Constraining charged Higgs bosons from measurements of $B^\pm \to \tau^\pm \nu_\tau$ and $B_c^\pm \to \tau^\pm \nu_\tau$ at Giga Z

A. G. Akeroyd

School of Physics and Astronomy, University of Southampton
Highfield, Southampton, SO17 1BJ, UK

(ricevuto il 20 Luglio 2011; pubblicato online il 19 Ottobre 2011)

Summary. — The measurement of $B^\pm \to \tau^\pm \nu_\tau$ at the $B$ factories provides important constraints on the parameter $\tan \beta/m_{H^\pm}$ in the context of models with two Higgs doublets. Limits on this decay from the LEP1 experiment (i.e. $e^+e^-$ collisions at the $Z$ peak) were sensitive to the sum of $B^\pm \to \tau^\pm \nu_\tau$ and $B_c^\pm \to \tau^\pm \nu_\tau$. We point out that a future $e^+e^-$ Linear Collider operating at the $Z$ peak (the Giga Z option) could constrain $\tan \beta/m_{H^\pm}$ from the sum of these processes with a precision comparable to that anticipated at proposed high-luminosity $B$ factories from $B^\pm \to \tau^\pm \nu_\tau$ alone. This work is an updated summary of the publication Phys. Rev. D, 77 (2008) 115018, by Akeroyd, Chen and Recksiegel.

PACS 12.60.Fr – Extensions of electroweak Higgs sector.
PACS 13.20.He – Decays of bottom mesons.

1. – Introduction

The purely leptonic decay $B^\pm \to \tau^\pm \nu_\tau$ has been observed at the $B$ factories. The measured branching ratio (BR) is in agreement with the Standard Model (SM) prediction within theoretical and experimental errors. The average of the BELLE [1, 2] and BABAR [3, 4] measurements is given as [5]

\begin{equation}
BR(B^\pm \to \tau^\pm \nu_\tau) = (1.63 \pm 0.39) \times 10^{-4}.
\end{equation}

A significantly improved precision for $BR(B^\pm \to \tau^\pm \nu_\tau)$ would require a High Luminosity ($L \geq 10^{35} \text{ cm}^{-2} \text{s}^{-1}$) $B$ factory [6-12]. As of June 2011, the prospects for two such facilities (in Japan and Europe, respectively) are very promising, with concurrent operation likely by the year 2020. In the context of the SM the decay $B^\pm \to \tau^\pm \nu_\tau$ provides a direct measurement of the combination $f_B |V_{ub}|$, where $V_{ub}$ is a CKM matrix element and $f_B$ is the decay constant, which can only be calculated by non-perturbative techniques such as lattice QCD. Charged Higgs bosons ($H^\pm$), which are present in the Two Higgs Doublet Model (2HDM) and the Minimal Supersymmetric SM (MSSM), would also mediate
The decays $B^\pm \to \tau^\pm \nu_\tau$ [13] with the New Physics contribution being sizeably enhanced if $\tan \beta$ (the ratio of vacuum expectation values of the two Higgs doublets) is large [14]. The above measurements of $B^\pm \to \tau^\pm \nu_\tau$ now provide a very important constraint on the parameter $\tan \beta/m_{H^\pm}$ in the context of the 2HDM and the MSSM. Hence this decay is of much interest in both the SM and in models beyond the SM, and improved precision is certainly desirable.

Prior to the era of the $B$ factories, three LEP Collaborations searched for $B^\pm \to \tau^\pm \nu_\tau$ and obtained upper bounds within an order of magnitude of the SM prediction [15-17]. As pointed out in [18], such limits were actually sensitive to the sum of $B^\pm$ and obtained upper bounds within an order of magnitude of the SM prediction [15-17].

