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Transversity studies with a polarized ^{3}He target

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Summary. — A realistic study of the SiDIS process $^3\vec{\mathrm{He}}(\mathrm{e},\mathrm{e}'\pi)\mathrm{X}$ in the Bjorken limit is briefly reviewed, showing that the nuclear effects, present in the extraction of the neutron information, are largely under theoretical control, within an Impulse-Approximation approach. In view of the forthcoming experimental data, we shortly present a novel Poincaré covariant description of the nuclear target, implementing a Light-Front analysis at finite Q^2, within the Bakamijan-Thomas construction of the Poincaré generators. Furthermore, as a by-product of the introduction of a Light-Front spin-dependent spectral function for a $J = 1/2$ system, we straightforwardly extend our analysis to the quark-quark correlator, obtaining three new exact relations between the six leading-twist Transverse-Momentum–Dependent distributions.

PACS 13.60.Hb – Total and inclusive cross sections
(including deep-inelastic processes).
PACS 13.85.Hd – Inelastic scattering: many-particle final states.
PACS 13.85.Ni – Inclusive production with identified hadrons.

1. – Introduction

Due to ongoing and forthcoming challenging measurements in various Laboratories, the partonic structure of transversely polarized hadrons, not accessible in inclusive experiments, is widely studied by the hadronic Physics Community. In particular, semi-inclusive deep inelastic scattering (SiDIS) is one of the processes proposed to access transversity observables. The theoretical description of SiDIS implies a complicated formalism, accounting for the transverse motion of the quarks in the target [1-4]. In particular, the non-perturbative effects of the intrinsic transverse momentum $\vec{k}_T$ of the quarks inside the nucleon may induce significant hadron azimuthal asymmetries [5,6]. In order to describe such quantities, a central role is played by the Transverse-Momentum–Dependent parton distributions (TMDs), that depend, besides on the fraction x of longitudinal momentum of the target carried by the parton, as the standard parton distributions (PDs), also on its intrinsic transverse momentum. The number of TMDs, six at leading twist, is fixed by counting the scalar quantities allowed by hermiticity, parity and time-reversal invariance. However, the existence of leading twist final state interactions (FSI) allows for two additional time-reversal odd functions [7], namely, the Sivers and the Boer-Mulders functions [8, 9].

The experimental scenario which arises from the analysis of SiDIS off transversely polarized proton and deuteron targets is puzzling, showing a strong flavor dependence [10, 11]. With the aim of extracting the neutron information and clarifying the situation, a measurement of SiDIS off transversely polarized ^{3}He has been proposed [12], and an experiment, planned to measure azimuthal asymmetries in the production of leading $\pi^{\pm}$ from transversely polarized ^{3}He, has been just completed at Jefferson Lab (JLab), with a beam energy of 6 GeV [13]. Another experiment will be performed after upgrading the JLab beam to 12 GeV [14, 15]. In this contribution, a brief review of the realistic analysis of SiDIS off transversely polarized ^{3}He, as presented in ref. [16], is given and it is extended in order to implement both the Poincaré covariant treatment of the nuclear target and the finite values of the actual four-momentum transfer Q^2. In particular, the Light-front (LF) spin-dependent spectral function for a generic $J = 1/2$ target is introduced, exploiting the Bakamijan-Thomas (BT) construction of the Poincaré generators (see, *e.g.* [17] for a detailed review) and it is applied to the quark-quark correlator, obtaining new relations between the six leading-twist TMDs.

