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Unpolarized and polarized Fragmentation Functions

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Summary. — I give an overview of the present knowledge about nonperturbative functions parametrizing the fragmentation into one or two hadrons of (un)polarized light quarks in vacuum, including information on their transverse momentum dependence.

PACS `13.66.Bc` – Hadron production in e^+e^- interactions.
PACS `13.87.Fh` – Fragmentation into hadrons.
PACS `13.88.+e` – Polarization in interactions and scattering.

1. – Introduction

The fragmentation process describes the transition from a highly virtual parton i at a scale Q^2 to one hadron h carrying a fraction z of its energy. The information is encoded in the nonperturbative fragmentation function $D^h_{1,i}(z, Q^2)$, which must be extracted from experiments. A large amount of data has been collected in last 30 years by measuring hadron spectra in e^+e^- annihilations. Based on these data, several parametrizations of $D^h_{1,i}(z, Q^2)$ have been released. More recently, new measurements in Semi-Inclusive Deep-Inelastic Scattering (SIDIS) and in hadronic (p-p and p-$\bar{p}$) collisions were included in various fits. A very brief overview is given in sect. **2** and the most updated parametrizations of $D^h_{1,i}$ (only for light partons in vacuum, again for brevity) are compared in sect. **2**˙1.

The dependence of $D^h_{1,i}$ upon the transverse momentum k_T of the fragmenting parton is basically unknown. Most of the phenomelogical studies are based on a simple flavor- and z-independent Gaussian ansatz. But in several experimental results for hadron multiplicities the evidence emerges about transverse-momentum distributions depending on both energy and flavor of the detected hadron. This topic is directly addressed in sect. **2**˙2, and also in sect. **3** where the main three types of models of fragmentation functions are sketched.

The above considerations apply also to polarized fragmentation functions, actually to the only one that has been parametrized so far: the Collins function. In fact, only its first k_T moment could be extracted leaving the k_T dependence fully unconstrained (see sect. **2**˙3). The Collins effect in spin asymmetries in SIDIS is one crucial tool to address the so-called transversity parton distribution [1], a poorly known cornerstone in the knowledge of the (spin) partonic structure of the nucleon. That is why on one side models of the Collins function were developed and studied in detail (see sect. **3**˙1 and **3**˙3), and on the other side alternatives were considered in the "hunting for transversity".

The most promising alternative is based on a spin asymmetry in SIDIS with two hadrons detected in the final jet. The corresponding Di-hadron Fragmentation Functions (DiFF) are encoded in functions like $D_{1,i}^{h_1 h_2}(z_1, z_2, M_h^2, Q^2)$, which must depend also on the invariant mass of the hadron pair, M_h^2; the latter represents a second natural scale in the fragmentation, with $M_h^2 \ll Q^2$ [2]. Further details and some first results are presented in sect. **4**.

2. – Single-hadron Fragmentation Functions

In order to extract information on $D_{1,i}^h(z, Q^2)$ from data, the most suitable process is by far the electron-positron annihilation. Measuring the socalled scaled-energy distribution $(1/\sigma_{\text{tot}})\frac{\mathrm{d}\sigma^h}{\mathrm{d}z}$ gives direct access at leading order (LO) in α_s to the fragmentation function summed over all active flavors. Well established factorization theorems [3] allow to explore higher orders in terms of perturbatively calculable coefficient functions, that are known up to NNLO in the $\overline{MS}$ scheme [4]. A large amount of data has been collected in the last 30 years in a wide energy range and for various hadron species (see ref. [5] and references therein for a short review). Most experiments were able to disentangle the contribution of light quarks (u, d, s) from c and b jets. In particular, the OPAL collaboration released also an analysis with full flavor separation [6]. However, since at LO the e^+e^- annihilation leads to the back-to-back production of a quark and an antiquark jet, data only allow for the extraction of the flavor-inclusive fragmentation function $D_{1,q}^h + D_{1,\bar{q}}^h$. Moreover, the gluon fragmentation function $D_{1,g}^h$ can only be extracted from 3-jet events that by construction appear at NLO; hence, it is weakly constrained.

