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Process dependence and spin asymmetries in hadronic reactions

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Summary. — We study transverse-spin asymmetries in single inclusive particle production hadronic scattering in terms of the generalized parton model (GPM).

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PACS 13.85.Ni – Inclusive production with identified hadrons.
PACS 12.38.Bx – Perturbative calculations.

1. – Introduction

Single transverse-spin asymmetries (SSAs) in high energy lepton-hadron and hadronic scattering processes have received considerable attention from the experimental and theoretical communities [1]. Generally, they are defined as the ratio of the difference and the sum of the cross sections when the hadron's spin vector $S_\perp$ is flipped, $A_N \equiv (\sigma(S_\perp) - \sigma(-S_\perp))/(\sigma(S_\perp) + \sigma(-S_\perp)) \equiv \Delta\sigma/(2\sigma^{\rm unp})$. The SSAs for single inclusive particle production in proton-proton scattering are among the earliest processes studied [2] and remain extremely challenging to explain in the context of perturbative quantum chromodynamics (QCD) [3]. The trend of large SSAs in the pioneering fixed target experiments has been observed over a wide range of energies, and in the proton-proton collision experiments at Relativistic Heavy Ion Collider (RHIC) [4, 5]. Also, azimuthal and transverse-spin asymmetries have been observed in Drell-Yan (DY) processes [6], in semi-inclusive deep inelastic scattering (SIDIS) [7, 8] and in hadron pair production in $e^+ e^-$ scattering [9].

From a theoretical perspective SSAs are characterized by the interference between helicity flip and non-flip scattering amplitudes with a relative color phase. Two approaches have been proposed in the framework of perturbative QCD to account for these effects. One is the collinear factorization formalism at next-to-leading-power (twist-3) in the hard scale where SSAs are given by a convolution of universal non-perturbative quark-gluon-quark correlation functions and hard scattering amplitudes [10-12]. The other framework relies on factorization in terms of a hard scattering cross section and transverse-momentum dependent (TMD) parton distribution and fragmentation functions (PDFs and FFs). Prominent examples are the quark Sivers function [13], which represents the azimuthal distribution of unpolarized quarks in a transversely polarized nucleon and the Collins fragmentation function [14], which describes the production of pseudo-scalar mesons (or unpolarized hadrons) from transversely polarized fragmenting quarks. In this approach color phases are given by initial and/or final state interactions (ISIs/FSIs) between the active quark and spectator remnants in the full scattering amplitude. The details of the ISIs and FSIs depend on the scattering process and for PDFs such as the Sivers function, these color phases are incorporated into the Wilson lines of the gauge invariant definition of TMD PDFs. It is a fundamental prediction of QCD factorization that the form of the gauge link depends on the hard sub-process [15] indicating that the Sivers function is *non-universal* [16]. The oft-discussed case is the difference between the FSIs in SIDIS and the ISIs in DY scattering which leads to the prediction of an opposite relative color factor [16,17]. Further, applying similar reasoning to hadron production in proton-proton collisions, typically the Sivers function has a more complicated color factor structure since both ISIs and FSIs contribute [15,18,19].

While TMD factorization has not been established for single hadron production in hadronic reactions [20], an extensive program of phenomenology has been carried out by including the correlations of intrinsic parton motion and transverse spin in the context of the so-called generalized parton model (GPM). Introduced [21] as a generalization of the collinear perturbative QCD approach, it has been used to describe the SSAs for inclusive particle production [22] where factorization has been assumed as a reasonable starting point for analyses. At the same time, the leading-twist naive time-reversal odd (T-odd) TMD PDFs have conditionally been assumed to be *universal*.

Recently, we have presented [19,23] an analysis of SSAs in proton-proton scattering taking into account the effects of ISIs and FSIs, that is allowing for process dependence within the framework of GPM thereby determining the process-dependent Sivers function. Further we find one can shift the process-dependence of the Sivers function to the squared hard partonic scattering amplitude under one-gluon exchange approximation, where these modified hard parts are very similar in form as those in the twist-3 collinear approach [12] in terms of Mandelstam variables $\hat{s}, \hat{t}, \hat{u}$. This suggests a close connection between this modified GPM formalism and the twist-3 approach [19]. Here we summarize the results of these analyses.

