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Probing the multi-gluon correlations through single-spin asymmetries

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Summary. — We study the single-spin asymmetries for the D-meson production, Drell-Yan lepton-pair production and the direct-photon production in the pp collision induced by the twist-3 three-gluon correlation functions in the transversely polarized nucleon. We present a corresponding polarized cross section formula in the leading-order with respect to the QCD coupling and a model calculation for the asymmetries, illustrating the sensitivity to the form of the correlation functions.

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1. – Introduction

Understanding the origin of the large single spin asymmetries (SSAs) observed in various high-energy semi-inclusive processes have been a big challenge during the past decades. The SSA can be generated as a consequence of the multiparton correlations inside the hadrons in the collinear factorization approach which is valid when the transverse momentum of the particle in the final state can be regarded as hard. Among such multiparton correlations, purely gluonic correlations have a potential importance, since gluons are ample in the nucleon. The best way to probe such gluonic correlations is the measurement of SSA for a heavy meson production in semi-inclusive deep inelastic scattering (SIDIS) and the pp collision [1, 2], since the heavy-quark pair one of which fragments into the final-state meson is mainly produced, respectively, by the photon-gluon and gluon-gluon fusion mechanisms. The measurement of SSA for the D-meson production in the pp collision is ongoing at RHIC [3].

In this report, we study the contribution of the three-gluon correlation functions representing such multigluonic effects to SSA in the pp collision for the D-meson production ($p^\uparrow p \to DX$) [4], Drell-Yan lepton-pair production ($p^\uparrow p \to \gamma^* X$) and the direct-photon production ($p^\uparrow p \to \gamma X$) [5]. We will present the corresponding single-spin dependent cross sections by applying the formalism developed for SIDIS, $ep^\uparrow \to eDX$ [2]. We will also present a model calculation of the SSAs for $p^\uparrow p \to DX$ and $p^\uparrow p \to \gamma X$ induced by the three-gluon correlation functions in comparison with the RHIC preliminary data for the former [3].

2. – Three-gluon correlation functions

Three-gluon correlation functions in the transversely polarized nucleon are defined as the color-singlet nucleon matrix element composed of the three gluon's field strength tensors $F^{\alpha\beta}$. Corresponding to the two structure constants for the color $SU(3)$ group, d_{bca} and f_{bca}, one obtains two independent three-gluon correlation functions $O(x_1, x_2)$ and $N(x_1, x_2)$ as [2]

$$
\begin{aligned}
(1)\qquad O^{\alpha\beta\gamma}(x_1,x_2) &= -g(i)^3 \int \frac{\mathrm{d}\lambda}{2\pi} \int \frac{\mathrm{d}\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \\
&\quad \times \langle pS| d_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) |pS \\
&= 2iM_N \left[O(x_1,x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} + O(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} \right. \\
&\quad \left. + O(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \right],
\end{aligned}
$$

$$
\begin{aligned}
(2)\qquad N^{\alpha\beta\gamma}(x_1,x_2) &= -g(i)^3 \int \frac{\mathrm{d}\lambda}{2\pi} \int \frac{\mathrm{d}\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \\
&\quad \times \langle pS| i f_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) |pS \\
&= 2iM_N \left[N(x_1,x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} - N(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} \right. \\
&\quad \left. - N(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \right],
\end{aligned}
$$

where S is the transverse-spin vector for the nucleon, n is the light-like vector satisfying $p \cdot n = 1$, and we have used the shorthand notation as $F^{\beta n} \equiv F^{\beta\rho} n_\rho$ etc. The gauge-link operators which restore gauge invariance of the correlation functions are suppressed in (1) and (2) for simplicity. The nucleon mass M_N is introduced to define $O(x_1, x_2)$ and $N(x_1, x_2)$ dimensionless.

