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Electroweak effective couplings for future precision experiments

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Summary. — The leading hadronic effects in electroweak theory derive from vacuum polarization which are non-perturbative hadronic contributions to the running of the gauge couplings, the electromagnetic $\alpha_{\rm em}(s)$ and the $SU(2)_L$ coupling $\alpha_2(s)$. I will report on my recent package **alphaQED**, which besides the effective fine structure constant $\alpha_{\rm em}(s)$ also allows for a fairly precise calculation of the $SU(2)_L$ gauge coupling $\alpha_2(s)$. I will briefly review the role, future requirements and possibilities. Applied together with the **Rhad** package by Harlander and Steinhauser, the package allows to calculate all SM running couplings as well as running $\sin^2\Theta$ versions with state-of-the-art accuracy.

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PACS 13.40.-f – Electromagnetic processes and properties.
PACS 12.15.Lk – Electroweak radiative corrections.
PACS 13.40.Ks – Electromagnetic corrections to strong- and weak-interaction processes.

1. – Introduction

Precise Standard Model (SM) predictions require to determine the $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ SM gauge couplings $\alpha_{\rm em}$, α_2 and $\alpha_s \equiv \alpha_3$ (QCD) as accurately as possible. Obviously, the predictability of theory is limited by the precision of its input parameters. This in particular requires to fight precision limitations due to non-perturbative hadronic contributions. Precise predictions confronting precise measurements are the basis for all SM precision tests, which allow us to unravel new physics from discrepancies between theory and experiment. An important test case, which requires as precise as possible running couplings, is the quest of gauge coupling unification in grand-unified extensions of the SM.

Key input parameter for ILC physics currently are known to precision:

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}, \qquad \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 1.6\text{–}6.8 \times 10^{-4}, \qquad (1)$$

$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}, \qquad \frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}.$$

We observe that the accuracy of $\alpha(M_Z)$ is roughly one order of magnitude worse than that of the next best M_Z! The loss in precision caused by non-perturbative strong interaction effects is 10^5 between the classical low energy α and $\alpha(M_Z)$. The requirement for ILC precision physics is

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5 \times 10^{-5}. \qquad (2)$$

A prominent example where theory may be obscured by lack of precision in the effective α is the indirect Higgs mass bound obtained from the precise measurement of $\sin^2\theta_{\rm eff}^{\rm lep}$. The required improvement could be achieved by dedicated efforts in cross-section measurements in the energy range from 1.2 to 3.2 GeV, and be adopting the Adler function controlled split in parts evaluated from data (from experiments or from lattice QCD simulations) and parts which can be calculated reliably in perturbative QCD (pQCD):

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} + \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} \qquad (3)$$

$$+ \left[\Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2)\right]^{\rm pQCD},$$

where s_0 can be optimized by adopting the Adler function as a monitor for the range of validity of pQCD [1, 2]. In the following we will present a description of the package `alphaQED` which allows state-of-the-art calculations of the SM running couplings, optionally, with their imaginary parts. Some emphasis is put on the not so straightforward determination of the running $SU(2)_L$ coupling $\alpha_2(s)$, which is important for the calculation of variants of the weak mixing parameter $\sin^2\Theta_W(s)$, an interesting quasi-observable and monitor of new physics particularly at ILC energy scales.

2. – Effective running coupling $\alpha_{\rm QED}$

The effective fine-structure "constant" $\alpha(E)$ depends on the energy scale because of charge screening by vacuum polarization:

$$\Delta\alpha(s) = -e^2\,\left[\mathrm{Re}\,\Pi'^{\gamma\gamma}(s) - \Pi'^{\gamma\gamma}(0)\right] \qquad (4)$$

which exhibit the leading hadronic non-perturbative part $\Delta^{(5)}_{\rm had}\alpha$. $\Pi(s) = \Pi(0) + s\,\Pi'(s)$ denotes the transversal current correlator, for the electromagnetic current $\Pi(0) = 0$. While electroweak effects (leptons, etc.) are calculable in perturbation theory, the calculation of the strong interaction effects (hadrons/quarks, etc.) by perturbative QCD fails. Fortunately, dispersion relations and the optical theorem allow us to perform rather accurate evaluations in terms of experimental e^+e^-–data encoded in

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} \,. \tag{5}$$

For the electromagnetic running coupling the dispersion integral reads

$$\Delta\alpha^{(5)}_{\text{had}}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\text{cut}}} \mathrm{d}s' \frac{R^{\text{data}}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\text{cut}}}^{\infty} \mathrm{d}s' \frac{R^{\text{pQCD}}_\gamma(s')}{s'(s'-s)}\right) \tag{6}$$