2. – The generalized Parton Model and process dependence

The GPM was introduced by Feynman and collaborators [21] as a generalization of the usual collinear pQCD approach. It was adapted and used to describe the SSAs for inclusive particle production [22,24,25], which has had considerable phenomenological success [24]. According to this approach, for the inclusive production of large P_{hT} hadrons (or photons), $A^{\uparrow}(P_A) + B(P_B) \to h(P_h) + X$, the differential cross section is written as

$$E_h \frac{\mathrm{d}\sigma}{\mathrm{d}^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{\mathrm{d}x_a}{x_a} \mathrm{d}^2 k_{aT} f_{a/A^{\uparrow}}(x_a, \vec{k}_{aT}) \int \frac{\mathrm{d}x_b}{x_b} \mathrm{d}^2 k_{bT} f_{b/B}(x_b, k_{bT}^2) \cdot \int \frac{\mathrm{d}z_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \tag{1}$$

where $S = (P_A + P_B)^2$, $f_{a/A^{\uparrow}}(x_a, \vec{k}_{aT})$ is the TMD parton distribution functions with k_{aT} the intrinsic transverse momentum of parton a with respect to the light-cone direction of hadron A, and $D_{h/c}(z_c)$ is the fragmentation function. Since we will only consider the SSAs generated from the parton distribution functions in this analysis, we neglect the k_T-dependence in the fragmentation function. $H^U_{ab\to c}(\hat{s},\hat{t},\hat{u})$ is the hard part coefficients with $\hat{s},\hat{t},\hat{u}$ the usual partonic Mandelstam variables. Equation (1) can also be used to describe direct photon production, in which one replaces the fragmentation function $D_{h/c}(z_c)$ by $\delta(z_c-1)$, and α_s^2 by $\alpha_{em}\alpha_s$.

Further specifying the kinematics, consider the center-of-mass frame of the two initial hadrons, in which one has $P_A^\mu = \sqrt{S/2}\,\bar{n}^\mu$ and $P_B^\mu = \sqrt{S/2}\,n^\mu$, with $\bar{n}^\mu = [1^+, 0^-, 0_\perp]$ and $n^\mu = [0^+, 1^-, 0_\perp]$ in light-cone components. We also include the definitions of the hadronic Mandelstam invariants, $T = (P_A - P_h)^2$ and $U = (P_B - P_h)^2$. The momenta of the partons in the partonic process $a(p_a) + b(p_b) \to c(p_c) + d(p_d)$ can be written as

$$p_a^\mu = \left[x_a\sqrt{\frac{S}{2}},\, \frac{k_{aT}^2}{x_a\sqrt{2S}},\, \vec{k}_{aT}\right], \qquad p_b^\mu = \left[\frac{k_{bT}^2}{x_b\sqrt{2S}},\, x_b\sqrt{\frac{S}{2}},\, \vec{k}_{bT}\right], \tag{2}$$

where the momentum of parton c is related to the final hadron as: $p_c = P_h/z_c$.

To study the SSAs, the PDFs $f_{a/A^\uparrow}(x_a,\vec{k}_{aT})$ in the transversely polarized hadron A can be expanded as [22, 24, 26]

$$f_{a/A^\uparrow}(x_a,\vec{k}_{aT}) = f_{a/A}(x_a,k_{aT}^2) + f_{1T}^{\perp a}(x_a,k_{aT}^2)\frac{\epsilon^{k_{aT}S_A n\bar{n}}}{M}, \tag{3}$$

where S_A is the transverse polarization vector, M is the mass of hadron A, $f_{a/A}(x_a,k_{aT}^2)$ is the spin-averaged PDFs, and $f_{1T}^{\perp a}(x_a,k_{aT}^2)$ is the Sivers functions. In the GPM, the spin averaged differential cross section is given by eq. (1) with $f_{a/A^\uparrow}(x_a,\vec{k}_{aT})$ replaced with $f_{a/A}(x_a,k_{aT}^2)$.