Fortunately, these drawbacks can be compensated by Semi-Inclusive Deep-Inelastic Scattering (SIDIS) data, where $D_{1,q}^h$ and $D_{1,\bar{q}}^h$ can be independently extracted, and by data on hadronic collisions, where the $D_{1,g}^h$ can be directly addressed.

SIDIS data have been collected in the last 15 years for both unidentified ($h^\pm$) and identified charged hadrons ($\pi^\pm, K^\pm$, and also $\Lambda, \bar{\Lambda}$) mostly in e^-p collisions at HERA (H1 [7], HERMES [8], and ZEUS [9] collaborations), and also at CERN with muonic (anti)neutrino beams (NOMAD [10]). The explored kinematical range, $1 \leq Q \leq 100\,\text{GeV}$ and $0.1 \leq z < 1$, significantly enlarges the phase space available, since the hard scale is not constrained at the cm energy as in e^+e^-.

Hadron spectra in hadronic collisions appeared more recently, thanks to high-precision $p-p$ measurements at RHIC (BRAHMS [11], PHENIX [12], and STAR [13] collaborations) and $p-\bar{p}$ ones by CDF [14] at the Tevatron.

2˙1. *Unpolarized fragmentation.* – The year 2007 represents a sort of turning point for the phenomenological work about extraction of $D_{1,i}^h(z, Q^2)$ from experiments. All parametrizations released before this date are based on e^+e^- data only, suffer from large uncertainties at large z and Q^2, and fail to reproduce the scaling violations diplayed by SIDIS data reported by the H1 collaboration [7].

On year 2007, two parametrizations (HKNS [15] and DSS [16, 17]), followed by AKK08 [18] one year later, have been released which include also data from SIDIS and

TABLE I. – *Main features of global fit analyses DSS* [16, 17]*, HKNS* [15]*, and AKK08* [18]*: data sample, kinematic range covered (Q^2 in GeV^2), technique for error analysis.*

DSS	HKNS	AKK08
e^+e^-, SIDIS, pp	e^+e^-	e^+e^-, pp, $p\bar{p}$
$0.05 \leq z$, $1 \leq Q^2 \leq 10^5$	$0.01 \leq z$, $1 \leq Q^2 \leq 10^8$	$0.05 \leq z$, $2 \leq Q^2 \leq 4 \times 10^4$
Lagrange multipliers	Hessian errors	in progress

hadronic collisions, and show an error analysis in the fit. Their main features are listed in table I. $SU(2)$ isospin symmetry is always assumed for the unfavoured (sea) channel at the starting scale; AKK08 and HKNS assume it also for the favoured one. AKK08 further includes the resummation of leading (LL) and next-to-leading (NLL) logarithms for $z \to 1$ both in the evolution equations and in the coefficient functions of the factorization formula. In fact, AKK08 and DSS produce very similar results but at large z [18], where the effect of large logarithms is dominant; HKNS gives a doubtful softer gluon $D_{1,g}^{\pi^\pm}$ because it lacks the constraint from RHIC pp data [15]. Remarkably, all sets fail to reproduce the STAR data for $\Lambda, \bar{\Lambda}$ production in pp collisions by almost one order of magnitude [19].

2˙2. *Tranverse-momentum dependence.* – The completely unknown dependence upon the transverse momentum of partons is usually parametrized in terms of a Gaussian ansatz. In SIDIS, the Gaussian width is fixed, for example, by reproducing the data for the average transverse momentum squared $\langle \mathbf{P}_{h\perp}^2 \rangle$ of final hadron h with respect to the virtual photon direction in the lab [20]. A newer combined analysis of recent SIDIS data (including azimuthally asymmetric $\cos\phi$ and $\cos 2\phi$ modulations in the cross section) and Drell-Yan data has lead to a new parametrization where the $\langle \mathbf{P}_{h\perp}^2 \rangle$ distribution broadens linearly with the cm energy s (see ref. [21] and references therein).

