IL NUOVO CIMENTO Vol. 34 C, N. 6 Novembre-Dicembre 2011
DOI 10.1393/ncc/i2011-11069-6

Colloquia: IFAE 2011

Masses and mixings in grand-unified models

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(ricevuto il 29 Luglio 2011; pubblicato online il 6 Dicembre 2011)

Summary. — We make a general study of $SO(10)$ models with type-II see-saw dominance and show that an excellent fit can be obtained for fermion masses and mixings, also in comparison with other realistic $SO(10)$ models.

PACS 12.10.Dm – Unified theories and models of strong and electroweak interactions.

1. – Introduction

In the last twenty years we achieved a rather precise knowledge of the leptonic mixing angles, which, within the experimental accuracy, are consistent with the Tri-Bimaximal (TB) pattern [1] and, as such, are very different from the quark mixing angles. In fact the quark flavour structure is characterized by hierarchical masses and small mixing angles, while the lepton sector presents a milder hierarchy in the neutrino masses and two large and one small mixing angles.

It is well known that with the see-saw mechanism the very small neutrino masses point to a very high energy theory of lepton flavour, such as a Grand Unified Theory (GUT). In particular in this context, among the possible unified groups, $SO(10)$ is very interesting because the right-handed neutrinos are naturally introduced and are not gauge singlets, unlike the Standard Model or $SU(5)$. A still open and challenging problem is that of formulating a natural $SO(10)$ grand-unified model leading to a good description of quark masses and mixing and, at the same time, with a TB lepton mixing structure built-in in a well-defined first approximation, due, for example, to an underlying (broken) flavour symmetry. In $SO(10)$ the main added difficulty with respect to $SU(5)$ is clearly that all fermions in one generation belong to a single 16-dimensional representation, so that one cannot separately play with the properties of the $SU(5)$-singlet right-handed neutrinos in order to explain the striking difference between quark and neutrino mixing.

2. – A class of models

A promising strategy in order to separate charged fermions and neutrinos is to assume a renormalizable $SO(10)$ model with dominance of type-II see-saw [2] (with respect to type-I see-saw) for the light neutrino mass matrix. In renormalizable $SO(10)$ models the fermion masses are generated by Yukawa couplings with Higgs fields transforming as **10**, $\overline{\mathbf{126}}$ (both symmetric) and **120** (antisymmetric) [3]

$$W_Y = h\,\psi\psi H_{10} + f\,\psi\psi H_{120} + h'\,\psi\psi H_{\overline{126}}, \tag{1}$$

where the symbol ψ stands for the **16** dimensional representation of $SO(10)$ and H_i are the Higgs fields. I note that in this analysis we assume an underlying "parity" symmetry (justified by the fact that, as we shall see, the resulting fit is very good) that implies that all mass matrices obtained from h, h' and f are Hermitian [4]. The resulting Yukawa mass matrices for the different fermions are

$$\begin{aligned} M_u &= (h + r_2 f + r_3 h')v_u, & M_d &= r_1(h + f + h')v_d, \\ M_e &= r_1(h - 3f + c_e h')v_d, & M_{\nu^D} &= (h - 3r_2 f + c_\nu h')v_u, \end{aligned} \tag{2}$$

and with type-II see-saw dominance the neutrino mass matrix is

$$m_\nu = f v_L. \tag{3}$$

So if type-II see-saw is responsible for neutrino masses, then the neutrino mass matrix (proportional to f) is separated from the dominant contributions to the charged fermion masses (h for example) and can therefore show a completely different pattern. This is to be compared with the case of type-I see-saw where the neutrino mass matrix depends on the neutrino Dirac and Majorana matrices and, in $SO(10)$, the relation with the charged fermion mass matrices is tighter.