The high energy tail is neatly calculable perturbatively by the virtue of asymptotic freedom of QCD. Errors of data imply theoretical uncertainties. Some of the data sets are old and of rather limited precision, especially in the range above 1.4 GeV to about 2.2 GeV, a range which is subject to new measurement at the VEPP 2000 facility at Novosibirsk. Data from different experiments are combined by standard methods as recommended by the Particle Data Group (see, *e.g.*, [3]). In recent years progress has been due to much better $\sigma(e^+e^- \to \textit{hadrons})$ determinations at Novosibirsk (CMD2, SND) [4, 5] and more recently by the novel radiative return high accuracy measurements by KLOE [6, 7] and BaBar [8] (see also [9, 10]). Typically, vacuum polarization leads to large corrections and in fact $\alpha(E)$ is steeply increasing at low E already. So the deviation of $\alpha(m_\mu)$ at the muon mass scale m_μ from α gives the big leading hadronic correction to the muon $g-2$ [11]. That is why we need to know the running of α_{QED} very precisely at all scales (see fig. 1). Non-perturbative hadronic effects in electroweak precision observables affect most SM predictions via non-perturbative effects in parameter shifts, typically:

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}\,, \tag{7}$$

where

$$\Delta r_i = \Delta r_i(\alpha, G_\mu, M_Z, m_H, m_{f\neq t}, m_t) \tag{8}$$

represent the quantum corrections from gauge boson self-energies, vertex– and box–corrections. Uncertainties obscure in particular the indirect bounds on the Higgs mass obtained from electroweak precision measurements. Basic observables like M_W [$\sin^2\Theta_W = 1 - M_W^2/M_Z^2$], g_2 [$\sin^2\Theta_g = e^2/g_2^2 = (\pi\alpha)/(\sqrt{2}\,G_\mu\,M_W^2)$] or the vector coupling v_f [$\sin^2\Theta_f = (4|Q_f|)^{-1}(1 - v_f/a_f), f \neq \nu$] are related to versions of $\sin^2\Theta_W$ obtained form (7) and the general form of Δr_i reads

$$\Delta r_i = \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\text{remainder}} \tag{9}$$

with a universal term $\Delta\alpha$ which affects the predictions for M_W, A_{LR}, A^f_{FB}, Γ_f, etc. Only the ρ parameter in the axial coupling a_f, which is renormalized by $\rho_f = 1/(1-\Delta\rho)$, is independent of leading non-perturbative hadronic effects.

One issue concerning running couplings concerns the question complex *vs.* real $\alpha(s)$. For $s \neq 0$ (4) provides the definition of a complex coupling if we relax from taking the real part only. A typical example where this happens is the vacuum polarization correction to

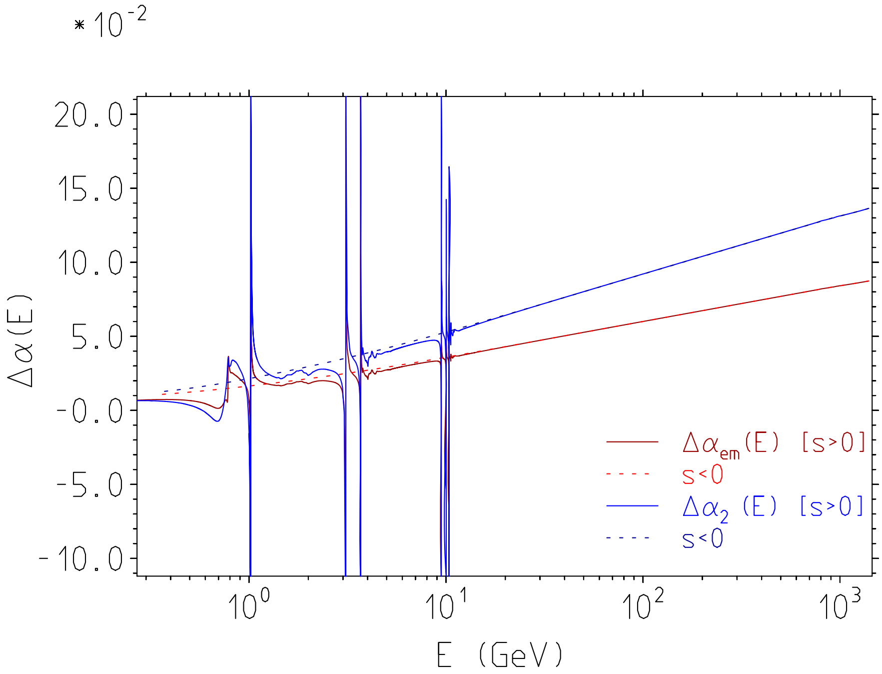

Fig. 1. – $\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.