As pointed out in [18], such limits were actually sensitive to the sum of $B^\pm$ and $B_\tau^\pm$ since the centre-of-mass energy ($\sqrt{s} = 91$ GeV) was above the $B^\pm$ production threshold (which is not the case at the $B$ factories). The strongest limits were set by the L3 Collaboration, which obtained $BR(B^\pm \to \tau^\pm \nu_\tau) < 5.7 \times 10^{-4}$ [15]. Since $BR(B^\pm \to \tau^\pm \nu_\tau)$ has been measured at the $B$ factories (see eq. (1)), the limit from the L3 Collaboration can now be used to constrain the product of the transition probability $f(b \to B_c)$ and $BR(B^\pm \to \tau^\pm \nu_\tau)$. A quantitative study of the magnitude of the contribution of $B_\tau^\pm \to \tau^\pm \nu_\tau$ and its impact on the LEP limits was first performed in [18].

In our earlier work [19] we updated the study in [18] by using the significant improvements in the measurements of the CKM matrix and calculations of $f_B$. Moreover, we pointed out that the measurements of the $B^\pm$ production cross section at the Fermilab Tevatron [20-23] provide the first measurements of the transition probability $f(b \to B_c)$, which suggest that its magnitude is considerably larger than the values used in the numerical analysis of [18]. Importantly, the LHC-b experiment has recently observed the $B_c$ meson, and thus more precise measurements of $f(b \to B_c)$ will be available in the next few years. In [19] we suggested that a future $e^+e^-$ Linear Collider operating at the $Z$ peak (the Giga $Z$ option [24-27]) could offer similar sensitivity to the parameter $\tan \beta/m_{H^\pm}$ from these leptonic decays as the proposed high luminosity $B$ factories. In this contribution we summarise and update our work in [19].

2. The decays $B^\pm \to \tau^\pm \nu$ and $B_\tau^\pm \to \tau^\pm \nu$

In the SM, the purely leptonic decays ($\ell^\pm \nu_\ell$) of $B^\pm$ and $B_\tau^\pm$ proceed via annihilation to a $W$ boson in the $s$-channel. The decay rate is given by (where $q = u$ or $c$)

$$\Gamma(B^\pm_q \to \ell^\pm \nu_\ell) = \frac{G_F^2 m_B^2 m_\ell^2 f_B^2_{B q}}{8\pi} |V_{q b}|^2 \left(1 - \frac{m_\ell^2}{m_{B q}^2}\right)^2. \tag{2}$$

Due to helicity suppression, the rate is proportional to $m_\ell^2$. Neglecting the suppression factor from phase space for $\ell = \tau$, one expects the following ratio of branching ratios:

$$BR(B^+_q \to \tau^+ \nu_\tau) : BR(B^+_q \to \mu^+ \nu_\mu) : BR(B^+_q \to e^+ \nu_e) = m_\tau^2 : m_\mu^2 : m_e^2. \tag{3}$$

These decays are relatively much more important for $B_\tau^\pm$ than for $B^\pm$ due to the enhancement factor $|V_{q b}/V_{u b}|^2(f_B/|f_B|^2)$. Using the input parameters given in table I, we obtain the SM predictions listed in table II.

The effect of $H^\pm$ in the 2HDM (Model II) on the decays $B^\pm \to \ell^\pm \nu_\ell$ was considered in [14] and the analogous analysis for $B_\tau^\pm \to \ell^\pm \nu_\ell$ was presented in [28]. In both cases the $H^\pm$ contribution modifies the SM prediction by a global factor $r_H^q$ where

$$r_H^q = [1 − tan^2 \beta (M_{B_q}/m_{H^\pm})^2]^2 \equiv [1 − R^2 M_{B_q}^2]^{-1}. \tag{4}$$
The $H^\pm$ contribution interferes destructively with that of $W^\pm$. There are two solutions for $r_H^q = 1$ which occur at $R = 0$ and $R \sim 0.27 \text{ GeV}^{-1}$ for $B^\pm \to \ell^\pm \nu_\ell$ ($R = 0$ and $R \sim 0.26 \text{ GeV}^{-1}$ for $B^\mp \to \ell^\pm \nu_\ell$). Since the current experimental measurement of $BR(B^\pm \to \tau^\pm \nu_\tau)$ in eq. (1) is in approximate agreement with the SM expectation, there are two possible values for $R$: $R \sim 0.27 \text{ GeV}^{-1}$ and $R \leq 0.1 \text{ GeV}^{-1}$, with the precision determined by the uncertainty both in the experimental measurement and in the input parameters ($V_{ub}$ and $f_B$).

