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Universality and evolution of TMDs

S. M. Aybat(1) and T. C. Rogers(2)

(1) *Nikhef Theory Group - Science Park 105, 1098XG Amsterdam, The Netherlands*
(2) *C.N. Yang Institute for Theoretical Physics, Stony Brook University - Stony Brook NY 11794, USA*

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Summary. — In this talk, we summarize how QCD evolution can be exploited to improve the treatment of transverse momentum dependent (TMD) parton distribution and fragmentation functions. The methods allow existing non-perturbative fits to be turned into fully evolved TMDs that are consistent with a complete TMD-factorization formalism over the full range of k_T. We argue that evolution is essential to the predictive power of calculations that utilize TMD parton distribution and fragmentation functions, especially TMD observables that are sensitive to transverse spin.

PACS 12.38.Bx – Perturbative calculations.
PACS 12.39.St – Factorization.
PACS 12.38.Cy – Summation of perturbation theory.

1. – Collinear *versus* TMD Factorization

The standard collinear QCD factorization theorems have set the standard for the use of perturbative QCD calculations to probe certain properties of the microscopic structure of matter. It will be instructive to recall the essential ingredients that lend the standard factorization treatments their predictive power:

- Unambiguous prescription for calculating perturbatively well-behaved higher-order corrections to the hard scattering.
- Correlation functions describing the non-perturbative factors are well defined and have universality properties so that, once measured, they can be useful for future phenomenological studies.
- Evolution equations, to allow the correlation functions to be compared at different scales.

The collinear PDFs and fragmentation functions have by now been parametrized by a wide range and variety of experimental data, and have become indispensable tools in general high-energy physics. Ideally, TMD-factorization, in which the PDFs and fragmentation functions also carry information about the intrinsic transverse momenta of partons, should follow a very similar framework. However, it is only very recently [1-3] that TMD-factorization has reached a level of logical completeness similar to the well-known collinear cases. As long there was ambiguity in the formulation of TMD-factorization, the procedure for correctly identifying TMDs in fits, as well as the correct procedure for implementing evolution has remained unclear.

In the absence of a full TMD-factorization treatment, there have been several popular but separate approaches for dealing with TMDs:

– Generalized Parton Model (GPM): A phenomenological approach is to extract TMDs from fits to data while assuming a literal parton model interpretation of the TMDs. One typically ignores evolution and therefore the fits correspond to specific scales.

– Resummation: Begin with a collinear treatment valid for large transverse momentum, and attempt to improve the treatment of lower transverse momentum by resuming logarithms of q_T/Q. A severe limitation of this approach is that it is bound to fail below some q_T, where many of the most interesting effects of TMD physics are expected to become important.

– Models: Non-perturbative models of TMDs can be used to study specific non-perturbative aspects of hadron structure (see A. Bacchetta's talk for a review of model calculations.). But there are ambiguities in how these TMDs are related to the ones used in actual perturbative QCD calculations of cross sections. In particular, it is unclear what hard scale they should correspond to.

– Lattice Calculations: Lattice calculations of TMDs (see, *e.g.*, [4]) describe the non-perturbative distribution of partons from first principles, but also require a clear definition for the TMDs, and a clear prescription for use in complete cross section calculations.

A useful TMD-factorization treatment should allow the advantages of each of these approaches to be unified within a single, clear formalism for relating TMD studies to observable cross sections. Fortunately, this is now possible following the recent work of ref. [1] (see, especially, chapts. 10 and 13). (A similar general approach was developed earlier by Ji, Ma, and Yuan [2,3], which built upon the Collins-Soper-Sterman (CSS) [5,6] formalism.) We will show how fits to TMDs can be constructed from existing work that follows the above tabulated approaches, but which include evolution and are consistent with a full TMD-factorization.

We apply the TMD-factorization method developed recently by Collins in ref. [1]. The factorization theorem for the Drell-Yan (DY) process, for example, is

$$
\begin{aligned}
(1)\quad W^{\mu\nu}_{\mathrm{DY}} = \sum_f |\mathcal{H}_f(Q^2,\mu)|^{\mu\nu} \int & \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} F_{f/P_1}(x_1,\mathbf{k}_{1T},\mu,\zeta_{1F}) F_{f/P_2}(x_2,\mathbf{k}_{2T},\mu,\zeta_{2F}) \\
& \times\delta^{(2)}(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T) + \mathrm{Y} + \mathcal{O}(m/Q).
\end{aligned}
$$

Note the similarity of the first term above to a GPM description; there is a hard part $\mathcal{H}_f(Q^2,\mu)|^{\mu\nu}$ and a convolution of two TMD PDFs, $F_{f/P_1}(x_1,\mathbf{k}_{1T},\mu,\zeta_{1F})$ and $F_{f/P_2}(x_2,\mathbf{k}_{2T},\mu,\zeta_{2F})$. However, in the full TMD-factorization treatment, they have acquired scale dependence through the renormalization group parameter μ and ζ_{1F} and

ζ_{2F} (which obey $\sqrt{\zeta_{1F}\zeta_{2F}} = Q^2$). Moreover, there is no explicit appearance of a soft factor. The first term in eq. (1) is appropriate for describing the small q_T region ($q_T \ll Q$). The Y term corrects the large q_T behavior and can be calculated in terms of normal integrated PDFs. Reference [1] derives very specific operator definitions for the TMD PDFs, which include the role of soft gluons and account for all spurious divergences which have hindered efforts to clearly define the TMD PDFs in the past.

