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Top quark physics at a future LC and the LHC

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Summary. — This lecture will give a brief review of top quark physics, concentrating on recent and some not so recent, mainly theoretical developments relevant for a future linear collider, and some related topics of top physics at hadron colliders.

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PACS 13.66.Jn – Precision measurements in e^-e^+ interactions.
PACS 12.38.Bx – Perturbative calculations.

1. – Introduction

The top quark, observed first at the Tevatron in 1995 [1], is the heaviest elementary particle observed so far. With a mass of about 173 GeV [2] it is far heavier than all other fermions of the Standard Model (SM), and even heavier than the Higgs boson as preferred by electroweak precision fits of the SM. It therefore plays a special role in testing the Yukawa sector and possible extensions of the SM with new heavy particles.

Since the beginning of the studies for a future e^+e^- international linear collider (ILC) more than 20 years ago, precision studies of the top sector have been a very important part of the physics scenario for the ILC [3], and a lot of work has been invested to better understand the theory of top quark production and decay at threshold and in the continuum. While the time scale for the realisation of the ILC has slipped and is still unclear, the Tevatron has achieved impressive measurements of the top quark's properties, and at the LHC already more than 2500 top events have been collected.

2. – $t\bar{t}$ top-peaks

Compared to top production in hadron collisions, $t\bar{t}$ production at an e^+e^- linear collider (LC) (or at a muon collider) is a "clean" process because of the colourless initial state with well-defined energy (the smearing of the beam energy is only due to electromagnetic effects like initial state radiation, but see the discussion of the experimental

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simulation below), relatively low multiplicity final states and low backgrounds. This will allow for a full and precise reconstruction of top production and decay and hence give access to the top mass m_t and the top couplings in a unique way.

2˙1. *Threshold scan at a Linear Collider*. – Of special interest is the possibility to study the production of $t\bar{t}$ in the threshold region, $\sqrt{s} \sim 2m_t$, by adjusting the e^+e^- beam energy. While traditional spectroscopy is prohibited by the fast decay $t \to bW$ ($\Gamma_{t\to bW} \sim 1.5\,\text{GeV}$ is the completely dominant partial decay width in the SM), the sharp rise of the cross section $\sigma(e^+e^- \to t\bar{t})$, with the remnant of the $1S$ "toponium" resonance, is the ideal observable to measure the top mass. A threshold scan is basically a simple counting experiment of $bW^+\bar{b}W^-$ colour singlet states, thus circumventing uncertainties from the jet energy scale and soft and collinear gluon emissions (connected to the problem of the correct mass definition), which are limiting the accuracy of the top mass determination from kinematic reconstructions. A threshold scan at a lepton collider allows for a top mass determination at per mille accuracy, about one order of magnitude better than measurements at hadron colliders which are limited by systematic uncertainties. To achieve such a goal, the theoretical predictions have to match the expected experimental accuracy.

Top pair production at threshold is a multi-scale process, where different aspects are described by the hierarchical scales: i) top mass m_t, ii) top momentum $p_t \sim m_t v$, iii) binding energy of the quasi-bound state $E \sim m_t v^2$ and top width Γ_t which is of the same size as E numerically. The parameter v is the non-relativistic velocity and acts, simultaneously with the strong coupling α_s, as an expansion parameter. Due to the top's large mass and fast decay (which effectively cuts off any hadronisation effects), $t\bar{t}$ production can be predicted in perturbative QCD, though additional electroweak corrections have to be taken into account. To achieve a systematic description within perturbation theory, various versions of effective field theories have been formulated on the basis of non-relativistic QCD (NRQCD), which allow to disentangle the different scales and simplify the calculations, see, *e.g.*, the review [4].