Importantly, these constraints on $R$ are from a tree-level process and so are complementary to analogous constraints which are obtained from processes which are induced only at higher-orders in perturbation theory, such as $b \to s\gamma$, $B_s \to B_s$ mixing and $B_{d,s} \to \mu^+\mu^-$. Certainly, improved precision for $BR(B^\pm \to \tau^\pm \nu_\tau)$ is desirable and relevant even in the era of the LHC in which the plane $[\tan \beta, m_{H^\pm}]$ will be probed via direct production of $H^\pm$. Currently, only high-luminosity $B$ factories operating at the $\Upsilon(4S)$ are discussed when considering future facilities which could offer improved precision for $BR(B^\pm \to \tau^\pm \nu_\tau)$.

3. The Giga $Z$ option at a future $e^+e^-$ Linear Collider

A possible option of a future $e^+e^-$ Linear Collider is operation at the $Z$ peak ($\sqrt{s} \sim 91\text{ GeV}$). When such a “Giga $Z$” option is discussed it is usually assumed that the incident beams of $e^+$ and $e^-$ are polarised. For a measurement of the process $B^\pm/B_s^\pm \to \tau^\pm \nu$ it is not necessary to have polarised beams, and thus for our purposes we will consider Giga $Z$ to be a high-luminosity version of the LEP1 experiment with superior detectors. We note that a period of operation at the $Z$ peak (even without polarised beams) would be beneficial in order to re-measure many $Z$ peak observables, which would provide a valuable test of the detectors of the Linear Collider.

Assuming that a Linear Collider has a luminosity of $5 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, approximately $10^9$ $Z$ bosons could be produced in 50–100 days of operation [24-27]. This corresponds to roughly 1000 times the number of $Z$ bosons recorded at each detector during the LEP1 experiment. Historically, limits on $BR(B^\pm \to \tau^\pm \nu)$ from $Z$ decays have been comparable.

Table II. – Standard Model predictions for the branching ratios (central values).

<table>
<thead>
<tr>
<th>$BR(B^\pm \to \tau^\pm \nu)$</th>
<th>$BR(B^\pm \to \mu^\pm \nu_\mu)$</th>
<th>$BR(B^\pm \to e^\pm \nu_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{e}$</td>
<td>$1.2 \cdot 10^{-4}$</td>
<td>$5.5 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>$B_{\tau}$</td>
<td></td>
<td>$9.3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$B_{\mu}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table I. – Input parameters used in this work, unless indicated otherwise in the text.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_F$</td>
<td>$1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$</td>
</tr>
<tr>
<td>$m_u$</td>
<td>$0.10566 \text{ GeV}$</td>
</tr>
<tr>
<td>$m_c$</td>
<td>$0.511 \text{ MeV}$</td>
</tr>
<tr>
<td>$m_t$</td>
<td>$1.777 \text{ GeV}$</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
</tr>
<tr>
<td>$m_{B_u}$</td>
<td>$5.279 \text{ GeV}$</td>
</tr>
<tr>
<td>$m_{B_c}$</td>
<td>$6.271 \text{ GeV}$</td>
</tr>
<tr>
<td>$f_{B_u}$</td>
<td>$0.216(22) \text{ GeV}$</td>
</tr>
</tbody>
</table>
to (if not stronger than) those at \( \Upsilon(4S) \) for the same number of \( Z \) bosons and \( B \) mesons. For example, the CLEO Collaboration obtained \( BR(B^\pm \rightarrow \tau^\pm \nu) < 8.4 \times 10^{-4} \) with \( 9.7 \times 10^6 \) \( B \) mesons [29], while L3 obtained \( BR(B^\pm \rightarrow \tau^\pm \nu) < 5.7 \times 10^{-4} \) [15] with 1.5 \( \times 10^6 \) hadronic decays of the \( Z \) boson.

