the state of zero momentum of a finite fraction of quasiparticles implies a spectrum of low-energy excitations very similar to that assumed by Landau. Bogolubov's contribution was recognized much later as the first step towards a micro-

⁽¹⁹⁾ In his celebrated text, *Superfluids*, in 1950, F. London gave reasons to affirm that helium at zero pressure cannot maintain any structure ordered in space; therefore, he proposed an alternative form of order to justify the λ transition.

⁽²⁰⁾ In spite of its relevance, Bogolubov's paper was not properly quoted in the important subsequent works by Yang, Lee, Feynman. Instead, Fritz London reviewed the work, as kindly pointed out to me by N. N. Achasov (who acknowledged Bogolubov as the author of the theory of quantum fluids from the first principles [46]). The difficulty of communication was surely due to the war years climax, but Griffin also mentions the question of the still not fully assimilated methods of field theory and the presence of approximations with non-conserved number of particles in Bogolubov's work [44].

scopic theory of quantum fluids. However, at the time, the phenomenological approach again proved to be extremely fruitful in dealing with superconductivity.

Since the late Thirties, the most reliable description of superconductive behavior (in the case of weak magnetic fields) was based on a set of phenomenological equations proposed in 1935 by the two brothers Fritz and Heinz London [48]. In order to provide the behavior of perfect diamagnetism in a superconductor, they assumed the principle that the superconductive density currents \mathbf{J}_s are always determined by the local magnetic field, according to the relation $\mathbf{J}_s = -\frac{e^2}{mc} n_s \mathbf{A}$, where \mathbf{A} is the potential vector; n_s is the concentration of the superconducting electrons; m and e are mass and charge of electron⁽²¹⁾. In the concluding remarks of the paper, the Londons tried to justify their phenomenological relation from quantum mechanics considerations, starting from the well-known Gordon equation of the quantum current in the presence of a magnetic field $\mathbf{J} = \frac{he}{4\pi im} (\psi \nabla \psi^* - \psi^* \nabla \psi) - \frac{e^2}{mc} \psi \psi^* \mathbf{A}$, where ψ is the wave function of a single electron. In the case of normal conductors without magnetic field the total current is zero, because summing over all the electron currents the first term vanishes for symmetry reasons. In case of superconductors, the Londons assumed the existence of an *energy gap* (due to non-well-specified electron coupling) between the ground state and the first energy levels of the internal motions. This means that, with sufficiently weak magnetic field, the electron wave functions remain essentially unperturbed⁽²²⁾. Thus, summing over all the electron currents, the first term is always vanishing and, interpreting $\sum \psi \psi^*$ as n_s , the London relation results.

In 1950, in a fundamental paper [49], L. Landau and V. Ginzburg elaborated a theoretical structure to explain the superconductive behavior and deduce the London equation, through an appropriate application of the Landau's theory of phase transition. The starting point was that, having assumed the existence of a continuous transition between the superconductive and the normal phase, an order parameter must exist. Now, it is clearly possible to develop a phenomenological theory without establishing exactly the correspondence of any parameter to physical quantities at a microscopic level; however, the theory has to be structured so that such a connection could be eventually found, in a *first-principle* approach. Therefore they considered the order parameter (denoted with the symbol Ψ) as a sort of an *effective wave function* of the superconductive electrons which takes into account the overall behavior of the particles. In so doing, they give to the order parameter a new fundamental development assuming it as a *complex function*. The connection with microscopic quantum mechanics was proposed to be realized through the *density matrix* ρ , a statistical object introduced in the early Twenties independently by Landau and von Neumann as a quantum analogue of the phase space probability measure. Ginzburg and Landau considered the one-particle density matrix $\rho_1(\mathbf{r}, \mathbf{r}') = \int \psi^*(\mathbf{r}, \mathbf{r}'_i) \psi(\mathbf{r}', \mathbf{r}'_i) d\mathbf{r}'_i$. In this expression, $\psi(\mathbf{r}, \mathbf{r}'_i)$ is the true wave function of the N electrons in the metal, depending on the coordinates of all electrons \mathbf{r}_i ($i = 1, 2, \dots, N$); the integral is extended to the coordinates \mathbf{r}'_i of all the electrons except the one considered; the density matrix refers to the two points given by the coordinates \mathbf{r} and \mathbf{r}' . Below the transition point, the order parameter Ψ must assume a

⁽²¹⁾ London's equation can be considered as a sort of magnetic Ohm's law for superconductors, in which the vector potential replaces the electric field.