However, there are several indications in SIDIS measurements that the Gaussian widths should depend at least on z and on the flavor content of the final hadrons. For example, the COMPASS 2004 data for $\langle \mathbf{P}_{h\perp}^2 \rangle$ show a clear dependence on z^2 and on the hadron charge, while the fit based on constant Gaussian widths can only marginally account for them [22]. Similarly, the HERMES collaboration reports that the asymmetry between deuteron and proton targets of multiplicities for pions and kaons are very sensitive to the different flavor content of different targets [23].

Finally, in ref. [24] a first attempt in describing the evolution of transverse-momentum dependent (TMD) nonperturbative functions, either initial distributions or final fragmentations, was put forward in the context of a proper factorization theorem. It turns out that even at LO, for the test case $D_{1,u}^{\pi^+}$ of interest here, the transverse momentum distribution strongly depends on the hard scale Q^2 even at very low values of momenta, getting broader and broader with increasing Q^2.

2˙3. *Polarized fragmentation.* – The only polarized fragmentation function extracted from experimental data so far is the socalled Collins function $H_1^\perp$ [1]. It is related to the probability density of having a distorted $\mathbf{P}_{h\perp}$ distribution of the final hadron h depending on the direction of the transverse polarization $\mathbf{S}_q$ of the fragmenting quark. It can appear in $e^+e^- \to q^\uparrow q^\downarrow \to h^+h^-X$ events or, more interestingly, in SIDIS on transversely polarized targets. In fact, a specific azimuthally asymmetric modulation of the leading-twist SIDIS cross section contains the convolution $h_1^q \otimes H_{1,q}^{\perp\, h}$ on the transverse momenta of the initial and final quarks, where h_1 is the so-called transversity parton distribution, a poorly known cornerstone in the knowledge of the (spin) partonic structure of the nucleon (for a review, see for example ref. [25]).

Both azimuthal asymmetries in e^+e^- and in SIDIS have been recently measured with increased precision by the BELLE [26] and HERMES and COMPASS [27, 28] collaborations, respectively. A simultaneous fit of the three data sets made it possible for the first time to extract a parametrization for the transversity h_1 [29] with the interesting byproduct that $H_{1,u}^{\perp\,\pi^-} \approx -H_{1,u}^{\perp\,\pi^+}$, *i.e.* that the unfavoured fragmentation is opposite to and as large as the favoured one [27].

The azimuthal asymmetry in e^+e^- does not constrain the transverse-momentum dependence of $H_1^{\perp}$. The convolution in the SIDIS cross section is, then, computed assuming again a Gaussian behaviour independently of kinematics and of the involved flavor; therefore, the extraction of h_1 in ref. [29] is affected by a model dependence. Moreover, a full treatment of TMD evolution between the e^+e^- scale ($Q^2 = 100\,\mathrm{GeV}^2$) and the SIDIS scale ($Q^2 = 2.5\,\mathrm{GeV}^2$) is still missing [30].

3. – Models

Since the extraction of fragmentation functions from experimental data is sometimes affected by large uncertainties, it is desirable that this phenomenology is supported by model speculations. In the following, we sketch three main classes of models that appeared in the recent literature.

3˙1. *Spectator approximation.* – The spectator approximation amounts to describe the fragmentation as the decay of a parton with momentum k into the observed hadron h with momentum P_h leaving a residual system in an on-shell state with a specific mass. The latter condition grants that most of the calculations can be performed analytically, including the expression for the off-shellness $k^2(z)$ of the fragmenting parton. The drawback is that only the favoured channel can be taken into account. For the typical $u \to \pi^+$ channel, two main choices have been adopted in the literature for the quark-pion-spectator vertex: the pseudoscalar coupling $g_{\pi q}\gamma_5$ [31-34] and the pseudovector coupling $g_{\pi q}\gamma_5\gamma_\mu P_h^\mu$ [32]. In all cases the coupling was assumed to be point-like except in refs. [34, 33], where a Gaussian form factor was used with a z-dependent cut-off.