There are two assumptions in the GPM approach: one is that the spin-averaged and spin-dependent differential cross sections can be factorized in terms of TMD PDFs as in eqs. (1) and (4), and the other one is that the Sivers functions are assumed to be universal and equal to those in SIDIS process, $f_{1T}^{\perp a}(x_a,k_{aT}^2) = f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a,k_{aT}^2)$. We adopt the framework of the GPM approach, assuming that TMD factorization is a reasonable phenomenological starting point. However, we also take into account the initial- and final-state interactions. Since both ISIs and FSIs contribute for single inclusive particle production in hadronic collisions, in principle the Sivers functions should be different from those probed in SIDIS and DY. We account for this process dependence by calculating the contributions of coming from the ISIs and FSIs for all the partonic scattering processes relevant to single inclusive particle production to determine the proper Sivers functions to be used in the formalism. To signify this process dependence in the GPM, the spin-dependent cross section is generalized to [19],

$$\begin{aligned} E_h\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}&\int\frac{\mathrm{d}x_a}{x_a}\mathrm{d}^2k_{aT}\,f_{1T}^{\perp a,ab\to cd}(x_a,k_{aT}^2)\frac{\epsilon^{k_{aT}S_A n\bar{n}}}{M}\\ &\cdot\int\frac{\mathrm{d}x_b}{x_b}\mathrm{d}^2k_{bT}\,f_{b/B}(x_b,k_{bT}^2)\\ &\cdot\int\frac{\mathrm{d}z_c}{z_c^2}D_{h/c}(z_c)H^U_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \end{aligned} \tag{4}$$

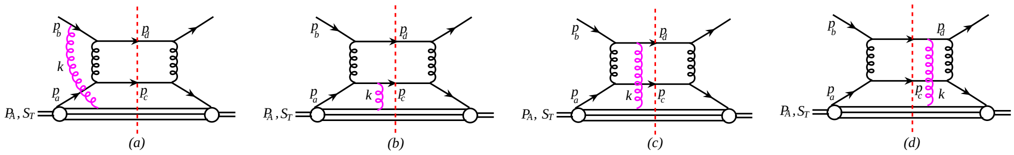

Fig. 1. – Initial- and final-state interactions in $qq' \to qq'$: (a) initial-state interaction, (b) final-state interaction, (c) and (d) the final-state interactions for the unobserved particle.

in which a *process-dependent Sivers function* denoted as $f_{1T}^{\perp a, ab\to cd}(x_a, k_{aT}^2)$ is used rather than that from SIDIS $f_{1T}^{\perp a,\text{SIDIS}}(x_a, k_{aT}^2)$ as in the conventional GPM approach [22].

2$\cdot$1. *Initial and final state interactions.* – Here, we discuss how to formulate the ISIs and FSIs. The crucial point is that the existence of the Sivers function in the polarized nucleon relies on the initial- and final-state interactions between the struck parton and the spectators from the polarized nucleon through the gluon exchange. Analyzing these interactions, one can determine the process dependent Sivers function $f_{1T}^{\perp a, ab\to cd}(x_a, k_{aT}^2)$ to be used in eq. (4) for the corresponding partonic scattering $ab \to cd$.

By way of example we consider the partonic sub-process $qq' \to qq'$. Here the initial-quark q is from the polarized nucleon, and the final-quark q fragments to the final-state hadron. The one-gluon exchange approximation for the initial- and final-state interactions are shown in fig. 1. Under the eikonal approximation, for ISI fig. 1(a),

$$\frac{i(\not{p}_b + \not{k})}{(p_b+k)^2 + i\epsilon}(-ig)\gamma^- T^a u(p_b) = \left[\frac{-g}{-k^+ - i\epsilon}T^a\right] u(p_b). \tag{5}$$