be performed on $R(s)$ before it can be used in (6): $R_{\rm physical} \to R^{(0)} \doteq (\alpha/\alpha(s))^2\, R_{\rm physical}$. Usually, $\alpha(s)$ is take to be real, *i.e.* $(\alpha/\alpha(s))^2 = |1 - {\rm Re}\,\Pi'(s)|^2$ ($\Pi'(0)$ subtracted). More precisely, one should subtract $|1 - \Pi'(s)|^2 = (\alpha/|\alpha_c(s)|)^2$ where $\alpha_c(s)$ denotes the complex version of running α. Typically, corrections from imaginary parts given by $1 - |1 - \Pi'(s)|^2/(\alpha/\alpha(s))^2$, are small $\lesssim 0.1\%$ in non-resonance regions. However, at resonances corrections are of order $\sim 1/\Gamma_R$ and thus are large for narrow resonances.

3. – The coupling α_2, M_W and $\sin^2\Theta_f$

Unlike for the electromagnetic coupling, for the $SU(2)_L$ coupling the hadronic shift cannot be directly obtained by integration of measured data. There is however a pretty clean way to evaluate $\Delta^{(5)}_{\rm had}\alpha_2$, contributing to

$$\Delta\alpha_2 = -\frac{e^2}{\sin^2\Theta_W}\,\left[{\rm Re}\,\Pi'^{3\gamma}(s) - \Pi'^{3\gamma}(0)\right] \tag{10}$$

which has been proposed long ago in [12]. The surprising fact is that the evaluation of α_2 does not require to separate all individual flavor contributions to recombine them in the proper way. In fact, up to perturbative or very small contributions the hadronic shift of α_2 is proportional to the self-energy correlation amplitude $\Pi^{3\gamma}$ where 3 refers to the

3rd component of the weak isospin current and γ to the electromagnetic current. For the non-perturbative low energy range, it implies that the contribution corresponding to the u, d and s flavors actually requires no flavor separation in the $SU(3)$ limit. This makes it possible to calculate $\Delta\alpha_2$ reliably, because the other heavier flavors may be safely separated by relying on pQCD weighting. The assumption is that for $N_f > 3$ the $N_f - 1$ lighter flavors above the N_f flavor threshold can be evaluated by pQCD. A detailed discussion of the approximations made is given in appendix C of [12]. Given $\Pi^{\gamma\gamma}_{\rm con} = \Pi^{\gamma\gamma}_{(uds)} + \Pi^{\gamma\gamma}_{(c)} + \Pi^{\gamma\gamma}_{(b)}$ for the continuum and $\Pi^{\gamma\gamma}_{\rm res} \simeq \Pi^{\rho} + \Pi^{\omega} + \Pi^{\phi} + \Pi^{J/\psi} + \Pi^{\Upsilon}$ for the narrow resonances, we have the relations

$$\Pi^{3\gamma}_{\rm con} \simeq \frac{1}{2}\,\Pi^{\gamma\gamma}_{(uds)} + \frac{3}{8}\,\Pi^{\gamma\gamma}_{(c)} + \frac{3}{4}\,\Pi^{\gamma\gamma}_{(b)} \tag{11}$$

for the background contribution and

$$\Pi^{3\gamma}_{\rm res} \simeq \frac{1}{2}\,\Pi^{\rho} + \frac{3}{4}\,\Pi^{\phi} + \frac{3}{8}\,\Pi^{J/\psi} + \frac{3}{4}\,\Pi^{\Upsilon} \tag{12}$$

for the resonance contributions. The $\rho - \omega$ mixing contribution usually included in the Π^{ρ} taking into account the isospin $I = 0$ component $\omega \to \pi\pi$ in the $\gamma \to \pi\pi \to \gamma$ channel is to be subtracted via the Gounaris-Sakurai parametrization (by setting to zero the corresponding mixing parameter). The coupling α_2 can be "measured" in a charged current channel via M_W $(g \equiv g_2)$:

$$M_W^2 = \frac{g^2\,v^2}{4} = \frac{\pi\,\alpha_2}{\sqrt{2}\,G_\mu} \tag{13}$$

or via the neutral current channel $\sin^2\Theta_f$. In fact here running $\sin^2\Theta_f(E)$ connects the LEP scale mixing parameter to the one of low energy $\nu_e e$ scattering

$$\sin^2\Theta_e(M_Z) = \left\{\frac{1-\Delta\alpha_2(M_Z)}{1-\Delta\alpha(M_Z)} + \Delta_{\nu_\mu e,{\rm vertex+box}} + \Delta\kappa_{e,{\rm vertex}}\right\}\sin^2\Theta_{\nu_\mu e}. \tag{14}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $(1{-}\Delta\alpha_2)/(1{-}\Delta\alpha)$ can be taken to be 100% correlated and thus largely cancel.

Above results allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self-energies. Gauge boson self-energies potentially are very sensitive to new physics (oblique corrections), which, however, may be obscured by uncertainties of the non-perturbative hadronic effects. For complete analytic expressions for electroweak parameter shifts at one-loop see [13, 14]. Another interesting version of running $\sin^2\Theta_W(Q^2)$ is found in *polarized Moeller scattering asymmetries* as advocated by Czarnecki and Marciano [15]. It includes specific bosonic contribution $\Delta\kappa_b(Q^2)$ such that

$$\kappa(s=-Q^2) = \frac{1-\Delta\alpha_2(s)}{1-\Delta\alpha(s)} + \Delta\kappa_b(Q^2) - \Delta\kappa_b(0), \tag{15}$$

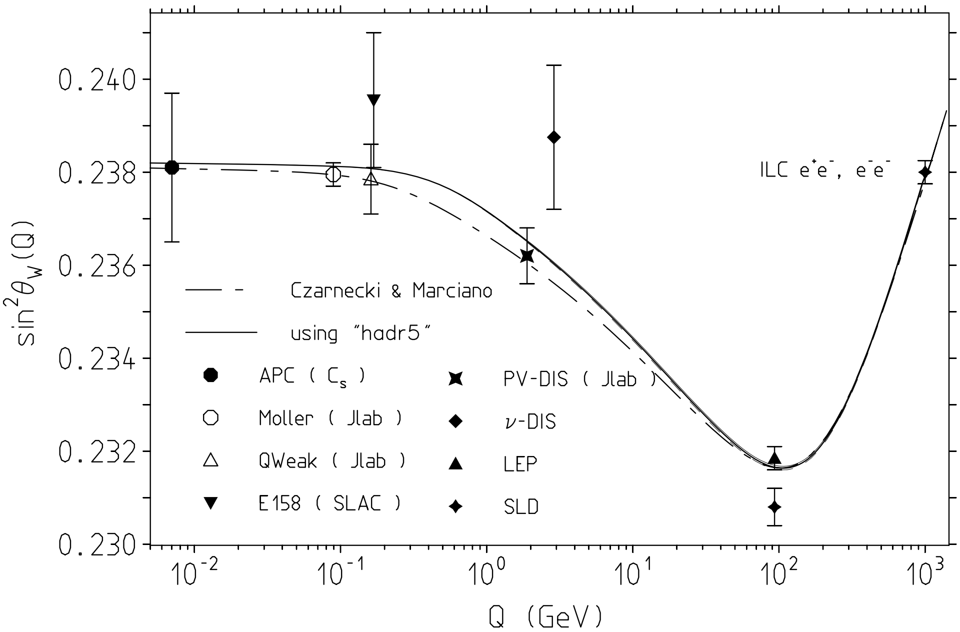

Fig. 2. – $\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23156(21)$ are not shown. Future ILC measurements at 1 TeV would be sensitive to Z', H^{--}, etc.

where([1]), in our low energy scheme, we require $\kappa(Q^2) = 1$ at $Q^2 = 0$. Explicitly [15],

$$\begin{aligned}(16)\quad \Delta\kappa_b(Q^2) = &\\ &-\frac{\alpha}{2\pi\, s_W}\left\{-\frac{42\, c_W+1}{12}\ln c_W + \frac{1}{18} - \left(\frac{r}{2}\ln\xi - 1\right)\left[(7-4z)\, c_W + \frac{1}{6}(1+4z)\right]\right.\\ &\left. -z\left[\frac{3}{4} - z + \left(z - \frac{2}{3}\right) r\,\ln\xi + z(2-z)\ln^2\xi\right]\right\},\end{aligned}$$

$$(17)\quad \Delta\kappa_b(0) = -\frac{\alpha}{2\pi\, s_W}\left\{-\frac{42\, c_W+1}{12}\ln c_W + \frac{1}{18} + \frac{6\, c_W+7}{18}\right\},$$

with $z = M_W^2/Q^2$, $r = \sqrt{1+4z}$, $\xi = \frac{r+1}{r-1}$, $s_W = \sin^2\Theta_W$ and $c_W = \cos^2\Theta_W$. Results obtained in [15] based on one-loop perturbation theory using light quark masses $m_u = m_d = m_s = 100\,\mathrm{MeV}$ are compared with results obtained in our non-perturbative approach in fig. 2.

