Colloquia: LaThuile11

Comments on CPT(*)

V. Novikov

ITEP - Moscow, Russia

Summary. — We present a class of interacting nonlocal quantum field theories, in which the CPT invariance is violated while the Lorentz invariance is not. This result rules out a previous claim in the literature that the CPT violation implies the violation of Lorentz invariance.

PACS 11.10.-z – Field theory.
PACS 11.30.Er – Charge conjugation, parity, time reversal, and other discrete symmetries.

1. – Introduction

Lorentz symmetry and the CPT invariance are two of the most respectable symmetries in Nature. The individual symmetries C, P and T have been observed to be violated. The combined product —CPT— remarkably remains as an exact symmetry.

2. – A brief history of CPT

As far as I know, J. Schwinger was the first one who mentioned the CPT theorem in his paper in 1951. He considered CPT theorem as a rather self-evident statement. The first Proof of CPT was done by Lüders [2], by Pauli [3] and by John Bell (but I have never seen his paper) within the Hamiltonian formulation of quantum field theory with local and Lorentz-invariant interaction. Later Jost [4] gave a General Proof of CPT within the axiomatic formulation of quantum field theory. The great deal was that the "local commutativity" condition was relaxed to "weak local commutativity".

Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian QFT and in the axiomatic QFT. The main idea was that the reflection of all 4 axes is equvalent to the rotation for Euclidean space with even dimensions. Lorentz boost

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in Minkowsky space-time is more or less equivalent to rotation in Euclidean time. The problem was in accurate continuation of Minkowsky space to Euclidean one.

Violation of Lorentz symmetry and CPT was considered in the literature for decades. A long list of references includes the great names of Coleman, Glashow, Okun, Colladay, Kostelecky, Cohen, Lehner.

It is important to clarify the relation between CPT and Lorentz invariance. Does the violation of any of the symmetries automatically imply the violation of the other one? This issue has recently become a topical one due to the growing phenomenological importance of CPT-violating scenarios in neutrino physics and in cosmology.

The first phenomenological consideration was made by Murayama and Yanagida [5]. They introduced a CPT-violating quantum field theory with a mass difference between neutrino and antineutrino. Later Barenboim *et al.* [6] and then Greenberg [7] investigated theoretical aspects of this assumption. Greenberg conclusion is: CPT violation implies violation of Lorentz invariance.

The dispute on the validity of the theorem is the subject of this talk.

3. – CPT-violating free-field model

To formulate the CPT-violating free-field model we use commutation relations for particle $a(p), a^+(p')$ with mass m; Bose commutation relations for antiparticle $b(p), b^+(p')$ with mass $\tilde{m}$ and the considered Hamiltonian as a sum over free oscillators.

Greenberg arguments were that the propagator of free particles is not Lorentz covariant, unless the masses of particle and antiparticle coincide. The theory is nonlocal and acausal: the $\Delta(x, y)$-function, *i.e.* the commutator of two fields, does not vanish for space-like separation, unless the two masses are the same, thus violating the Lorentz invariance. These arguments support a general "theorem" that interacting fields that violate CPT symmetry necessarily violate Lorentz invariance.

I would like to point out that such theory can not be considered as a quantum field theory. There are no differential equations of motion. Canonical conjugate momenta do not exist and, as a result, there are no canonical equal-time commutation relations "Free fields" separated by a space-like distance do not commute. They do not anticommute as well. One has no rule whether to apply commutation or anticommutation relations in quantizing the fields! There does not exist any reasonable field theory formulation of a model where particle and antiparticle have different masses.

4. – CPT-violating, Lorentz-invariant nonlocal model

We propose a model which preserves Lorentz invariance and breaks the CPT symmetry through a (nonlocal) interaction.

In this model the free-field theory is a local one. Nonlocal field theories appear, in general, as effective field theories of a larger theory.

Consider a field theory with nonlocal interaction Hamiltonian of the type

$$(1) \qquad \mathcal{H}_{int}(x) = g \int \mathrm{d}^4 y \, \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi(y) + \text{h.c.},$$

where $\phi(x)$ is a Lorentz-scalar field and θ is the Heaviside step function, with values 0 or 1, for its negative and positive argument, respectively. The combination $\theta(x_0 - y_0)\theta((x-y)^2)$ ensures the Lorentz invariance, *i.e.* invariance under the proper orthochronous Lorentz

transformations, since the order of the times x_0 and y_0 remains unchanged for time-like intervals, while for space-like distances the interaction vanishes.

Also, the same combination makes the nonlocal interaction causal at the tree level, which dictates that there is no interaction when the fields are separated by space-like distances and thus there is a maximum speed of $c = 1$ for the propagation of information.

On the other hand, it is clear that C and P invariance are trivially satisfied in (1), while T invariance is broken due to the presence of $\theta(x_0 - y_0)$ in the integrand.

One can always insert into the Hamiltonian (1), without changing its symmetry properties, a weight function or form factor $F((x - y)^2)$, for instance of a Gaussian type:

$$(2) \qquad F = \exp\left(-\frac{(x - y)^2}{l^2}\right),$$

with l being a nonlocality length in the considered theory. Such a weight function would smear out the interaction and would guarantee the desired behaviour of the integrand in (1); in the limit of fundamental length $l \to 0$ in (2), the Hamiltonian (1) would correspond to a local, CPT- and Lorentz-invariant theory.