The Collins function is given by the interference of different channels. In the spectator approximation, these final-state interactions can be achieved by considering the interference with the amplitude at tree level and by including loop insertions (self-energies, vertex corrections,. . .) involving pions and/or gluons. As an example, in fig. 1 the transverse-momentum integrated $\frac{1}{2}$-moment $H_{1,u}^{\perp\,\pi^+(1/2)}$ (normalized to $D_{1,u}^{\pi^+}$) from ref. [33] is plotted as a function of z for $Q^2 = 0.4$ (solid line), $Q^2 = 2.4$ (dashed), and $Q^2 = 110\,\mathrm{GeV}^2$ (dot-dashed). The dashed line must be compared with the uncertainty band corresponding to the parametrization of ref. [35] at $Q^2 = 2.5\,\mathrm{GeV}^2$.

3˙2. *Nambu–Jona-Lasinio jet model.* – In the Nambu–Jona-Lasinio (NJL) jet model [36], the fragmentation is represented as a sequence of quark → quark+meson splittings. The fragmentation $D_{1,q}^m(z)$ of a quark q in the meson m can be obtained by solving a set of coupled integral equations where each splitting step is described by an elementary fragmentation function d_q^m and all allowed intermediate states Q in the

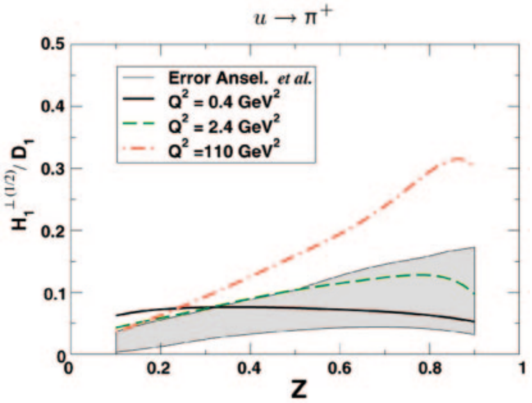

Fig. 1. – The spectator model result for $H_{1,u}^{\perp\,\pi^+(1/2)}(z)/D_{1,u}^{\pi^+}(z)$ from ref. [33] (see text). Solid (black), dashed (green), and dot-dashed (red) lines for $Q^2 = 0.4, 2.4, 110\,\mathrm{GeV}^2$, respectively. Uncertainty band for the phenomenological extraction of ref. [35] at $Q^2 = 2.5$.

cascade are taken into account. In each splitting, the d_q^m depends on the quark-meson coupling g_{qmQ}, which is determined from the residue (at the pole of the meson mass) in the quark-antiquark T matrix [37].

The above framework is justified only in the Bjorken limit where the quark initiating the cascade has an infinite momentum and produces an infinite number of hadrons. Moreover, solving the coupled integral equations is sometimes a heavy computational task. For these reasons, the quark-cascade description of the fragmentation has been approached using the Monte Carlo technique [38]. In this context, the fragmentation function is deduced by calculating the average number of hadrons of type h produced in the cascade for a predefined number of steps N_{step}. Each step is randomly sampled using the d_q^h calculated in the NJL jet model, and the entire cascade is simulated N_{sim} times with N_{sim} large enough to stabilize the average.

Several results have been presented in ref. [38] for different types of hadrons and for (un)favoured channels. Overall, there is a qualitative agreement with the HKNS and DSS parametrizations except for low z, where the model results for $D_i^h(z)$ tend to diverge too rapidly.

3˙3. *Recursive model.* – Present Monte Carlo event generators of quark and gluon jets do not include spin in the elementary degrees of freedom. Therefore, in order to have a guide in studying azimuthal asymmetries like the Collins effect, a new quantum approach to polarized quark fragmentation was suggested in ref. [39]. As in the previous case, it is based on a repeated sequence of quark → quark+(pseudo-scalar meson) splittings. But the overall amplitude is estimated in the socalled multiperipheral model where each quark propagator is approximated by an expression resembling the meson-nucleon scattering amplitude, *i.e.* with a non–spin-flip complex function μ and a spin-flip part.