Close to threshold, gluon exchanges lead to a Coulomb-like binding force responsible for the strong enhancement of the $t\bar{t}$ cross section (or bound state production in the case of stable quarks). In the language of the perturbative expansion in v and α_s, this corresponds to terms of order $(\alpha_s/v)^n$ and leads to the well-known Coulomb singularity. As, in the threshold region, v is of the same order as α_s, already at leading order (LO) all such terms must be summed to obtain the correct cross section. This is achieved by solving a Schrödinger-like equation with a QCD potential for the $t\bar{t}$ Green function, thus accounting for the binding effects from the gluon exchanges. Higher-order perturbative corrections come with additional powers of v or α_s, and the power-counting for the $t\bar{t}$ cross section is given schematically by

$$\sigma(e^+e^- \to t\bar{t}) = v \cdot \sum_n \left(\frac{\alpha_s}{v}\right)^n \left[\text{LO}\{1\}, \text{NLO}\{v, \alpha_s\}, \text{NNLO}\{v^2, v\alpha_s, \alpha_s^2\}\right], \tag{1}$$

where NLO and NNLO are the next-to-leading and next-to-next-to-leading order corrections, respectively. Within the effective field theory, the cross section is factorised in matching coefficients and current correlators, which both can be calculated perturbatively. Already some time ago, several groups have independently calculated the $t\bar{t}$ cross section at NNLO, see [5] for a detailed discussion and references.

However, even at NNLO, the theoretical prediction is not very well under control, as signalled by both the large size of the corrections and the scale dependence of the

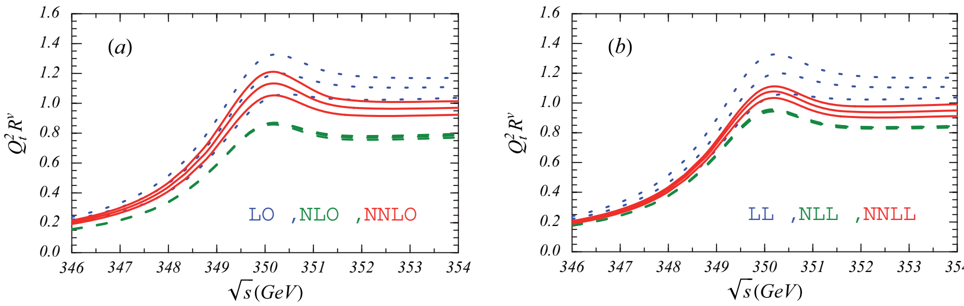

Fig. 1. – Cross section for $t\bar{t}$ production as predicted in different orders of perturbative NRQCD: (a) fixed order results, (b) RG improved results within vNRQCD. For each order, as labelled on the plots, the curves show the dependence on the renormalisation scale. Figures from [10].

cross section. Therefore, during the last years, the most important building blocks to predict the cross section at next-to-next-to-next-to-leading order (NNNLO) have been calculated, see the short reviews [6] and references therein, and [7] for recent results. This includes also the recently completed three-loop corrections to the QCD potential [8], to which more than 20000 Feynman diagrams contribute. While these results lead to a further stabilisation of the perturbative prediction, the size of the NNNLO corrections is still large, casting doubt on the stability of the fixed order expansion. One reason for the large corrections in the perturbative expansion is the appearance of large logarithms of scale ratios like, *e.g.*, $\ln(m_t^2/E^2) \sim 8$. Such logarithms can be predicted using methods based on the Renormalisation Group (RG). The resulting RG improved calculations sum, in addition to the Coulomb singular terms, also logarithms of the form $(\alpha_s \ln v)^k$ to all orders and have therefore, compared to eq. (1), a modified power counting. Such predictions are called leading logarithmic (LL), next-to-leading logarithmic (NLL), etc. and show, compared to the fixed order approach, an improved convergence [9]. This is demonstrated in fig. 1, where predictions in fixed order (left panel; LO, NLO, NNLO) are compared to RG improved results (right panel; LL, NLL, NNLL) obtained in an effective field theory called velocity NRQCD (vNRQCD) [4]. The bands formed by the curves show the dependence of the cross section predictions on the renormalisation scale.