3. – D-meson production in pp collision

Applying the formalism for the contribution of the three-gluon correlation functions to SSA developed in [2], the twist-3 cross section for $p^\uparrow(p, S_\perp) + p(p') \to D(P_h) + X$ with the center-of-mass energy $\sqrt{S}$ can be obtained in the following form [4]:

$$
\begin{aligned}
(3)\quad P_h^0 \frac{\mathrm{d}\sigma^{\mathrm{tw3},D}}{\mathrm{d}^3 P_h} &= \frac{\alpha_s^2 M_N \pi}{S} \epsilon^{P_h p n S_\perp} \sum_{f=c\bar{c}} \int \frac{\mathrm{d}x'}{x'} G(x') \\
&\times \int \frac{\mathrm{d}z}{z^2} D_f(z) \int \frac{\mathrm{d}x}{x} \delta\left(\tilde{s} + \tilde{t} + \tilde{u}\right) \frac{1}{z\tilde{u}} \left[\delta_f \left\{ \left(\frac{\mathrm{d}}{\mathrm{d}x} O(x,x) - \frac{2O(x,x)}{x} \right) \right. \right. \\
&\times \left. \hat{\sigma}^{O1} + \left(\frac{\mathrm{d}}{\mathrm{d}x} O(x,0) - \frac{2O(x,0)}{x} \right) \hat{\sigma}^{O2} + \frac{O(x,x)}{x} \hat{\sigma}^{O3} + \frac{O(x,0)}{x} \hat{\sigma}^{O4} \right\} \\
&+ \left\{ \left(\frac{\mathrm{d}}{\mathrm{d}x} N(x,x) - \frac{2N(x,x)}{x} \right) \hat{\sigma}^{N1} + \left(\frac{\mathrm{d}}{\mathrm{d}x} N(x,0) - \frac{2N(x,0)}{x} \right) \hat{\sigma}^{N2} \right. \\
&+ \left. \left. \frac{N(x,x)}{x} \hat{\sigma}^{N3} + \frac{N(x,0)}{x} \hat{\sigma}^{N4} \right\} \right],
\end{aligned}
$$

where $\delta_c = 1$ and $\delta_{\bar{c}} = -1$, $D_f(z)$ represents the $c \to D$ or $\bar{c} \to \bar{D}$ fragmentation functions, $G(x')$ is the unpolarized gluon density, p_c is the four-momentum of the c (or

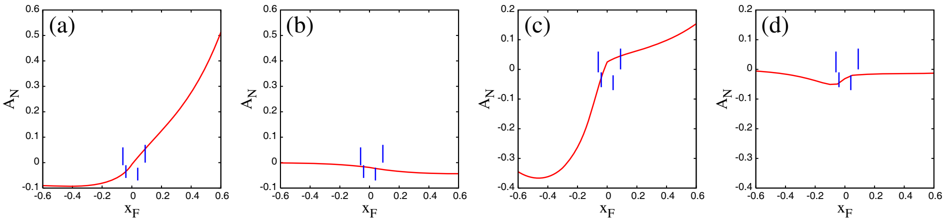

Fig. 1. – (a) A_N^D for D^0 and (b) A_N for $\bar{D}^0$ for Model 1 in (4) with $O(x) = N(x)$ and $K_G = 0.004$, and (c) A_N^D for D^0 and (c) A_N for $\bar{D}^0$ for Model 2 in (5) with $O(x) = N(x)$ and $K'_G = 0.001$. Short bars denote the RHIC preliminary data taken from [3].

$\bar{c}$) quark (mass m_c) fragmenting into the final D (or $\bar{D}$) meson and $\tilde{s}$, $\tilde{t}$, $\tilde{u}$ are defined as $\tilde{s} = (xp + x'p')^2$, $\tilde{t} = (xp - p_c)^2 - m_c^2$, $\tilde{u} = (x'p' - p_c)^2 - m_c^2$. The hard cross sections $\hat{\sigma}^{O1,O2,O3,O4}$ and $\hat{\sigma}^{N1,N2,N3,N4}$ are listed in [4]. The cross section (3) receives contributions from $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$ separately, which differs from the previous result [1].

We perform numerical estimate for A_N based on (3). For the RHIC kinematics, we found that the terms with $\hat{\sigma}^{O3,O4,N3,N4}$ are negligible compared with those with $\hat{\sigma}^{O1,O2,N1,N2}$ and that $\hat{\sigma}^{O1} \simeq \hat{\sigma}^{O2} \sim \hat{\sigma}^{N1} \simeq -\hat{\sigma}^{N2}$. One can thus regard the cross section as a function of the correlation functions $O(x) \equiv O(x,x) + O(x,0)$ and $N(x) \equiv N(x,x) - N(x,0)$ to a very good approximation. We assume the relation $O(x) = \pm N(x)$ together with $O(x,x) = O(x,0)$ and $N(x,x) = -N(x,0)$ for simplicity. For the functional form of each function, we employ the following two models:

$$\text{Model 1}: \quad O(x) = K_G\, x\, G(x), \tag{4}$$

$$\text{Model 2}: \quad O(x) = K'_G\, \sqrt{x}\, G(x), \tag{5}$$

where K_G and K'_G are the constants to be determined so that the calculated asymmetry is consistent with the RHIC data [3].