A weight function such as (2) would make the acausality of the model (see the next section) restricted only to very small distances, of the order of l. The latter could be looked upon as being a characteristic parameter relating the effective field theory to its parent one, for instance the radius of a compactified dimension when the parent theory is a higher-dimensional one. Furthermore, with such a weight function, the interaction vanishes at infinite $(x - y)^2$ separations and thus one can envisage the existence of in- and out-fields.

There exists a whole class of such CPT-violating, Lorentz-invariant field theories involving different, scalar, spinor or higher-spin interacting fields. Typical simplest examples are

$$(3) \qquad H_{int}(x) = g_1 \int \mathrm{d}^4 y \phi_1^*(x)\phi_1(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi_2(y) + \text{h.c.},$$

$$H_{int}(x) = g_2 \int \mathrm{d}^4 y \, \bar{\psi}(x)\psi(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi(y) + \text{h.c.},$$

$$H_{int}(x) = g_3 \int \mathrm{d}^4 y \, \phi(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi^2(y) + \text{h.c.}$$

5. – Quantum theory of nonlocal interaction

The S-matrix in the interaction picture is obtained as solution of the Lorentz-covariant Tomonaga-Schwinger equation:

$$(4) \qquad i\frac{\delta}{\delta\sigma(x)}\Psi[\sigma] = H_{int}(x)\Psi[\sigma],$$

with σ a space-like hypersurface, and the boundary condition

$$(5) \qquad \Psi[\sigma_0] = \Psi,$$

where H_{int} is for instance the Hamiltonian (3) with the fields in the interaction picture. Then eq. (4) with the boundary condition (5) represents a well-posed Cauchy problem.

The existence of a unique solution for the Tomonaga-Schwinger equation is ensured if the integrability condition

$$\text{(6)} \qquad \frac{\delta^2 \Psi[\sigma]}{\delta\sigma(x)\delta\sigma(x')} - \frac{\delta^2 \Psi[\sigma]}{\delta\sigma(x')\delta\sigma(x)} = 0,$$

with x and x' on the surface σ, is satisfied. The integrability condition (6), inserted into (4), requires that the commutator of the interaction Hamiltonian densities vanishes at space-like separation:

$$\text{(7)} \qquad [\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] = 0, \quad \text{for } (x-y)^2 < 0.$$

Since in the interaction picture the field operators satisfy free-field equations, they automatically satisfy Lorentz-invariant commutation rules. The Lorentz-invariant commutation relations are such that (7) is fulfilled only when x and y are space-like separated, $(x-y)^2 < 0$, *i.e.* when σ is a space-like surface. As a result, the integrability condition (7) is equivalent to the microcausality condition for local relativistic QFT. When the surfaces σ are hyperplanes of constant time, the Tomonaga-Schwinger equations reduce to the single-time Schrödinger equation. Inserting the expression (3) into (7), we have

$$\text{(8)} \qquad [\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] =$$
$$\int \mathrm{d}^4 a \mathrm{d}^4 b\, \theta((x-a)^2)\theta(x^0 - a^0)\theta((y-b)^2)\theta(y^0 - b^0)$$
$$\times [\phi(x)\phi^2(a) + \text{h.c.}, \phi(y)\phi^2(b) + \text{h.c.}].$$

The commutator on the r.h.s. will open up into a sum of products of field at the points x, y, a, b, multiplied by commutators of free fields like $[\phi(x), \phi(y)]$, $[\phi(x), \phi(b)]$, $[\phi(a), \phi(y)]$, $[\phi(a), \phi(b)]$. In order for the commutator (8) to vanish, all the coefficients of the products of fields in the expansion have to vanish, since the fields at different space-time points are independent. Clearly, the terms with the coefficient $\Delta(x-y) = [\phi(x), \phi(y)]$ vanish for $(x-y)^2 < 0$. However, the commutator (8) does not vanish for $(x-y)^2 < 0$. In order to show this, it is enough to show that one independent product of fields has nonzero coefficient. Let us consider the products which contain the fields $\phi(x)$, $\phi(y)$, $\phi(a)$, $\phi(b)$

A straightforward calculation shows that the terms containing these fields are:

$$\text{(9)} \qquad \int \mathrm{d}^4 a\, \mathrm{d}^4 b\, \theta((x-a)^2)\theta(x^0 - a^0)\theta((y-b)^2)\theta(y^0 - b^0)$$
$$\times 2\Delta(a-b)\{\phi(a), \phi(b)\}\phi(x)\phi(y) + \text{h.c.}$$

A closer study of the expression (9) shows that it does not vanish at space-like distances between x and y and thus the causality condition (7) is not satisfied.

This, in turn, implies that the field operators in the Heisenberg picture, $\Phi_H(x)$ and $\Phi_H(y)$, do not satisfy the locality condition

$$\text{(10)} \qquad [\Phi_H(x), \Phi_H(y)] = 0, \quad \text{for } (x-y)^2 < 0,$$

when the quantum corrections are taken into account. This is in accord with the requirement of locality condition (10) for the validity of CPT theorem both in the Hamiltonian

proof (Luders, Pauli) and in the axiomatic one (Jost, Bogoliubov), taking into account that there is no example of a QFT, which satisfies the weak local commutativity condition (WLC) but not the local commutativity (LC).

6. – Conclusions

Let me summarize the results. We have presented a very simple class of interacting nonlocal quantum field theories, which violate CPT invariance and preserve Lorentz invariance. This result invalidates a general claim made previously by Greenberg, that "CPT violation implies violation of Lorentz invariance". Violation of Lorentz invariance does not necessarily lead to CPT violation.

We hope that we have made a step in the right direction.

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