Note that the cross section predictions shown in fig. 1 were obtained using the $1S$ mass scheme [11] for the top mass, which is advantageous for the threshold region. This mass is defined as half of the perturbatively defined mass of the $1S$ peak. If one would use the standard pole mass definition, the energy of the $1S$ peak, and with it m_t^{pole} as determined from a threshold scan, would be strongly dependent on the order of perturbation theory. While this by itself is not necessarily a problem, the translation from the on-shell pole mass([1]) to any other scheme, like the $\overline{\text{MS}}$ scheme usually used in theoretical calculations at high energy, suffers from large uncertainties. These uncertainties stem from the infrared (IR) regime and do not diminish at higher orders. In contrast, the conversion between different "short distance" masses like the $1S$, the $\overline{\text{MS}}$ or the so-called "potential subtracted" mass does not suffer from this problem (see [5] for a discussion and references). This can be understood from the absence of IR "renormalon" ambiguities, which are absent in properly defined short distance masses.

([1]) While the pole mass is infrared finite and gauge invariant to all orders, it is not an observable and defined only up to infrared uncertainties of the order of Λ_{QCD}.

In addition to QCD effects, also electroweak corrections have to be taken into account in the prediction of the $t\bar{t}$ cross section. As already discussed, the large decay width Γ_t is of crucial importance. At leading order of the non-relativistic expansion, it is accounted for by the simple inclusion of Γ_t as a complex part of the non-relativistic energy, $E \to E + i\Gamma_t$ (similar to absorptive effects in optics). However, at higher order this description is not sufficient. In principle, the effective field theory has to account not only for strong interaction effects in $e^+e^- \to t\bar{t} \to W^+bW^-\bar{b}$, but for all Feynman diagrams which contribute to the same final state in a given order in the power-counting (which then has to include electroweak couplings as well). This includes genuine electroweak corrections to the production and decay vertices, but also single- and non-resonant corrections from off-shell top quarks or with no top quarks at all in the intermediate states. No complete calculation including all these effects exists so far, but important progress has been achieved, see [12].

The threshold cross section for $t\bar{t}$ production, shown in fig. 1 as a function of $\sqrt{s}$, depends strongly on the top quark's mass m_t, its width Γ_t and the strong coupling α_s, but also, to a lesser extent, on the top Yukawa coupling y_t which enters through virtual Higgs corrections. Therefore, by measuring the full cross section shape in a threshold scan, it will be possible to determine not only the top mass, but also its couplings. To make best use of the anticipated experimental data, also other observables (differential distributions as opposed to the total cross section shape) can be measured, such as the momentum distribution of the top quarks, $\mathrm{d}\sigma/\mathrm{d}p_t$, or the forward-backward asymmetry, A_{FB}. The added information can help to disentangle the correlations of the different parameters. While, *e.g.*, the peak of the momentum distribution is strongly dependent on m_t, variations in α_s modify mainly its normalisation. A_{FB}, on the other hand, depends significantly on Γ_t but less on α_s and is therefore a sensitive probe of deviations of the weak coupling from its SM value. Earlier studies, performing a multi-parameter fit using the total cross section, the momentum distribution and A_{FB}, have shown that very small (experimental) errors will be achievable at a future ILC, *i.e.* $\Delta m_t \sim 20\,\mathrm{MeV}$, $\Delta\Gamma_t \sim 30\,\mathrm{MeV}$, $\Delta\alpha_s \sim 0.0012$ and $\Delta y_t/y_t \sim 35\%$, see [13] for more details.

Due to the dominance of S wave production near threshold, the top quarks produced in e^+e^- collisions at threshold will already be polarised to about 40%. A future LC may also offer the option to run with polarised beams [14]. This would result in a very high degree of polarisation for the top quarks, which is transferred to the decay b quarks, thus enhancing the potential to study top couplings, including possible deviations from the SM such as a $V + A$ admixture to the $V - A$ structure of the weak interaction in the SM [15].

Note that in general also so-called rescattering corrections have to be taken into account, *i.e.* effects from gluon exchange between top and bottom quarks, or between the b and $\bar{b}$. These corrections are suppressed for the total cross section, but are sizeable for differential distributions and the polarisation [16]. Calculations on this exist in a framework suitable for the implementation in Monte Carlo generators.