⁽²²⁾ The property of *rigidity* of the ground-state wave function, supposed by F. London for superconductors, was later recognized by P. W. Anderson as one of the main macroscopic features of SSB [38].

value different from zero. In Landau and Ginzburg view that implies, like in a ferromagnet, the existence of long-range correlations, which is expressed by the condition $\lim_{\mathbf{r}-\mathbf{r}'\rightarrow\infty} \rho_1(\mathbf{r}, \mathbf{r}') \neq 0$. Thus, to establish the relation of electron wave functions with the order parameter they supposed the factorization $\lim_{\mathbf{r}-\mathbf{r}'\rightarrow\infty} \rho_1(\mathbf{r}, \mathbf{r}') = \Psi^*(\mathbf{r}) \Psi(\mathbf{r}')$ to be valid. Of course, an essential requirement of a phenomenological theory is the accountability of previous empirical laws (in this case, the London equations); in order to obtain that result Ginzburg and Landau imposed the normalization condition $|\Psi|^2 = n_s$.

The theory was built in such a way that it preserved the invariance for global variations of the phase of the order parameter (the so-called $U(1)$ gauge symmetry): to the expansion of free energy in series of $|\Psi|^2$ was added the gradient term $\frac{1}{2m} |\mathbf{p} - i\hbar \nabla \Psi - \frac{e}{c} \mathbf{A} \Psi|^2$ to take into account the interaction with the magnetic field by means of the quantum coupling rule. Minimizing the free energy with respect to Ψ^* and \mathbf{A} , Ginzburg and Landau derived a set of differential equations, whose solutions, providing the values of the order parameter and the potential vector, allow to calculate the superconductive current. In the case of weak magnetic fields, the London equation can be derived under the assumption that the zero field constant solution Ψ is not perturbed and so $\nabla \Psi = 0$ can be placed in the expression of the current.

Landau and Ginzburg theory allowed a satisfying quantitative description of superconductivity, which was valid for magnetic fields of any intensity. In 1959, the Soviet physicist Gor'kov [50] was able to derive, in the neighborhood of the critical point, the Ginzburg and Landau equations from the BCS (Baarden, Cooper, Schrieffer) microscopic theory⁽²³⁾. However, apart from the particular context of application, the main feature that later acquired enormous relevance was the extremely flexible structure of the theory, useful for the formalization of different theoretical fields [51], and the highly descriptive effectiveness of a SSB mechanism in which a complex function was used as an order parameter. Indeed, in the next evolution of the SSB concept, a very important role was assumed by the order parameter phase.

7. – Marching towards microscopic theories

Starting from the Fifties, the aim of a large part of condensed-matter physicists became the search of a microscopic justification of the phenomenological theories. First in superfluidity, then in superconductivity, the key was found in Bose-Einstein condensation and the symmetry properties of boson wave functions. A preliminary essential question was to be sure that BEC really occurred in helium. Paradoxically, the technical way to answer the question came just from the consideration of long range order correlations in the framework of density matrix method, despite Landau was not convinced of BEC occurrence.

In 1951, in Penrose seminal paper [52] the existence of long-range correlations in the configuration space below the λ -point (due to the extension of de Broglie wavelengths of the particles over the average interatomic distance) was expressed in terms of density matrix and showed to be equivalent, according to London's suggestion, to the condition of concentration of particles in momentum space. Moreover, the factorization of density matrix (which was previously proposed by Ginzburg and Landau) was formally justified⁽²⁴⁾ and the function Ψ recognized as the wave function of the single particle in the

⁽²³⁾ In order to get this result, the charge of the electron e was replaced with $2e$, which refers to the charge of the Cooper pairs.

⁽²⁴⁾ In his paper R. Penrose did not cite the work of Landau and Ginzburg. As a matter

condensate state. In this way, the hydrodynamic behavior of superfluids was related to SSB mechanism: the velocity of superfluid current resulted to be connected to the gradient of the phase of Ψ . This means that, although the laws of motions are gauge invariant, in a superfluid at constant velocity the phase of the wave function of the particles in the ground state is fixed at an arbitrary but coherent value for the whole condensate.

The work of Penrose was the first of a series of papers in which Onsager-Penrose [54] and Yang [53] established strict mathematical criteria to be satisfied in case of BEC (all based on density matrix behavior below the λ -point) which were denoted as the *Off Diagonal Long Range Order* (ODLRO) condition.

However, an essential element was still needed for a microscopic approach to superfluidity: a reliable ground-state wave function to test the ODLRO criterion. This came out from the prominent works by R. Feynman in the mid-Fifties [55]. The American physicist, judging the task of exactly calculating the wave function of low-energy states virtually impossible, ingeniously elaborated an *ansatz* to guess their formal structure, on the basis of the boson symmetry property. In such a way, not only the ODLRO criterion was showed to be satisfied by helium [54], but, moreover, Feynman and Cohen [56] were able to calculate a low-energy spectrum very similar to Landau's.⁽²⁵⁾

In superconductivity, the march to the microscopic theory was successful in 1957, with the celebrated BCS theory [57]. That is a very complex theory, but the main ideas involved, such as the role of the energy gap due to some coupling between the electrons and the relevance of low-energy spectrum of excitations, were already in London's mind since the end of the Thirties, as well as in Landau's description of a quantum fluid in 1941.