([1]) Here $\Delta\alpha$ = `dggvap(s,0.d0)` and $\Delta\alpha_2$ = `degvap(s,0.d0)` are provided by functions from the package `alphaQED`.

4. – Adler function controlled split data *vs.* pQCD

A strategy to exploit the rather precise perturbative QCD predictions in a optimal well controlled way is to monitor QCD predictions via the Adler function $D(Q^2)$ in the Euclidean region by comparing theory and data there.

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{\mathrm{d}}{\mathrm{d}s} \Delta\alpha_{\mathrm{had}}(s) = -\left(12\pi^2\right)\, s\, \frac{\mathrm{d}\Pi'_\gamma(s)}{\mathrm{d}s}\,, \tag{18}$$

$$D(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} \mathrm{d}s\, \frac{R(s)}{(s+Q^2)^2}\,. \tag{19}$$

Low energies, resonances and thresholds prevent us from making reliable and precise predictions of $R(s)$ in pQCD. Locally deviations between data and R-predictions can be huge. In contrast, the smooth function $D(Q^2)$ is easy to compare and deviations show up at low energies only. A detailed inspection of the time-like approach shows that pQCD works well in "perturbative windows" like 3.00 GeV–3.73 GeV, 5.00 GeV–10.52 GeV and 11.50 GeV–∞. In the space-like approach pQCD works well for $\sqrt{Q^2=-q^2} > 2.5$ GeV [1, 2]. Theory is based on results by Chetyrkin, Kühn *et al.* [16, 17]. One thus requires data to calculate

$$\Delta\alpha_{\mathrm{had}}(-s_0) = \frac{\alpha}{3\pi} \int_0^{s_0} \mathrm{d}Q'^2\, \frac{D(Q'^2)}{Q'^2} \tag{20}$$

up to $s_0 = (2.5\,\mathrm{GeV})^2$. Equivalently, $\Delta\alpha_{\mathrm{had}}(-s_0)$ can be directly calculated by (6) and used in (3). One obtains [1, 2]

$$\Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)^{\mathrm{data}} = 0.007337 \pm 0.000090,$$

$$\Delta\alpha^{(5)}_{\mathrm{had}}(-M_Z^2) = 0.027460 \pm 0.000134,$$

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027498 \pm 0.000135.$$

The result includes a shift +0.000008 from the 5-loop contribution. The error ±0.000103 in the perturbative part is added in quadrature. QCD parameters used are $\alpha_s(M_Z) = 0.1189(20)$, $m_c(m_c) = 1.286(13)\,[M_c = 1.666(17)]$ GeV, and $m_b(m_c) = 4.164(25)\,[M_b = 4.800(29)]$ GeV based on a complete 3–loop massive QCD analysis [18]. The latter results are in agreement with results from lattice QCD [19-21]. Results based on the Adler controlled split are $\Delta\alpha^{(5)}_{\mathrm{hadrons}}(M_Z^2) = 0.027498 \pm 0.000135\,[0.027510 \pm 0.000218]$ or $\alpha^{-1}(M_Z^2) = 128.962 \pm 0.018\,[128.961 \pm 0.030]$ in braces for comparison the results obtained by the standard approach.

A comparison of error profiles between $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$, $\Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)$ and a_μ may be found in [2]. Note that our approach, with a conservative cut of $\sqrt{s_0} = 2.5$ GeV, does not rely substantially more on pQCD than standard analyses by Davier, Höcker *et al.* [9] and others (see table I). Further progress is possible due to progress in methods to include the hadronic τ–decay data [22, 23].