If $\mathrm{Im}(\mu) \neq 0$, this imaginary part can act as a source (or sink) of transverse polarization even if the quark was unpolarized or longitudinally polarized at the previous step [39]. This means also that during the cascade the helicity of a quark can be partly converted to its transversity or viceversa. In this way, the Collins effect can be repeatedly generated with alternate sign, thus respecting the experimental finding of $H_1^{\perp\,\mathrm{unf}} \approx -H_1^{\perp\,\mathrm{fav}}$ [27]. Moreover, it is possible to recover the jet handedness of ref. [40] as a two-step process,

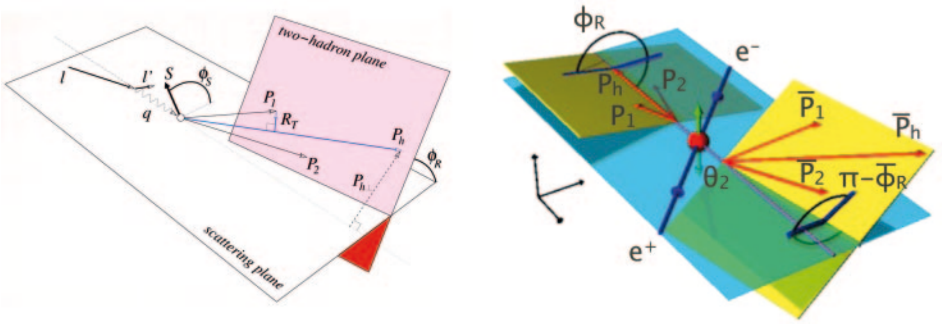

Fig. 2. – The kinematics for the SIDIS process $ep^\uparrow \to e'(h_1h_2)X$ (left panel) and for the process $e^+e^- \to (h_1h_2)(h_1'h_2')X$ (right panel, adapted from ref. [42]).

where a transverse polarization is created by some helicity at previous step, followed by a Collins effect [39].

Further work is needed to promote the multiperipheral model to a realistic Monte Carlo event generator, by including, *e.g.*, antiquarks in the fragmentation cascade or exploring the interference with amplitudes showing different orderings. Nevertheless, experimental results about an almost vanishing Collins effect for K^- [41] already appeared, that cannot be easily accommodated in this multiperipheral model.

4. – Di-hadron Fragmentation Functions (DiFF)

A complementary approach in the extraction of h_1 is provided by the semi-inclusive process $ep^\uparrow \to e'(h_1h_2)X$ where the two unpolarized hadrons with momenta P_1 and P_2 emerge from the fragmentation of the same quark. The kinematics is similar to the single-hadron SIDIS except for the final state, where the hadron pair carries a fractional energy $z = z_1 + z_2$ with a total momentum $P_h = P_1 + P_2$ and a relative momentum $R = (P_1 - P_2)/2$ (see fig. 2, left panel). In the underlying mechanism, the transverse polarization of the fragmenting quark q is transferred to the relative orbital angular momentum of the hadron pair. Contrary to the Collins effect, this mechanism survives after integrating away the transverse momentum of each particle and can be analyzed in the collinear factorization scheme [2]. The probabilistic weight for this to happen is represented by the polarized DiFF $H_{1,q}^{\sphericalangle\, h_1h_2}(z, M_h^2, Q^2)$, where $P_h^2 = M_h^2 \ll Q^2$ is the pair invariant mass and represents a new soft scale in the process.