For the numerical calculation, we use GJR08 [6] for $G(x)$ and KKKS08 [7] for $D_f(z)$. We calculate A_N for the D and $\bar{D}$ mesons at the RHIC energy at $\sqrt{S} = 200\,\text{GeV}$ and the transverse momentum of the D-meson $P_T = 2\,\text{GeV}$. We set the scale of all the distribution and fragmentation functions at $\mu = \sqrt{P_T^2 + m_c^2}$ with the charm quark mass $m_c = 1.3\,\text{GeV}$.

Figure 1 shows the result of A_N for the D^0 and $\bar{D}^0$ mesons with the relation $O(x) = N(x)$ together with the preliminary data [3] denoted by the short bars. The sign of the contribution from $O(x)$ changes between D^0 and $\bar{D}^0$ as shown in (3) and works constructively (destructively) for D^0 ($\bar{D}^0$) for the case $O(x) = N(x)$. The values $K_G = 0.004$ and $K'_G = 0.001$ have been determined so that A_N does not overshoot the RHIC data. If one adopts the relation $O(x) = -N(x)$, the result for the D^0 and $\bar{D}^0$ mesons will be interchanged. The rising behavior of A_N at large x_F as shown in figs. 1(a) and (c) is originated from the derivative of $O(x)$ and $N(x)$, as in the case of the soft-gluon-pole (SGP) contribution for the quark-gluon correlation function. By comparing the results for the models 1 and 2 in figs. 1(a) and (c), one sees that the behavior of A_N at $x_F < 0$ depends strongly on the small-x behavior of the three-gluon correlation functions. Therefore A_N at $x_F < 0$ is useful to get constraint on the small-x behavior of the three-gluon correlation functions.

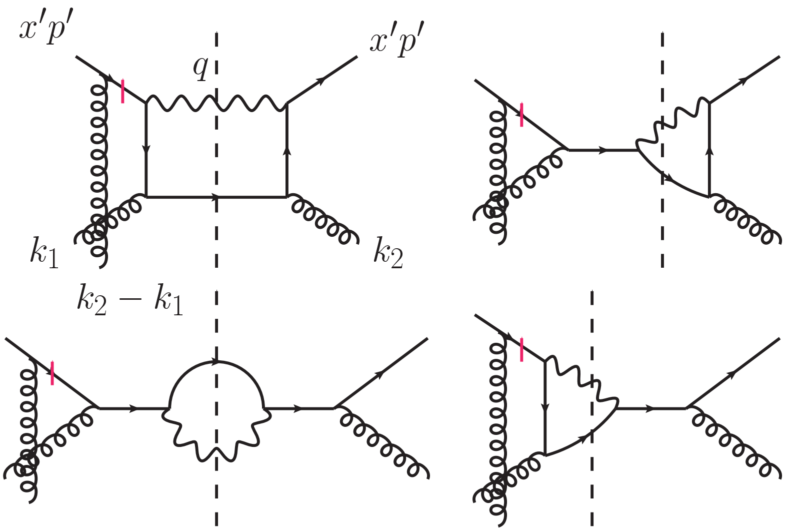

Fig. 2. – The diagrams for the three-gluon contribution to the twist-3 cross section for $p^{\uparrow}p \to \gamma^* X$. In the collinear limit, the momenta k_1 and k_2 coming from the polarized nucleon are set to $k_i = x_i p$ $(i = 1, 2)$ and the pole contribution from the bared propagator gives rise to the SSA.