It was pointed out in [18] that searches for \( B^\pm \rightarrow \tau^\pm \nu \) from \( Z \) decays are also sensitive to the decay \( B^\pm_c \rightarrow \tau^\pm \nu \). Assuming that the detection efficiencies are the same(1) the ratio of \( \tau^\pm \nu \) events originating from \( B^\pm \rightarrow \tau^\pm \nu \) and \( B^\pm_c \rightarrow \tau^\pm \nu \) is given by

\[
\frac{N_c}{N_u} = \left| V_{cb} \right|^2 f(b \rightarrow B^+_c) \left( \frac{f_{B_c}}{f_B} \right)^2 \frac{M_{B_c}}{M_B} \frac{\tau_{B_c}}{\tau_B} \left( \frac{1 - m^2_{f_B}}{m^2_{f_B}} \right)^2.
\]

The largest uncertainty in the determination of \( N_c \) is from the transition probability \( f(b \rightarrow B^+_c) \) and the decay constant \( f_{B_c} \). The magnitude of \( N_c \) is suppressed by the small \( f(b \rightarrow B^+_c) \) but this can be compensated by the large ratio \( \left| V_{cb} \right|^2 f(b \rightarrow B^+_c)/f_B \). Consequently \( N_c \) can be similar in magnitude to \( N_u \). The main uncertainty in the ratio \( N_c/N_u \) is from \( f(b \rightarrow B^+_c) \), whose magnitude can be extracted (although with a sizeable uncertainty) from the measurement of ratio of \( B^+_c \rightarrow J/\psi \ell^+ \nu_\ell \) to \( B^\pm \rightarrow J/\Psi K^\pm \) which is defined by

\[
R_\ell = \frac{\sigma(B^+_c \rightarrow J/\psi \ell^+ \nu_\ell)}{\sigma(B^+ \rightarrow J/\Psi K^+)}.
\]

Tevatron Run II data gives \( R_\ell = 0.28 \pm 0.07 \) [22], and the denominator in eq. (6) has been measured precisely by various experiments. The transition probability \( f(b \rightarrow B_c) \) determines \( \sigma(B^+_c) \) and several theoretical calculations are available for \( BR(B^+_c \rightarrow J/\psi \ell^+ \nu_\ell) \). The LHC-b experiment will significantly improve the precision in the measurement of \( R_\ell \).

For measurements of \( B^\pm/B^+_c \rightarrow \tau^\pm \nu \) at the \( Z \) peak, one can define an “effective branching ratio” defined by

\[
BR_{\text{eff}} = BR(B^\pm \rightarrow \tau^\pm \nu) \left( 1 + \frac{N_c}{N_u} \right).
\]

For searches at the \( \Upsilon(4S) \) (i.e. at the \( B \) factories) clearly \( N_c = 0 \) and \( BR_{\text{eff}} = BR(B^\pm \rightarrow \tau^\pm \nu) \). Three LEP Collaborations searched for the decay \( B^\pm \rightarrow \tau^\pm \nu \) using data taken at the \( Z \) peak. L3 [15] used around \( 1.5 \times 10^6 \) hadronic decays of the \( Z \) boson which corresponds to about half of their total data [30]. DELPHI [16] and ALEPH [17] used their full data samples of around \( 3.6 \times 10^6 \) hadronic decays of the \( Z \) boson. The best sensitivity was from the L3 experiment, which set the upper limit \( BR(B^\pm \rightarrow \tau^\pm \nu) < 5.7 \times 10^{-4} \). The L3 limit is of particular interest since it could be improved if the full data sample of \( \sim 3.6 \times 10^6 \) hadronic \( Z \) boson decays were used.