2. – Evolution of TMDs

The evolution of the individual TMDs in transverse coordinate space is governed by the Collins-Soper (CS) equation [5],

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu), \tag{2}$$

and the renormalization group equations,

$$\frac{\mathrm{d}\tilde{K}(b_T, \mu)}{\mathrm{d}\ln \mu} = -\gamma_K(g(\mu)), \qquad \frac{\mathrm{d}\ln \tilde{F}(x, b_T, \mu, \zeta)}{\mathrm{d}\ln \mu} = -\gamma_F(g(\mu), \zeta/\mu^2). \tag{3}$$

In eq. (2), $\tilde{K}(b_T, \mu)$ is the kernel for evolution with respect the energy variable ζ, while $\gamma_F(g(\mu), \zeta/\mu^2)$ and $\gamma_K(g(\mu))$ are the anomalous dimensions. These can all be calculated in perturbation theory, though $\tilde{K}(b_T, \mu)$ becomes non-perturbative at large b_T and needs to be fit. It is a universal function, however, both with respect to different processes and with regard to PDFs *versus* fragmentation functions.

3. – Specific fits

Working within a GPM approach, Gaussian parametrizations have been fit to various TMDs in, for example, ref. [7] from low energy SIDIS measurements. The non-perturbative information corresponding to $\tilde{K}(b_T, \mu)$ at large b_T in eq. (2) has been extracted from more traditional applications of the CSS method, such as in ref. [8]. With the non-perturbative parts constrained, specific tables for the TMDs in eq. (1) can be generated for any arbitrary scale Q once the perturbative contributions have been calculated.

We have carried this out explicitly for the unpolarized TMD PDFs and fragmentation functions in ref. [9], and have made the results available at the website: `https://projects.hepforge.org/tmd/`.

An example showing the unpolarized up quark PDF for a selection of Q values is the graph in fig. 1.

Note that, once the Y term is included in eq. (1), the factorization formula exactly valid over the whole range of k_T from $k_T = 0$ to Q.

4. – Conclusions and future directions

A crucial element of the TMD-factorization approach is that the TMDs are well-defined and, therefore, can be taken to be genuinely universal. One consequence of this is that different theories of the non-perturbative properties of hadron structure can be compared with one another, and with experimental results over a range of hard scales.

Ultimately, in order to take full advantage of the predictive power of TMD-factorization, requires a concerted effort to improve upon non-perturbative fits, to calculate

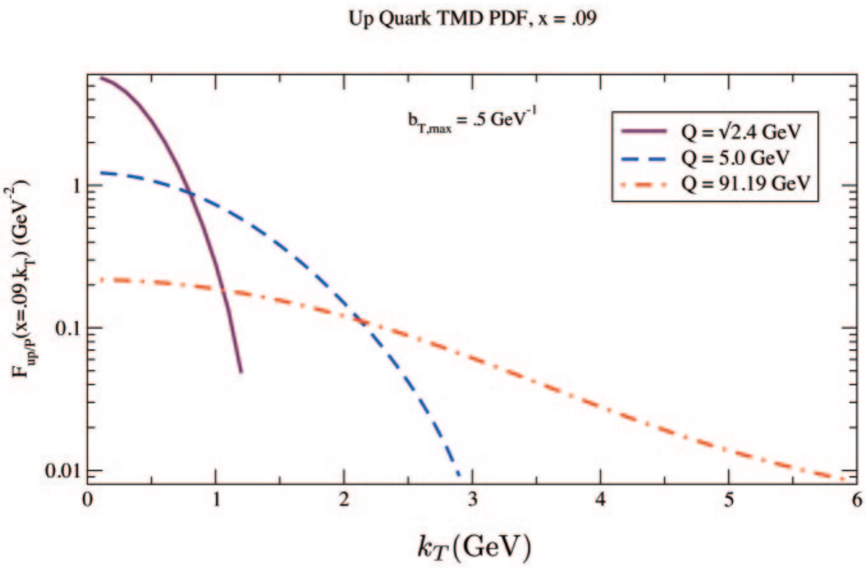

Fig. 1. – Unpolarized TMD PDF for an up-quark, for scales $Q = \sqrt{2.4}$, 5 and 91.19 GeV.

higher orders to the anomalous dimensions, K, the coefficient functions, and the hard parts, and to provide numerical implementations of evolution in transverse momentum space similar to what has been done in ref. [9], but applied to other TMD PDFs and fragmentation functions, including interesting spin dependent ones. We have recently been involved in efforts to extend evolution to the Sivers function [10], and plan to continue this work to generate evolved fits for most of the various spin-dependent TMDs.

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REFERENCES

[1] Collins J. C., *Foundations of Perturbative QCD* (Cambridge University Press, Cambridge) 2011.
[2] Ji X. D., Ma J. P. and Yuan F., *Phys. Rev. D*, **71** (2005) 034005.
[3] Ji X. D., Ma J. P. and Yuan F., *Phys. Lett. B*, **597** (2004) 299.
[4] Musch B. U., Hagler P., Negele J. W. and Schafer A., arXiv:1011.1213 [hep-lat].
[5] Collins J. C. and Soper D. E., *Nucl. Phys. B*, **193** (1981) 381; **213** (1983) 545(E).
[6] Collins J. C., Soper D. E. and Sterman G., *Nucl. Phys. B*, **250** (1985) 199.
[7] Schweitzer P., Teckentrup T. and Metz A., *Phys. Rev. D*, **81** (2010) 094019.
[8] Landry F., Brock R., Nadolsky P. M. and Yuan C. P., *Phys. Rev. D*, **67** (2003) 073016.
[9] Aybat S. M. and Rogers T. C., *Phys. Rev. D*, **83** (2011) 114042.
[10] Aybat S. M., Collins J. C., Qiu J. W. and Rogers T. C., to be published in *Phys. Rev. D*, arXiv:1110.6428 [hep-ph].