2˙2. *Experimental simulations*. – A lot of effort has already been invested in simulations of the top quark threshold. For this, various Monte Carlo codes have been developed within the American, European and Asian study groups. Tagging of $t\bar{t}$ events and background suppression should not pose a severe problem with the planned detector designs, and the expected statistics should be sufficient for a high-precision determination of the top mass, width, α_s and possibly even the top Yukawa coupling. However, most simulations so far assumed a very good knowledge of the e^+, e^- beam spectrum, including the average centre-of-mass energy $\sqrt{s}$. Recent studies have shown [17] that this can not be taken for granted, but that it should be possible. The precise determination of the average energy $\sqrt{s}$ will be done with energy spectrometers and can also be calibrated by radiative return to the Z. The luminosity spectrum, $\mathrm{d}L/\mathrm{d}\sqrt{s}$, depends strongly on the machine design and parameters. It is influenced by the initial beam spread, the beamstrahlung (interaction of the beams close to the interaction point) and the usual initial state photon radiation. These effects lead not only to a significant loss of the effective luminosity, but also to a smearing of the observed $t\bar{t}$ excitation curve and hence to a possibly sizeable shift of the determined top mass. Therefore it will be crucial to understand the luminosity spectrum as precisely as possible. For this, calculations of the beam dynamics and measurements of Bhabha scattering (acollinearity) will be crucial(^{2}). The experimental simulations done so far indicate that a top mass determination at the per mille level should definitely be possible, but that a thorough understanding of the beam dynamics is mandatory. Therefore, for the best physics reach, it will be important to study further the influence of and optimise the machine and interaction point design, and to carefully plan the scan strategy (*i.e.* at which energies to run with which statistics).

2˙2.1. Threshold scans in related cases. The top threshold can be seen as the benchmark case for similar threshold scans possible at a future LC, like, *e.g.*, the W^+W^- or possibly SUSY thresholds. In the case of W pair production, a dedicated threshold scan at a future LC could lead to a determination of the W mass with an accuracy of less than 6 MeV. Recent calculations of the corresponding dominant NNLO corrections within an effective theory of unstable particles show that such an accuracy can be achieved from the theoretical side [19].

If supersymmetry is realised with new particles in reach of a future LC, precision studies of their masses and couplings will become a very strong motivation for such a machine. In this context, stop (the scalar partner of the top quark) pair production at threshold has been studied theoretically and simulated experimentally. In contrast to the case of $t\bar{t}$, stop pair production in e^+e^- collisions is dominated by the P wave and hence suppressed by order v^3 at threshold. Nevertheless the threshold corrections due to gluon exchange are very important and can be predicted within effective field theories [20]. Simulations indicate that a dedicated threshold scan could lead to a stop mass determination with an accuracy of less than one percent [21].

2˙3. *Top quarks at the Tevatron and the LHC*(^{3})

2˙3.1. Mass determination at hadron colliders. Top production at hadron colliders is dominated by quark (for the Tevatron) and gluon (for the LHC) fusion. In theoretical predictions, the required convolution of the initial parton luminosities with the partonic cross section integrates over a range of partonic centre-of-mass energies. This leads, in comparison with a lepton collider, to additional uncertainties and smears out pronounced structures in the partonic cross section. In addition, separation of the top signal from the background is a serious issue. Nevertheless, the numbers of top events for LHC:Tevatron:ILC are scaling roughly as 1000:10:1, and the LHC will be a true top factory. The formidable performance of the Tevatron and its experiments CDF and D0 has led already to a very

(^{2}) Recent work on higher-order predictions of Bhabha scattering [18] have partly been driven by these physics studies.
(^{3}) For a detailed discussion of top quark physics at the LHC the reader is referred to [22].