TABLE I. – *How much pQCD?* $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$ *pQCD part only.*

Method	range [GeV]	pQCD		
Standard approach:	5.2 - 9.5	33.50(0.02)		
My choice	13.0 - ∞	115.69(0.04)	$\rightarrow$	149.19 (0.06)
Standard approach:	2.0 - 9.5	72.09(0.07)		
Davier *et al.*	11.5 - ∞	123.24(0.05)	$\rightarrow$	195.33 (0.12)
Adler function controlled:	5.2 - 9.5	3.92(0.00)		
	13.0 - ∞	1.09(0.00)		
	$-\infty$ - -2.5	201.23(1.03)		
	$-M_Z \rightarrow M_Z$	0.38(0.00)	$\rightarrow$	206.62 (1.03)

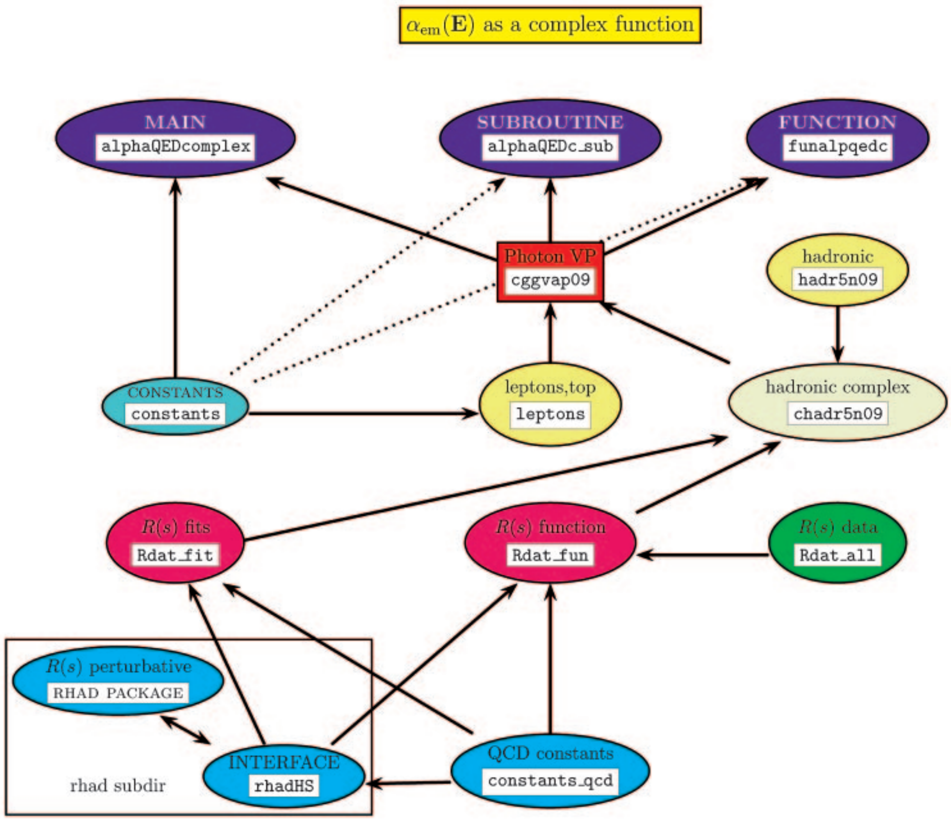

Fig. 3. – Structure of `alphaQEDcomplex`. The corresponding diagram for `alphaQEDreal` is much simpler as it involves the upper part only.

5. – The FORTRAN package `alphaQED`

The FORTRAN package `alphaQED.tar.gz` [24] for calculating the SM effective couplings includes two versions:

– alphaQEDreal [FUNCTION funalpqed] providing the real part of the subtracted photon vacuum polarization including hadronic, leptonic and top quark contributions as well as the weak part (relevant at ILC energies). Hadronic, leptonic, top and weak contributions are accessible separately via common blocks

```
common /resu/dalept,dahadr,daltop,Dalphaweak1MSb
common /resg/dglept,dghadr,dgetop,Dalpha2weak1MSb
```

– alphaQEDcomplex [FUNCTION funalpqedc] provides in addition the corresponding imaginary parts. See fig. 3.

– corresponding options alpha2SMreal and alpha2SMcomplex are available for the $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$.

The functions are available for the space-like and the time-like region. The complex versions require to install the `Rhad` package of Harlander and Steinhauser [25] (FORTRAN package version rhad-1.01 (March 2009 issue)). The latter also provides the QCD coupling $\alpha_3(s) = \alpha_s(s)$. The imaginary part given by the bare $R^{(0)}(s)$ is provided in parametrized form by Chebyshev polynomial fits. For sample plots I refer to the package description on my web page `http://www-com.physik.hu-berlin.de/~fjeger/`. The "organigram" of the program is shown in fig. 3.

* * *

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