4. – Drell-Yan lepton-pair production in pp collision

The analysis of the previous section can be extended to A_N for the Drell-Yan lepton-pair production induced by the three-gluon correlation function. The corresponding twist-3 diagrams for the hard part are shown in fig. 2, which give rise to SGP contribution at $x_1 = x_2$ due to the initial-state interaction between the extra incoherent gluon from the polarized proton and the quark coming out of the unpolarized proton. From these diagrams, one obtains for the single-spin-dependent cross section for the Drell-Yan process, $p^{\uparrow}(p, S_\perp) + p(p') \to \gamma^*(q) + X$, with the invariant mass $Q^2 = q^2$ for the lepton-pair as [5]

$$
(6) \quad \frac{\mathrm{d}\sigma^{\mathrm{tw3,DY}}}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}^2\vec{q}_\perp} = \frac{2M_N\alpha_{em}^2\alpha_s}{3SQ^2}\int\frac{\mathrm{d}x}{x}\int\frac{\mathrm{d}x'}{x'}\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)\epsilon^{qpnS_\perp}\frac{1}{\hat{u}}\sum_a e_a^2 f_a(x')
$$
$$
\times\Bigg[\delta_a\left\{\left(\frac{\mathrm{d}}{\mathrm{d}x}O(x,x)-\frac{2O(x,x)}{x}\right)\hat{\sigma}_1+\left(\frac{\mathrm{d}}{\mathrm{d}x}O(x,0)-\frac{2O(x,0)}{x}\right)\hat{\sigma}_2\right.
$$
$$
\left.+\frac{O(x,x)}{x}\hat{\sigma}_3+\frac{O(x,0)}{x}\hat{\sigma}_4\right\}
$$
$$
-\left(\frac{\mathrm{d}}{\mathrm{d}x}N(x,x)-\frac{2N(x,x)}{x}\right)\hat{\sigma}_1+\left(\frac{\mathrm{d}}{\mathrm{d}x}N(x,0)-\frac{2N(x,0)}{x}\right)\hat{\sigma}_2
$$
$$
-\frac{N(x,x)}{x}\hat{\sigma}_3+\frac{N(x,0)}{x}\hat{\sigma}_4\Bigg],
$$

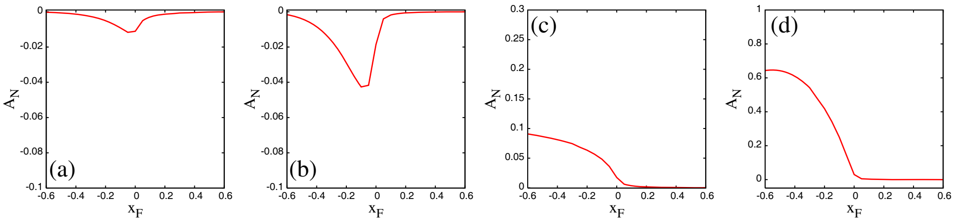

Fig. 3. – (a) A_N for Case 1 with Model 1. (b) A_N for Case 1 with Model 2. (c) A_N for Case 2 with Model 1. (d) A_N for Case 2 with Model 2.

where y is the rapidity of the virtual photon, $\alpha_{em} \simeq 1/137$ is the QED coupling constant, and the partonic hard cross sections are defined as

$$(7) \qquad \hat{\sigma}_1 = \frac{2}{N}\left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} + \frac{2Q^2\hat{t}}{\hat{s}\hat{u}}\right), \qquad \hat{\sigma}_2 = \frac{2}{N}\left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} + \frac{4Q^2\hat{t}}{\hat{s}\hat{u}}\right),$$
$$\hat{\sigma}_3 = -\frac{1}{N}\frac{4Q^2(Q^2+\hat{t}\,)}{\hat{s}\hat{u}}, \qquad \hat{\sigma}_4 = -\frac{1}{N}\frac{4Q^2(3Q^2+\hat{t}\,)}{\hat{s}\hat{u}},$$

with the number of colors $N = 3$ and $\hat{s} = (xp + x'p')^2$, $\hat{t} = (xp - q)^2$ and $\hat{u} = (x'p' - q)^2$. For a large Q^2, $\hat{\sigma}_1$ differs from $\hat{\sigma}_2$ significantly, and $\hat{\sigma}_{3,4}$ are not negligible. Therefore the cross section (6), in general, depends on the four functions $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$ independently unlike the case for $p^{\uparrow}p \to DX$ in the previous section, where the twist-3 cross section can be regarded as a function of the combination $O(x,x) + O(x,0)$ and $N(x,x) - N(x,0)$.

The A_N for the Drell-Yan process receives contribution not only from the three-gluon correlation functions but also from the quark-gluon correlation functions, for which the twist-3 cross section have been derived in [8-10]. The sum of (6) and those from the quark-gluon correlation functions gives the complete leading-order cross section for the asymmetry.