2. – A realistic description of ^{3}He Single Spin Asymmetries

In ref. [16], within an impulse approximation (IA) framework, the formal expressions of the Collins [18] and Sivers [8] contributions to the azimuthal Single Spin Asymmetry (SSA) for the production of leading pions off ^{3}He have been derived. In particular, the initial transverse momentum of the struck quark has been included, and, in order to give a realistic description of the nuclear dynamics, ^{3}He SSAs have been calculated by using the nucleon spin-dependent spectral function (SF), that yields the probability distribution to find a nucleon with given three-momentum and missing energy inside a target nucleus with polarization $\mathbf{S}_A$. The adopted SF was the one obtained in ref. [19] within a non-relativistic framework and corresponding to the AV18 nucleon-nucleon interaction [20]. Moreover, the nucleonic structure, *i.e.* PDs and fragmentation functions, has been described by proper parametrizations of data [21] or suitable model calculations [22]. In ref. [16], the crucial issue of extracting the neutron information from $^3\vec{\rm He}$ data has been also discussed and it was showing that, even for SiDIS, where fragmentation functions are present beside PD's, one can adopt the same extraction scheme already successfully applied to DIS. In this case, in order to take effectively into account the momentum and energy distributions of the polarized bound nucleons in ^{3}He, it was adopted a model independent procedure, based on the realistic evaluation of the proton and neutron polarizations in ^{3}He [23], p_p and p_n, respectively. According to this extraction procedure, the neutron SSA, A_n^i, is obtained from the nuclear one, $A_3^{exp,i}$, as follows:

$$A_n^i \simeq \frac{1}{p_n d_n}\left(A_3^{exp,i} - 2p_p d_p A_p^{exp,i}\right), \tag{1}$$

where $d_n(d_p)$ is the neutron (proton) dilution factor. For a neutron n (proton p) in ^{3}He, the diluition factor, experimentally known, is given by

$$d_{n(p)}(x,z) = \frac{\sum_q e_q^2 f^{q,n(p)}(x) D^{q,h}(z)}{\sum_{N=p,n} \sum_q e_q^2 f^{q,N}(x) D^{q,h}(z)}\,, \tag{2}$$

where $f^{q,N}(x)$ are the standard parton distributions and $D^{q,h}(z)$ the fragmentation functions with $z = E_h/\nu$ (see [16] for details). In eq. (1), the nucleon effective polarizations, p_p and p_n take properly care of all the effects in ^{3}He, like Fermi motion and binding, that produce a depolarization of the bound neutron. In ref. [16], it has been shown that, for any experimentally relevant x and z, the extraction scheme of eq. (1) is quite effective. This important result is due to the peculiar kinematics of the JLab experiments, which helps in two ways. Firstly, it favors the emission of fast pions from the current-quark fragmentation, (z has been chosen in the range $0.45 < z < 0.6$, which means that only high-energy pions are observed). Secondly, the pions are detected in a narrow cone around the direction of the momentum transfer, making nuclear effects in the fragmentation functions rather small [16]. Then, the leading nuclear effects result to be the ones affecting the parton distributions, as already found in inclusive DIS [23], and they can be taken into account in the usual way, *i.e.* by using eq. (1) for the extraction of the neutron information. In conclusion, eq. (1) represents a valuable tool for the experimental data analysis [13, 14].

While the analysis in [16] was performed assuming the experimental set-up of the experiment described in ref. [13], but using DIS kinematics, an analysis at the actual, finite values of the momentum and energy transfers, Q^2 and ν, is in progress [24], and some of the results are anticipated in the next section. Moreover, in order to improve the relativized framework where the non relativistic SF of ref. [19] was embedded, the new analysis is based on a fully Poincaré covariant description of the nuclear effects, by introducing a LF spin-dependent SF (see below).

3. – A Light-Front description of $^3\vec{\mathrm{He}}$

In the previous Section, it has been reported that the extraction procedure of the neutron information, successfully applied in DIS, nicely works also in SiDIS, though the calculation has been performed [16] in the Bjorken limit, using a non relativistic spin-dependent SF. Therefore, it is natural to test the extraction procedure in the actual JLab kinematics, rather far from the asymptotic one, and to investigate relativistic effects at nuclear level, in a more consistent way. In order to achieve this, we adopt the Relativistic Hamiltonian Dynamics (RHD) framework [25, 17], suitable for describing an interacting system in a Poincaré covariant way.