DiFF were introduced in ref. [43] and studied for the polarized case in refs. [44-46]. The decomposition of the SIDIS cross section in terms of parton distributions and DiFF was carried out at leading twist in ref. [47] and to sub-leading twist in ref. [48]. Given the angle θ between P_1 in the pair cm frame and the direction of P_h in the lab frame [49], the leading-twist cross section shows an azimuthally asymmetric modulation proportional to $\sin(\phi_R + \phi_S)\sin\theta$, where ϕ_R, ϕ_S, are defined in fig. 2. The proportionality coefficient contains the product $h_1^q\, H_{1,q}^{\sphericalangle\, h_1h_2}$ [45, 47, 2, 49]: the advantage of working in collinear factorization scheme reflects in a very simple relation with no convolution on transverse momenta, as in the case of the Collins effect.

The $H_{1,q}^{\sphericalangle\, h_1h_2}$ is sensitive to the interference between the fragmentation amplitudes into hadron pairs in relative s wave and in relative p wave [49]. The corresponding unpolarized

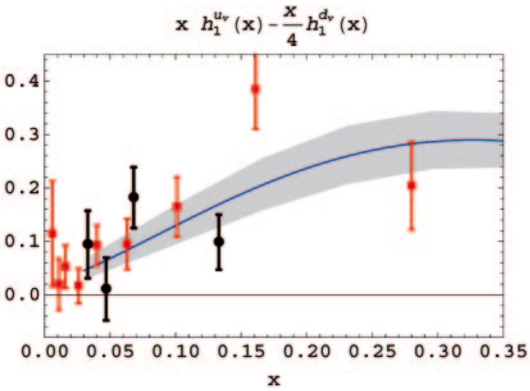

Fig. 3. – The combination of valence u, d, flavors for transversity. Black circles for the SIDIS data from HERMES [51], grey (red) squares from COMPASS [56]. The error bars are obtained by propagating the statistical errors of each term in $A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}$. The uncertainty band represents the same observable as deduced from the parametrization of ref. [29].

partner $D_{1,q}^{h_1 h_2}$ is averaged over quark polarization and hadron pair orientation. Similarly to the single-hadron SIDIS, the unknown DiFF must be independently determined from e^+e^- annihilation producing, in this case, two hadron pairs (see right panel of fig. 2, adapted from ref. [42]). The relevant signal is similar to that of the Collins function, except that each transverse polarization of the $q\bar{q}$ pair is now correlated to the azimuthal orientations $\phi_R, \phi_{\bar{R}}$, of the planes formed by the momenta of the corresponding hadron pairs. In the leading-twist cross section, this correlation shows up as a modulation proportional to $\cos(\phi_R + \phi_{\bar{R}})$ [50].

The $A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}$ spin asymmetry in SIDIS production of a $\pi^+\pi^-$ pair off a transversely polarized proton target was measured for the first time by the HERMES Collaboration [51], showing compatibility with predictions based on the spectator approximation [52, 53] (see also the later ref. [54]). Preliminary results are available also from the COMPASS collaboration using deuteron [55] and proton [56] targets. The $A^{\cos(\phi_R+\phi_{\bar{R}})}$ asymmetry in e^+e^- annihilation was recently measured by the BELLE collaboration [42]. By combining these data, the transversity h_1 was extracted for the first time in a collinear factorization scheme, properly including LO evolution effects in the behaviour of DiFF at different scales [57]. The result is shown in fig. 3 and compared with the parametrization of ref. [29]. We deduce that there is a substantial agreement between the two extractions of h_1 obtained from two independent methods.

5. – Outlooks

There are several interesting ongoing developments in each of the fields touched in previous sections. As for single-hadron fragmentations, a NNLO analysis of evolution effects is becoming available for $D_i^h(z, Q^2)$ [58, 59]. For $h = K$, nonsinglet fragmentation functions for $K^\pm$ can be directly extracted from data in a model independent way [60], the present limitation being due to weak constrains coming from a not enough large data set. As for two-hadron fragmentations, new data from the COMPASS collaboration have been released [61], which demand for a more refined analysis. Finally, some data were also released from the PHENIX collaboration about the $pp^\uparrow \to (\pi^+\pi^-)X$ process [62], which should help in separating the antiquark components of dihadron fragmentation functions and, consequently, of transversity [63].