Likewise, for the FSI fig. 1(b), we have

$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c-k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon}T^a\right]. \tag{6}$$

While, both interactions contribute to the phase $-i\pi\delta(k^+)$, which is the same as in the SIDIS process [19] they will have different color flow. To extract the color factors for fig. 1(a) and (b) as compared to the usual $qq' \to qq'$ without gluon attachments, we resort to the method developed in [11, 12, 27, 28]. We obtain the color factors C_I (C_{F_c}) for initial (final)-state interaction $C_I = -1/2N_c^2$ and $C_{F_c} = -1/4N_c^2$ while the color factors for unpolarized cross section is given by $C_u = (N_c^2 - 1)/4N_c^2$. In other words, the Sivers function in $qq' \to qq'$ should be the one as shown in fig. 2, which comes from the sum of the ISIs and FSIs with the corresponding color factors C_I and C_{F_c}, respectively. Note that by comparing the imaginary part of the eikonal propagators for SIDIS and those in eqs. (5) and (6) for ISI and FSI for $qq' \to qq'$, we immediately find the Sivers function probed in $qq' \to qq'$ process is related to those in SIDIS as follows:

$$f_{1T}^{\perp a, qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} f_{1T}^{\perp a,\text{SIDIS}}. \tag{7}$$

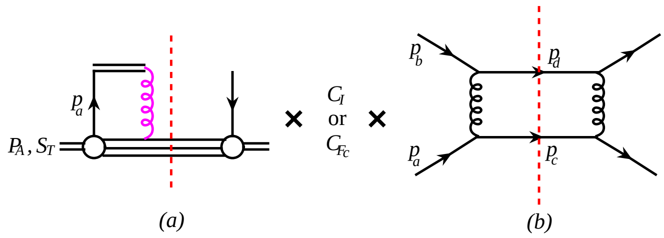

Fig. 2. – Sivers function in $qq' \to qq'$ from ISIs and FSIs, with the corresponding color factors C_I and C_{F_c}, respectively.

Thus in the GPM model, using the process dependent Sivers function, one should replace $f_{1T}^{\perp a,\text{SIDIS}} H^U_{qq'\to qq'} \equiv f_{1T}^{\perp a,\text{SIDIS}}[C_u h_{qq'\to qq'}]$, by the following form:

$$f_{1T}^{\perp a,qq'\to qq'} H^U_{qq'\to qq'} = f_{1T}^{\perp a,\text{SIDIS}} \left[C_I h_{qq'\to qq'} + C_{F_c} h_{qq'\to qq'}\right],$$

where $h_{qq'\to qq'}$ is the partonic cross section without color factors included. For $qq' \to qq'$, one has $h_{qq'\to qq'} = 2(\hat{s}^2 + \hat{u}^2)/\hat{t}^2$.

However, one can take an alternative view of the process dependence by associating the color factors with the square of the hard scattering amplitudes. That is, one can use $f_{1T}^{\perp a,\text{SIDIS}}$ for the single inclusive particle production while accounting for the process-dependence by shifting the color factors to the hard parts. In other words, instead of using $H^U_{qq'\to qq'}$ in eq. (4) for the spin-dependent cross section, can write, $H^{\text{Inc}}_{qq'\to qq'} \equiv H^{\text{Inc-I}}_{qq'\to qq'} + H^{\text{Inc-F}}_{qq'\to qq'}$, where

$$(8) \qquad H^{\text{Inc-I}}_{qq'\to qq'} = C_I h_{qq'\to qq'} \qquad \text{and} \qquad H^{\text{Inc-F}}_{qq'\to qq'} = C_{F_c} h_{qq'\to qq'},$$

are the corresponding hard parts related to initial- and final-state interactions, respectively. There are many other partonic processes contributing to the single inclusive particle production. Similar to the analysis in $qq' \to qq'$, one needs to analyze each individual Feynman diagram accordingly, moving the extra factors (process-dependence) from the corresponding Sivers function to the hard parts, thus obtaining $H^{\text{Inc-I}}_{ab\to cd}$ and $H^{\text{Inc-F}}_{ab\to cd}$ for each channel. The details can be found in ref. [19].