good measurement of the top quark mass, $m_t^{\text{Tevatron}} = 173.3 \pm 1.1\,\text{GeV}$ [2], from reconstructing top decays. However, the reconstruction of a colour octet decay is plagued by its sensitivity to IR effects and will depend on details of the analysis including the treatment of the parton shower (soft and collinear gluon radiation) in Monte Carlo event generators like PYTHIA or HERWIG(4). To go beyond the already achieved precision will therefore require to better define *which* mass is measured. It is not unexpected that at the LHC a value for m_t different from m_t^{Tevatron} will be measured when using a similar method. To address this problem, so-called "jet-mass schemes" have been introduced, see [25] for a recent review and references therein. They are based on the soft-collinear effective theory (SCET) and a factorisation formula using perturbatively calculable heavy-quark jet functions (including evolution and decay) and a universal non-perturbative soft function which parametrises the radiation between jets and their fragmentation products. It is anticipated that these conceptual works will put the top mass determination from kinematic reconstruction on firmer ground, though at present the formalism does not include gluonic initial state radiation and is therefore not fully applicable for hadroproduction but for e^+e^- collisions only.

An alternative way to determine the top mass which avoids these problems is to measure m_t from the total $t\bar{t}$ cross section. By using the $\overline{MS}$ scheme for the theoretical cross section prediction, a well-defined running mass $m_t(m_t)$ can be determined, which is free from the above-mentioned uncertainties. On the down-side, the error of the experimental cross section measurement and uncertainties in the theoretical calculation due to the truncation of the perturbative series and the errors of the PDF input, combined with the weak sensitivity of the cross section on the mass, are limiting this method. Using the existing cross section measurements from the Tevatron yields, after translation to the pole mass scheme, a top mass compatible with the one from kinematic reconstructions [26].

2˙3.2. Threshold enhancement in $t\bar{t}$ hadroproduction. Due to the attractive gluon exchange, the colour-singlet contribution to $t\bar{t}$ hadroproduction is enhanced near (the partonic) threshold. This leads to significant threshold corrections of the differential cross section $\mathrm{d}\sigma/\mathrm{d}M_{t\bar{t}}$ at the LHC, where gluon fusion dominates and the colour singlet channel is sizeable. Recently independent calculations, based on a formalism similar to the case of top pair production in e^+e^- collisions, have been performed to predict these threshold effects, see [27]. The corrections lead to contributions below the nominal threshold and a shift towards lower $M_{t\bar{t}}$. Kinematic distributions are also affected, see [28]. These effects could be relevant for future high-precision determinations of the top mass and require further studies.

A closely related process is stop pair production in hadron collisions, which may be observed at the LHC if low-energy supersymmetry is a reality. A sizeable enhancement of the cross section using a combined resummation of soft and Coulombic gluon corrections and adding bound state contributions below threshold has been found, see [29].

3. – Top couplings

3˙1. *$t\bar{t}H$ production.* – In order to fully establish the mechanism of mass generation in the SM via spontaneous electroweak symmetry breaking and the Higgs boson, also the

(4) For a detailed study of these problems see [23] and references therein, and [24] for the discussion of "top-jets" at the LHC.

fermion Yukawa couplings to the Higgs should be measured. While, for light fermions, this will be impossible for the foreseeable future, the top quark's large Yukawa coupling will be measurable at a LC in $t\bar{t}H$ production if the Higgs mass is in the range as expected by the electroweak precision fits of the SM. The experimental analyses will be challenging due to the complicated final states and the low signal compared to the high background rates. Early analyses had found a rather limited accuracy of $\Delta y_t/y_t \sim 23\%$, for a light Higgs of 120 GeV mass and assuming a baseline design with $\sqrt{s} = 500\,\text{GeV}$ and integrated luminosity of $1000\,\text{fb}^{-1}$, whereas the reach would be better for a 800 GeV collider. However, at $\sqrt{s} \sim 500\,\text{GeV}$ $t\bar{t}H$ production will be non-relativistic and be dominated by a threshold dynamics similar to the case of $t\bar{t}$ production discussed above. This leads to a large enhancement, which was calculated in [30]. The resulting expected event rates will be larger by more than a factor of two, and a similar gain could be achieved if the ILC would be operated with polarised beams. In such a case, an accuracy as high as $\Delta y_t/y_t \sim 10\%$ would be expected for a Higgs with $m_H = 120\,\text{GeV}$ at a 500 GeV collider assuming $1\,\text{ab}^{-1}$ and e^+, e^- beam polarisations of +30 and −80%, respectively [31].