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REFERENCES

[1] Collins J. C., *Nucl. Phys. B*, **396** (1993) 161.
[2] Radici M., Jakob R. and Bianconi A., *Phys. Rev. D*, **65** (2002) 074031.
[3] Collins J. C. and Soper D. E., *Nucl. Phys. B*, **194** (1982) 445.
[4] Mitov A. and Moch S., *Nucl. Phys. B*, **751** (2006) 18.
[5] Arleo F., *Eur. Phys. J. C*, **61** (2009) 603.
[6] Abbiendi G. *et al.* (OPAL Collaboration), *Eur. Phys. J. C*, **16** (2000) 407.
[7] Aaron F. D. *et al.* (H1 Collaboration), *Phys. Lett. B*, **654** (2007) 148.
[8] Hillenbrand A., Ph.D. Thesis (2005).
[9] Chekanov S. *et al.* (ZEUS Collaboration), *JHEP*, **06** (2008) 061.
[10] Altegoer J. *et al.* (NOMAD Collaboration), *Phys. Lett. B*, **445** (1999) 439.
[11] Arsene I. *et al.* (BRAHMS Collaboration), *Phys. Rev. Lett.*, **98** (2007) 252001.
[12] Adare A. *et al.* (PHENIX Collaboration), *Phys. Rev. D*, **76** (2007) 051106.
[13] Adams J. *et al.* (STAR Collaboration), *Phys. Rev. Lett.*, **97** (2006) 152302.
[14] Acosta D. E. *et al.* (CDF Collaboration), *Phys. Rev. D*, **72** (2005) 052001.
[15] Hirai M., Kumano S., Nagai T. H. and Sudoh K. (HKNS), *Phys. Rev. D*, **75** (2007) 094009.
[16] De Florian D., Sassot R. and Stratmann M. (DSS), *Phys. Rev. D*, **75** (2007) 114010.
[17] De Florian D., Sassot R. and Stratmann M. (DSS), *Phys. Rev. D*, **76** (2007) 074033.
[18] Albino S., Kniehl B. A. and Kramer G. (AKK08), *Nucl. Phys. B*, **803** (2008) 42.
[19] Abelev B. I. *et al.* (STAR Collaboration), *Phys. Rev. C*, **75** (2007) 064901.
[20] Collins J. C. *et al.*, *Phys. Rev. D*, **73** (2006) 014021.
[21] Schweitzer P., Teckentrup T. and Metz A., *Phys. Rev. D*, **81** (2010) 094019.
[22] Rajotte J. F. (COMPASS Collaboration), in *Proceedings of the Conference Symmetries and Spin (Spin-Praha 2010)*, to be published in *Eur. Phys. J. ST*, arXiv:1008.5125.
[23] Joosten S. (COMPASS Collaboration), in *Proceedings of the XIX International Workshop on Deep-Inelastic Scattering and Relates Subjects (DIS 2011)*, to be published in *AIP Conf. Proc.* (2011).
[24] Rogers T. and Aybat M., *Phys. Rev. D*, **83** (2011) 114042.
[25] Barone V., Drago A. and Ratcliffe P. G., *Phys. Rep.*, **359** (2002) 1.
[26] Seidl R. *et al.* (BELLE Collaboration), *Phys. Rev. D*, **78** (2008) 032011.
[27] Diefenthaler M. *et al.* (HERMES Collaboration), arXiv:0706.2242.
[28] Alekseev M. G. *et al.* (COMPASS Collaboration), *Phys. Lett. B*, **673** (2009) 127.
[29] Anselmino M. *et al.*, in *Proceedings of the XVI International Workshop on Deep-Inelastic Scattering and Relates Subjects (DIS 2008)*; doi: 10.3360/dis.2008.224.
[30] Kang Z., *Phys. Rev. D*, **83** (2011) 036006.
[31] Jakob R., Mulders P. J. and Rodrigues J., *Nucl. Phys. A*, **626** (1997) 937.
[32] Amrath D., Bacchetta A. and Metz A., *Phys. Rev. D*, **71** (2005) 114018.
[33] Bacchetta A., Gamberg L., Goldstein G. R. and Mukherjee A., *Phys. Lett. B*, **659** (2008) 234.
[34] Gamberg L., Goldstein G. R. and Oganessyan K. A., *Phys. Rev. D*, **68** (2003) 051501(R).
[35] Anselmino M. *et al.*, *Phys. Rev. D*, **75** (2007) 054032.