The RHD, introduced by Dirac [25], can be combined with the Bakamijan-Thomas explicit construction of the Poincaré generators [26] for obtaining a description of SiDIS off ^{3}He which: i) is fully Poincaré covariant; ii) has a fixed number of on-mass-shell constituents; iii) allows one to use the Clebsch-Gordan coefficients to decompose the wave function (a benefit from the BT construction). Obviously, this approach is not explicitly covariant, since locality is lost. Between the three possible forms of RHD, the *Light-Front* one has several advantages: i) seven Kinematical generators: three LF-boosts (at variance with the dynamical nature of the Instant-form boosts), three components of the LF-momentum, $\tilde{\mathbf{P}} \equiv \{P^+, \mathbf{P}_\perp\}$, and the rotation around the z-axis; ii) the LF-boosts have a subgroup structure, so that the separation of the intrinsic motion is trivially achieved (as in the NR case); iii) $P^+ \geq 0$ leads to a meaningful Fock expansion, in presence of massive boson exchanges; iv) there are no square roots in the operator P^-, propagating the state in the LF-time; v) the Infinite Momentum frame description of DIS is easily included. The main drawback is that the transverse LF-rotations are dynamical, but within the BT construction, one can define a *kinematical*, intrinsic angular momentum, reobtaining the rotational invariance even in presence of a truncated Fock space. This is a peculiar feature of RHD, at variance with Quantum Field Theory where infinite degrees of freedom are present due to the locality. The LF Hamiltonian Dynamics (LFHD) framework has been proven to be very suitable for phenomenological studies (see, *e.g.* [27, 28]).

For SiDIS, the nuclear hadronic tensor plays a central role, and in IA it reads

$$(3)\quad \mathcal{W}^{\mu\nu}(Q^2, x_B, z, \tau_{hf}, \hat{\mathbf{h}}, S_{\rm He}) \propto \sum_{\sigma,\sigma'} \sum_{\tau_{hf}} \int\!\!\!\!\!\!\sum_{\epsilon_S^{min}}^{\epsilon_S^{max}} \mathrm{d}\epsilon_S \int_{M_N^2}^{(M_X - M_S)^2} \mathrm{d}M_f^2 \int_{\xi_{lo}}^{\xi_{up}} \frac{\mathrm{d}\xi}{(2\pi)^3}$$
$$\times \frac{1}{\xi^2(1-\xi)} \int_{P_\perp^{min}}^{P_\perp^{max}} \frac{\mathrm{d}P_\perp}{\sin\theta} \left(P^+ + q^+ - h^+\right) w^{\mu\nu}_{\sigma\sigma'}\left(\tau_{hf}, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}}\right) \mathcal{P}^{\tau_{hf}}_{\sigma'\sigma}(\mathbf{k}, \epsilon_S, S_{\rm He}).$$

All the formal steps for obtaining eq. (3) and the explicit expression of the integration limits will be reported elsewhere [24]. Let us describe here only the two crucial terms appearing in the above equation. The first one is $w^{\mu\nu}_{\sigma\sigma'}(\tau_{hf}, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}})$, the SiDIS nucleonic tensor, that depends not only on its spins σ, σ' and its LF-momentum, $\tilde{\mathbf{P}}$, but also on the isospin τ_{hf} and LF-momentum $\tilde{\mathbf{h}}$ of the produced pseudo-scalar meson. The second one is $\mathcal{P}^{\tau_{hf}}_{\sigma'\sigma}(\mathbf{k}, \epsilon_S, S_{\rm He})$, *i.e.*, the LF spin-dependent SF, describing a nucleon with Cartesian momentum $\mathbf{k}$ inside a ^{3}He with polarization $S_{\rm He}$, when the spectator pair has an excitation energy ϵ_S. The LF spectral function is defined as