Indeed, the microscopic explanation of the superconductive phase was made showing that the so-called Cooper pairs (electrons interacting through the mediation of phonons in the lattice) can form a stable state of lower energy than electrons in normal phase⁽²⁶⁾. At low temperatures, no excitations were found in the energy spectrum from the ground state with vanishing energy (energy gap). This explained the superconductive behavior, through the Meissner effect. Because of the gap vanishing at a critical point, the continuous transition resulted⁽²⁷⁾.

At the end of the Fifties, the time was right for the SSB passage to the physics of elementary interactions. In 1958, N. Bogolubov reformulated the BCS theory building the ground state and the low-energy excitation states through the coherent superposition of electron and hole wave functions (Bogolubov's *quasiparticles* [58]). In 1961, after Heisenberg's works about the spontaneous breaking of isotopic spin symmetry in a nonlinear theory of elementary particles [59], Y. Nambu proposed an SSB mechanism to generate

of fact, the Ginzburg and Landau factorization was showed to be wrong in the context of superconductivity, because of the fermionic wave functions involved [53]. In that case, a two-particle density matrix must be considered. Anyway, the phenomenological theory showed to be effective beyond the physical interpretation of the order parameter.

⁽²⁵⁾ As a matter of fact, Onsager and Penrose found that about only one tenth of the helium atoms is in the condensate at zero temperature, even though the whole liquid is superfluid. That means that there exists a subtle relation between the superfluid density and the condensate fraction, and it sounds as a justification for Landau's opinions.

⁽²⁶⁾ The ground state of the BCS theory can be associated with the boson Cooper pairs condensation. However, the phenomenon of BEC in BCS was not much emphasized in the early years and became totally accepted only in the Eighties [44].

⁽²⁷⁾ In Gor'kov derivation of Ginzburg-Landau theory from BCS, the energy gap was found to be proportional to the order parameter Ψ [31].

nucleons masses from a massless bare fermion theory [60], at the same way the energy gap arises in BCS theory. The crucial point was the strict analogy between the Bogolubov equations of excited states and the Dirac's equation of quantum electrodynamics. In this analogy, a correspondence was found between the spontaneous breaking of charge symmetry in Bogolubov's ground state and the violation of chirality symmetry in particle theory; the energy gap and the observed nucleon mass; the collective excitations of quasiparticles and the bound nucleon pairs, or mesons.

A new way of thinking was beginning, in theoretical physics.

8. – Long live the phenomenology!

As a matter of fact, the SSB concept was elaborated through a combined action of first principle and phenomenological physics, in a dialectical relationship that can be traced back to that of the late nineteenth century between thermodynamics and kinetic theory [61]. In the present reconstruction, it has been showed the role of the phenomenological approach: it was decisive to *introduce* the main ideas involved in SSB. Let us now briefly summarize in some points the chief features which characterized that way of making physics.

1) Accepting the Kuhnian repartition of science historical periods, it can be said that the phenomenological approach developed in a *normal* phase of physics history. Its natural background was the physics of problems, rather than the physics of principles.

2) A typical frame of mind was the neglectfulness of philosophical questions, following the Landau's remark: "Think less about foundations!" [62].

3) The phenomenological approach was mainly based on a *bottom-up* method, that is to say, the elaboration of a theoretical model starting from the empirical manifestations of phenomena, in order to provide a structural synthesis of its fundamental relations [63].

4) The leading way to reach this result was to create flexible mathematical structures, in which parameters could be fixed according to experimental results.

5) The aim was that of looking at the state of things, as far as possible, neglecting the microscopic details. In this way, mathematical difficulties in treating elementary interactions were bypassed in favor of a *universal* description.

6) In this framework, a capital relevance was acquired by Statistical Mechanics, inasmuch as it allows to approach complex problems with the maximum of generality.

7) From the point of view of the research organization, the *physics of problems* was particularly cultivated in the context of schools (Sommerfeld in Munich, Heisenberg in Leipzig, Landau in Kharkov, P. Ehrenfest in Leiden), marked by common features of informality, broad interests, attractiveness to young people, links with experimentalists. In these communities, a major emphasis was given to didactics, as a decisive instrument to train a generation of physicists in the new techniques (it is hardly necessary to remind the monumental Landau-Lifshitz course [64]).

8) Ultimately, it can be said that phenomenology not only oriented the first principle physics towards well-definite theoretical targets, but also provided general ideas to enlighten at the same time different fields of research.

Anyway, SSB was the yield of a physics started to become very different from what was before.

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