5. – Direct photon production in pp collision

Taking the $q \to 0$ limit of (6), the twist-3 cross section for the direct photon production, $p^{\uparrow}(p, S_\perp) + p(p') \to \gamma(q) + X$, induced by the three-gluon correlation functions can be obtained as [5]

$$(8) \quad E_\gamma \frac{\mathrm{d}\sigma^{\mathrm{tw3,DP}}}{\mathrm{d}^3q} = \frac{4\alpha_{em}\alpha_s M_N \pi}{S}\sum_a e_a^2 \int \frac{\mathrm{d}x'}{x'} f_a(x') \int \frac{\mathrm{d}x}{x}\delta(\hat{s}+\hat{t}+\hat{u})\epsilon^{qpnS_\perp}$$
$$\times\left[\delta_a\left(\frac{\mathrm{d}}{\mathrm{d}x}O(x,x) - \frac{2O(x,x)}{x} + \frac{\mathrm{d}}{\mathrm{d}x}O(x,0) - \frac{2O(x,0)}{x}\right)\right.$$
$$\left. -\frac{\mathrm{d}}{\mathrm{d}x}N(x,x) + \frac{2N(x,x)}{x} + \frac{\mathrm{d}}{\mathrm{d}x}N(x,0) - \frac{2N(x,0)}{x}\right]\left(\frac{1}{N}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}^2}\right),$$

where $f_a(x')$ is the twist-2 unpolarized quark density and $\delta_a = 1(-1)$ for quark (anti-quark). As shown in (8), the combinations $O(x) \equiv O(x,x) + O(x,0)$ and $N(x) \equiv N(x,x) - N(x,0)$ appear in the cross section accompanying the common partonic hard cross section which is the same as the twist-2 hard cross section for the $qg \to q\gamma$ scattering. The origin of this simplification can be clearly understood in terms of the master formula for the contribution from the three-gluon correlation functions developed in [4,5,11]. We also note that the above result (8) differs from the previous study in [12].

We have performed a numerical calculation for A_N^γ based on the models used for $p^\uparrow p \to DX$ in sect. **3**. For the models 1 and 2, we calculate the asymmetry A_N^γ for the two cases: Case 1; $O(x) = N(x)$ and Case 2; $O(x) = -N(x)$. We use GJR08 [6] for $f_q(x')$ and the models (4) and (5) with $K_G = 0.004$ and $K'_G = 0.001$ which are consistent with the RHIC A_N^D data. We calculate A_N^γ at the RHIC energy at $\sqrt{S} = 200\,\text{GeV}$ and the transverse momentum of the photon $q_T = 2\,\text{GeV}$, setting the scale of all the distribution functions at $\mu = q_T$.

Figure 3 shows the result for A_N^γ for each case. One can see A_N at $x_F > 0$ becomes almost zero regardless of the magnitude of the three-gluon correlation functions, while A_N^γ at $x_F < 0$ depends strongly on the small-x behavior of the three-gluon correlation functions as in the case of $p^\uparrow p \to DX$. Even though the derivatives of $O(x)$ and $N(x)$ contribute, A_N is tiny at $x_F > 0$ due to the small partonic cross section. At $x_F < 0$, large-x' region of the unpolarized quark distributions and the small-x region of the three-gluon distributions are relevant. For the above case 1, only antiquarks in the unpolarized nucleon are active and thus lead to small A_N^γ as shown in figs. 3(a) and (b). On the other hand, for the case 2, quarks in the unpolarized nucleon are active and thus lead to large A_N^γ as shown in figs. 3(c) and (d). Therefore A_N^γ at $x_F < 0$ for the direct photon production could provides us with an important information on the relative sign between $O(x)$ and $N(x)$.

To summarize, we have studied the SSA for $p^\uparrow p \to DX$, $p^\uparrow p \to \gamma^* X$ and $p^\uparrow p \to \gamma X$ induced by the three-gluon correlation functions in the polarized nucleon. Combined with the known result for the contribution from the quark-gluon correlations, this complete the leading-order twist-3 cross sections for these processes. We have also presented a model calculation for the asymmetry for the $p^\uparrow p \to DX$ and $p^\uparrow p \to \gamma X$ at the RHIC energy, showing the sensitivity of the asymmetry to the form of the three gluon-correlation functions.

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