$$(4)\quad \mathcal{P}^{\tau}_{\sigma'\sigma}\left(\tilde{\mathbf{k}}, \epsilon_S, S_{\rm He}\right) \propto \sum_{\sigma_1 \sigma_1'} D^{\frac{1}{2}}\left[\mathcal{R}^\dagger_M(\tilde{\mathbf{k}})\right]_{\sigma'\sigma_1'} \mathcal{S}^{\tau}_{\sigma_1'\sigma_1}\left(\tilde{\mathbf{k}}, \epsilon_S, S_{\rm He}\right) D^{\frac{1}{2}}\left[\mathcal{R}_M(\tilde{\mathbf{k}})\right]_{\sigma_1\sigma},$$

with $D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})] = m + k^+ - \imath\boldsymbol{\sigma}\cdot(\hat{z}\times\mathbf{k}_\perp)/\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}$ the unitary Melosh Rotations and the instant-form SF given by

$$(5)\quad \mathcal{S}^{\tau}_{\sigma_1'\sigma_1}(\tilde{\mathbf{k}}, \epsilon_S, S_{\rm He}) = \sum_{J_S J_{zS} \alpha} \sum_{T_S \tau_S} \langle T_S, \tau_S, \alpha, \epsilon_S J_S J_{zS}; \sigma_1'; \tau, \mathbf{k}|\Psi_0 S_{\rm He}\rangle$$
$$\times \langle S_{\rm He}, \Psi_0|\mathbf{k}\sigma_1\tau; J_S J_{zS}\epsilon_S, \alpha, T_S, \tau_S\rangle = \left[B^\tau_{0,S_{\rm He}}(|\mathbf{k}|, E) + \boldsymbol{\sigma}\cdot\mathbf{f}^\tau_{S_{\rm He}}(\mathbf{k}, E)\right]_{\sigma_1'\sigma_1},$$

where $\mathbf{f}^\tau_{S_{\rm He}}(\mathbf{k}, E) = \mathbf{S}_A B^\tau_{1,S_{\rm He}}(|\mathbf{k}|, E) + \hat{k}(\hat{k}\cdot\mathbf{S}_A) B^\tau_{2,S_{\rm He}}(|\mathbf{k}|, E)$. It is tempting to approximate the instant-form SF by the one evaluated in a non relativistic framework, since the BT construction imposes the same constraints adopted for the non relativistic Hamiltonian [17].

From eqs. (4) and (5), it follows that, remarkably, the constituent SF for a $J = 1/2$ system is given in terms of only *three* independent functions (B_i, with $i = 0, 1, 2$), within a RHD framework.

From the phenomenological side, it will be very important to use the LF nuclear tensor, eq. (3), to evaluate the SSAs and to figure out whether or not the proposed extraction procedure still holds in the LF analysis. The preliminary calculations clearly indicate that the effect of considering the integration limits evaluated in the actual JLab kinematics at 6 GeV, and not in the Bjorken limit, as previously done, is negligible. From this point of view, the situation will become even better in the experiments planned at 12 GeV.

4. – The Light-Front quark-quark correlator

In general, the six leading-twist TMDs for a $J = 1/2$ system with 4-momentum P and polarization S are introduced as a proper parametrization of the so called quark-quark correlator, $\Phi(k, P, S)$ for a quark of 4-momentum k, in order to fulfill all the general symmetries (see, *e.g.*, ref. [1]). In particular, one can starts with the following combinations of Dirac structures and scalar functions, A_i and $\tilde{A}_i$:

$$\Phi(k,P,S) = \frac{1}{2}\Big\{ A_1 \not{P} + A_2 S_L \gamma_5 \not{P} + A_3 \not{P}\gamma_5 \not{S}_\perp + \frac{1}{M}\widetilde{A}_1 \vec{k}_\perp \cdot \vec{S}_\perp \gamma_5 \not{P} + \widetilde{A}_1 \frac{S_L}{M}\not{P}\gamma_5 \not{k}_\perp + \frac{1}{M^2}\widetilde{A}_1 \vec{k}_\perp \cdot \vec{S}_\perp \not{P}\gamma_5 \not{k}_\perp \Big\}. \tag{6}$$