[36] Ito T. *et al.*, *Phys. Rev. D*, **80** (2009) 074008.
[37] Matevosyan H. H., Thomas A. W. and Bentz W., *Phys. Rev. D*, **83** (2011) 074003.
[38] Matevosyan H. H., Thomas A. W. and Bentz W., *Phys. Rev. D*, **83** (2011) 114010.
[39] Artru X., presented at *XIII Workshop on High Energy Spin Physics (DSPIN 2009)*, arXiv:1001.1061.
[40] Efremov A.V., Mankiewicz L. and Törnqvist N.A., *Phys. Lett. B*, **284** (1992) 394.
[41] Airapetian A. *et al.* (HERMES Collaboration), *Phys. Rev. Lett.*, **103** (2009) 152002.
[42] Vossen A. *et al.* (BELLE Collaboration), *Phys. Rev. Lett.*, **107** (2011) 072004.
[43] Konishi K., Ukawa U. and Veneziano G., *Phys. Lett. B*, **78** (1978) 243.
[44] Collins J. C., Heppelmann S. F. and Ladinsky G. A., *Nucl. Phys. B*, **420** (1994) 565.
[45] Jaffe R. L., Jin X.-M. and Tang J., *Phys. Rev. Lett.*, **80** (1998) 1166.
[46] Artru X. and Collins J. C., *Z. Phys. C*, **69** (1996) 277.
[47] Bianconi A., Boffi S., Jakob R. and Radici M., *Phys. Rev. D*, **62** (2000) 034008.
[48] Bacchetta A. and Radici M., *Phys. Rev. D*, **69** (2004) 074026.
[49] Bacchetta A. and Radici M., *Phys. Rev. D*, **67** (2003) 094002.
[50] Boer D., Jakob R. and Radici M., *Phys. Rev. D*, **67** (2003) 094003.
[51] Airapetian A. *et al.* (HERMES Collaboration), *JHEP*, **06** (2008) 017.
[52] Bacchetta A. and Radici M., *Phys. Rev. D*, **74** (2006) 114007.
[53] Bacchetta A., Ceccopieri F. A., Mukherjee A. and Radici M., *Phys. Rev. D*, **79** (2009) 034029.
[54] She J., Huang Y., Barone V. and Ma B.-Q., *Phys. Rev. D*, **77** (2008) 014035.
[55] Martin A. (COMPASS Collaboration), *Czech J. Phys.*, **56** (2006) F33; arXiv:hep-ex/0702002.
[56] Wollny H. (COMPASS Collaboration), in *Proceedings of the XVII International Workshop on Deep-Inelastic Scattering and related subjects (DIS 2009) 26–30 April 2009, Madrid (Spain)*; doi: 10.3360/dis.2009.200; arXiv:0907.0961.
[57] Bacchetta A., Courtoy A. and Radici M., *Phys. Rev. Lett.*, **107** (2011) 012001.
[58] Almasy A. A., Vogt A. and Moch S., *Nucl. Phys. B*, **854** (2012) 133.
[59] Albino S., Bolzoni P. and Kniehl B. A., *Nucl. Phys. B*, **855** (2012) 801; arXiv:1108.3948.
[60] Albino S. and Christova E., *Phys. Rev. D*, **81** (2010) 094031.
[61] Braun C. (COMPASS Collaboration), these proceedings.
[62] Yang R. (PHENIX Collaboration), *AIP Conf. Proc.*, **1182** (2009) 569.
[63] Bacchetta A. and Radici M., *Phys. Rev. D*, **70** (2004) 094032.