By the time of operation of Giga \( Z \) the two main sources of uncertainty in \( N_c \) (and hence \( BR_{\text{eff}} \)) will have been substantially reduced. The error in \( f(b \rightarrow B_c) \) will be reduced from LHC-b measurements of the cross section in eq. (6), and improved lattice calculations of \( f_{B_c} \) and/or \( (f_{B_c}/f_B) \) would also reduce the error in \( N_c \). At present, we are

---

(1) In practice, the shorter lifetime of \( B^+_c \) would result in a slightly inferior detection efficiency.
only aware of one quenched lattice calculation, $f_{B_c} = 489 \pm 4 \pm 3 \text{MeV}$ [31]. In table III we present the required number of $B$ mesons and $Z$ bosons for a precision of 20% and 4% in the measurement of $BR(B^+/B_c^+ \rightarrow \tau^+\nu)$, assuming a signal of $BR_{\text{eff}} = 4 \pm 2 \times 10^{-4}$ at the LEP experiment L3. High-luminosity $B$ factories anticipate data samples of $10^{10}$ $B$ mesons, and the displayed numbers for the precision in the measurement $BR(B^+ \rightarrow \tau^+\nu)$ are taken from [11]. For the Giga $Z$ precision we assume a hypothetical signal of $BR_{\text{eff}} = 4 \pm 2 \times 10^{-4}$ (i.e. 50% error) at the L3 detector, obtained with the full data sample of $3.6 \times 10^6$ hadronic $Z$ decays. We then scale the experimental error by $1/\sqrt{N}$, where $N$ is the total number of $Z$ bosons at Giga $Z$ divided by the full L3 data sample of $\sim 5.1 \times 10^9$ $Z$ bosons. It is clear from table III that a Giga $Z$ facility might be capable of measuring $BR_{\text{eff}}$ in eq. (7) with a precision which is similar to that anticipated for $B^+ \rightarrow \tau^+\nu$ alone at high-luminosity $B$ factories. We believe that this competitiveness of the Giga $Z$ facility has not been pointed out for these leptonic $B$ decays, although it has been emphasised for the decay $B \rightarrow X_s\nu\bar{\nu}$ in [26]. If both facilities were realised, this would enable competitive and complementary probes of the parameter $\tan\beta/m_{H^\pm}$ in the context of models with $H^\pm$. Importantly, this indirect method (i.e. $B^+/B_c^+ \rightarrow \tau^+\nu$) can give sensitivity to values of $\tan\beta/m_{H^\pm}$ which are difficult to probe via the direct production of $H^\pm$ at the LHC.

### 4. Conclusions

The decay $B^+ \rightarrow \tau^+\nu$ has been observed at the $e^+e^-$ $B$ factories and is recognised as an important constraint on the parameter $\tan\beta/m_{H^\pm}$ in the context of models with Two Higgs doublets. Such an indirect probe of $H^\pm$ is complementary to the direct searches for $H^\pm$ at the LHC. We pointed out that the Giga $Z$ option of a future $e^+e^-$ collider could offer measurements of the combined signal of $B^+ \rightarrow \tau^+\nu$ and $B_c^+ \rightarrow \tau^+\nu$ with a precision which is comparable to that expected at high-luminosity $B$ factories for $B^+ \rightarrow \tau^+\nu$ alone. Importantly, such a measurement of $B^{\pm}/B_c^{\pm} \rightarrow \tau^\pm\nu$ does not need polarised beams. If there were an initial period of operation of a Linear Collider at the $Z$ peak, with the main purpose of redoing LEP1 measurements as a check of detector performance, the above measurement of $B^{\pm}/B_c^{\pm} \rightarrow \tau^\pm\nu$ could readily be performed.

** REFERENCES **