Then, the six leading-twist TMDs, f_1, g_{1L}, g_{1T}, h_1, h_{1L}, h_{1T}, are identified as follows:

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi) = f_1, \tag{7a}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi) = S_L g_{1L} + \frac{1}{M}\vec{k}_\perp \cdot \vec{S}_\perp g_{1T}, \tag{7b}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(i\sigma^{i+}\gamma_5\Phi) = S^i_\perp h_1 + \frac{S_L}{M} k^i_\perp h^\perp_{1L} - \frac{1}{M^2}\left(k^i_\perp k^j_\perp + \frac{1}{2}k^2_\perp g^{ij}_\perp\right) S_{\perp,j} h^\perp_{1T}. \tag{7c}$$

The LF SF introduced in the previous section for obtaining a Poincaré covariant description of the nucleon inside a polarized ^{3}He, can be formally adopted for describing a quark inside a polarized nucleon. Namely, *ceteris paribus*, the LF nucleon spectral function, $\mathcal{P}_N$, is the analogous of $\Phi(k, P, S)$, within a Poincaré covariant description of the nucleon. Then, one is able to perform the following identification:

$$\frac{1}{2}\,\mathrm{Tr}(\mathcal{P}_N I) = cB_0 = f_1^{LF}, \tag{8a}$$

$$\begin{aligned}\frac{1}{2}\,\mathrm{Tr}(\mathcal{P}_N \sigma_z) &= S_z\left[a(B_1 + B_2\cos^2\theta) + b\cos\theta\frac{|\mathbf{k}_\perp|^2}{k}B_2\right] \\ &\quad + \mathbf{S}_\perp\cdot\mathbf{k}_\perp\left[aB_2\frac{\cos\theta}{k} + b(B_1 + B_2\sin^2\theta)\right] \\ &= S_L g_{1L}^{LF} + \frac{1}{M}\mathbf{S}_\perp\cdot\mathbf{k}_\perp g_{1T}^{LF},\end{aligned} \tag{8b}$$

$$\begin{aligned}\frac{1}{2}\,\mathrm{Tr}(\mathcal{P}_N \sigma_y) &= S_y\left[\left(a + \frac{d}{2}|\mathbf{k}_\perp|^2\right)B_1 + \frac{1}{2}\left(a - b\frac{k^2_\perp\cos\theta}{k}\right)B_2\right] \\ &\quad + S_z k_y\left[a\frac{\cos\theta}{k}B_2 - b(B_1 + B_2\cos^2\theta)\right] \\ &\quad + \left(k_x k_y S_x - \frac{1}{2}k^2_\perp S_y\right)\left[\left(\frac{a}{k^2} - b\frac{\cos\theta}{k}\right)B_2 - dB_1\right] \\ &= S_y h_1^{LF} + \frac{S_L}{M}k_y h_{1L}^{\perp LF} + \frac{1}{M^2}\left(k_x k_y S_x - \frac{1}{2}k^2_\perp S_y\right)h_{1T}^{\perp LF}.\end{aligned} \tag{8c}$$

Therefore, the *six* TMDs depend actually upon *three* independent functions, within a LFHD framework. The quantities a, b, c and d, appearing in eqs. (8a)-(8c), are *kinematical* factors, *predicted* by the LF procedure (details will be given elsewhere [29]).

5. – Conclusions

A realistic study of $^3\vec{\mathrm{He}}(\mathrm{e},\mathrm{e}'\pi)\mathrm{X}$ in the Bjorken limit has been summarized, and the preliminary results of its generalization within a Light-Front analysis at finite Q^2 have been shortly discussed. The spin-dependent LF spectral function for a $J = 1/2$ system has been presented and, using the BT construction of the Poincaré generators, an intriguing simplification in the theoretical description of SiDIS is found. Three exact relations are established between the six T-even TMDs. As a future step, it will be investigated if similar relations can be found in other theoretical frameworks and eventually in the analysis of the experimental results.

A detailed analysis of these new relations will be given elsewhere [29].

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