3˙2. *Electroweak couplings and New Physics.* – Studies of the top quark's electroweak couplings have been performed at the Tevatron and have already started at the LHC. The Wtb coupling (and with it the CKM matrix element V_{tb}) can be measured in single top production([5]). In contrast, the Ztt coupling, which typically has an increased sensitivity to New Physics, will only become accessible at a LC. In a generic study, all SM and beyond the SM couplings have been parametrised by a set of gauge-invariant dimension-four operators [33]. It turns out that many four-fermion operators can only be tested at the ILC and not at the LHC, while for others the ILC would improve the accuracy achievable at the LHC. A topic which has attracted a lot of attention recently is the top charge asymmetry, which is a sensitive probe in many New Physics scenarios (see, *e.g.*, [34] and references therein, and [35] for a study of top quark anomalous couplings in the forward-backward asymmetry A_{FB} at the ILC). It has been calculated at order α_s^3 for hadron colliders [36] and leads to a forward-backward asymmetry at the percent level at the Tevatron. For the LHC with its symmetric pp initial state a smaller asymmetry is expected. Measurements at the Tevatron are in fair agreement with (about 2σ away from) the SM prediction and leave room for contributions beyond the SM. Such contributions could arise from axi-gluons, coloured scalars or additional Z' bosons with strong couplings to tops, originating from GUT models or as Kaluza-Klein modes appearing in extra-dimensional models. Studies at the LHC and the ILC will be able to scrutinize many of the scenarios put forward as possible extensions of the SM.

3˙3. *α_s with GigaZ.* – While it will be possible to measure the strong coupling α_s in top quark studies (*e.g.*, in a threshold scan as discussed above), using a very precise value as input will help those studies to increase the accuracy of other measurements. A most precise value of α_s is also a crucial ingredient in theoretical calculations related to the unification of the couplings (and masses) in supersymmetric or Grand Unified

([5]) Note that while the experimental resolution at hadron colliders is not sufficient for a direct measurement of the top quark width from mass distributions, Γ_t can be determined indirectly from single top production and using the branching fraction $\mathcal{B}(t \to Wb)$ as measured in $t\bar{t}$ production. In this way D0 at the Tevatron has found $\Gamma_t = 2.0^{+0.7}_{-0.6}\,\text{GeV}$ [32, 2], which is in agreement with the SM expectation.

Theory scenarios. One of the most precise and theoretically “clean” determinations of α_s could be achieved at the ILC by performing a dedicated low energy but very high statistics run at $\sqrt{s} = M_Z$, coined Giga$Z$. Similar to LEP1, observables related to the Z line-shape (like the hadronic and leptonic cross sections on the Z resonance, and the total and leptonic widths Γ_Z and $\Gamma_Z^{\rm lep}$) could be measured with unprecedented accuracy. For the determination of α_s, the observable R defined as the ratio of the hadronic over the leptonic cross section is most important. Its dependence on α_s can be predicted very reliably in perturbative QCD, and the accuracy which could be achieved for α_s with Giga$Z$ could be as high as $\Delta\alpha_s(M_Z^2) = 0.0005$ [37].

4. – Conclusions

Despite the uncertainty concerning the realisation of the ILC, the study of top quark physics at a LC is remaining a strong and active field and has triggered many theoretical developments. This includes the formulation of effective field theories and mass schemes. Theoretical predictions have been made up to (N)NNLO, but typically only for inclusive quantities. For Monte Carlo simulations, more and better tools need to be developed. For hadron colliders, complete predictions at NNLO are still missing. To fully exploit the top potential of the LHC, a better understanding of soft physics and jets will be required. While the LHC may eventually deliver more than we now think is possible, ultimately the ILC should be *the* next machine for precision studies of the top sector in the SM and beyond.

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