An important observation is that, without loss of generality, we can always go to a basis where the matrix f is of the TB type. In fact, if we start from a complex symmetric matrix f' not of the TB type, it is sufficient to diagonalise it by a unitary transformation U: $f'_{\text{diag}} = U^T f' U$ and then take the matrix

$$f = U^*_{\text{TB}} f'_{\text{diag}} U^\dagger_{\text{TB}} = U^*_{\text{TB}} U^T f' U U^\dagger_{\text{TB}}. \tag{4}$$

As a result the matrices f and f' are related by a change of the charged lepton basis induced by the unitary matrix $O = UU^\dagger_{\text{TB}}$ (in $SO(10)$ the matrix O rotates the whole fermion representations $\mathbf{16_i}$). Since TB mixing is a good approximation to the data we argue that this basis is a good starting point. In fact in this basis the deviations from TB mixing will be generated by the mixing angles from the diagonalisation of M_e which in $SO(10)$ are strongly related to the CKM angles and so are automatically small, while in general could be large.

3. – The analysis

An interesting question is to see to which extent the data are compatible with the constraints implied by this interconnected structure. So here we do not consider the problem of formulating a flavour symmetry or another dynamical principle that can lead to approximate TB mixing, but rather study the performance of the type-II see-saw $SO(10)$ model in fitting the data on fermion masses in comparison with other models architectures.

TABLE I. – *Fit results for each model as explained in the text.*

Model	d.o.f.	χ^2	χ^2/d.o.f.	d_{FT}	d_{Data}
DR	4	0.41	0.10	7.0×10^3	1.3×10^3
ABB	6	2.8	0.47	8.1×10^3	3.8×10^3
JLM	4	2.9	0.74	9.4×10^3	3.8×10^3
BSV	< 0	6.9	-	2.0×10^5	3.8×10^3
JK2	3	3.4	1.1	4.7×10^5	3.8×10^3
GK	0	0.15	-	1.5×10^5	3.8×10^3
T-IID	1	0.13	0.13	4.7×10^5	3.8×10^3

As comparison models we use a set of realistic $SO(10)$ theories with different features: renormalizable or not, with lopsided or with symmetric mass matrices, with various assumed flavour symmetries, with different types of see-saw and so on. Of course in these models TB mixing appears as accidental, and some dedicated parameters are available to fit the observed neutrino masses and mixing angles without a specific TB structure implemented. The models considered are [5]: Dermisek, Raby (DR); Albright, Babu, Barr (ABB); Ji, Li, Mohapatra (JLM); Bajc, Senjanovic, Vissani (BSV); Joshipura, Kodrani (JK2); Grimus, Kuhbock (GK).

Each model is compared with the same set of data on masses and mixing given at the GUT scale (except for DR that requires a large value of $\tan\beta$) [5]. The results of the analysis are shown in table I, where it is shown the χ^2 and the χ^2/d.o.f. obtained from the fit for each model. We also introduce as additional quality factor a parameter d_{FT} for a quantitative measure of the amount of fine-tuning of parameters which is needed in each model. This adimensional quantity is obtained as the sum of the absolute values of the ratios between each parameter and its error (defined for this purpose as the shift from the best-fit value that changes χ^2 by one unit with all other parameters fixed at their best-fit values), $d_{\mathrm{FT}} = \sum |\frac{par_i}{err_i}|$. It has to be compared with a similar number d_{Data} based on the data (*i.e.* the sum of the absolute values of the ratios between each observable and its error as derived from the input data), $d_{\mathrm{Data}} = \sum |\frac{obs_i}{err_i}|$.

In conclusion we have shown that a $SO(10)$ model with type-II see-saw dominance can achieve a very good fit of fermion masses and mixings also including the neutrino sector (provided that the representations **10**, $\overline{\mathbf{126}}$ and **120** are all included). The quality of the fit in terms of χ^2 and χ^2/d.o.f. is better than or comparable with any other realistic $SO(10)$ model that we have tested. However, the tight structure of the T-IID model implies a significantly larger amount of fine tuning with respect to more conventional models like the DR or the ABB and JLM models. But those models have no built-in TB mixing and in fact could accommodate a wide range of mixing angle values.

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