Some comments are in order. It appears that figs. 1(a), (b) can be factorized into a convolution of Sivers function and a hard part function [19]. However, this is not TMD factorization in the strict sense. Currently TMD factorization theorems have been established for both SIDIS and DY processes [29-32]. To the order we are studying, this means, the one-gluon exchange diagrams can be factorized into a convolution of a Sivers function $f_{1T}^{\perp a,\text{SIDIS}}(x_a, k_{aT}^2)$ and a hard part function $H(Q)$, as shown in fig. 2, where all the soft physics (those depending on k_{aT}) is absorbed into the Sivers function $f_{1T}^{\perp a}(x_a, k_{aT}^2)$, and the hard part function $H(Q)$ only depends on the hard scale Q, not k_{aT}. On the other hand, for $qq' \to qq'$, we write the corresponding diagram fig. 1(a) as a product of a Sivers function $f_{1T}^{\perp a,qq'\to qq'}(x_a, k_{aT}^2)$ and a hard part function $H_{qq'\to qq'}(\hat{s}, \hat{t}, \hat{u})$, as shown in fig. 2. Besides the k_{aT} dependence from the Sivers function, one needs to keep the k_{aT} dependence in the hard part functions $H_{qq'\to qq'}$, without which the SSAs will vanish in both the conventional GPM and this modified GPM formalism. While this is not TMD factorization, one surmises this formalism is a reasonable approximation. There are two reasons to suggest this might be the case. First, from phenomenological point of view, this formalism had some success [24]. Secondly, as is summarized in [19] and sect. **3** this formalism has a connection with the well-established collinear twist-3 approach [12]. In this respect, our identification of the color factors with the hard cross sections is reminiscent of the results of the twist-3 approach. Indeed we see that upon calculating all partonic processes that contribute from each channel, they have the same form in terms of Mandelstam variables $\hat{s}$, $\hat{t}$, $\hat{u}$, as compared to those in the twist-3 collinear factorization approach [12] (up to a prefactor associated with final state interactions) [19].

We want to emphasize that the above analysis holds true only under one-gluon exchange approximation. Going beyond one-gluon exchange, the Sivers functions are typically more complicated, there seems no simple relation (as extra color factors) to those in the SIDIS process [20, 33, 34].

3. – Single inclusive hadron production and twist-3 approach

Now taking into account both initial- and final-state interactions, and associating the color factors with the hard scattering cross sections the GPM formalism for spin-dependent cross section is written as

$$E_h \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{\mathrm{d}x_a}{x_a} \mathrm{d}^2 k_{aT} f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a, k_{aT}^2) \frac{\epsilon^{k_{aT} S_A n \bar{n}}}{M} \cdot \int \frac{\mathrm{d}x_b}{x_b} \mathrm{d}^2 k_{bT} f_{b/B}(x_b, k_{bT}^2) \cdot \int \frac{\mathrm{d}z_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \tag{9}$$

where we have a new hard part function $H_{ab\to c}^{\mathrm{Inc}}$ instead of $H_{ab\to c}^{U}$ used in the conventional GPM approach. Here the process dependence in the Sivers function has been absorbed into $H_{ab\to c}^{\mathrm{Inc}}$, which (as stated above) can be written as

$$H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u}) = H_{ab\to c}^{\mathrm{Inc\text{-}I}}(\hat{s}, \hat{t}, \hat{u}) + H_{ab\to c}^{\mathrm{Inc\text{-}F}}(\hat{s}, \hat{t}, \hat{u}). \tag{10}$$

The contributions for the various contributing partonic sub-processes are given in [19].

We also calculate the corresponding hard part functions for direct photon production, and they are given by

$$\begin{aligned} H_{qg\to\gamma q}^{\mathrm{Inc}} &= -H_{\bar{q}g\to\gamma\bar{q}}^{\mathrm{Inc}} = -\frac{e_q^2 N_c}{N_c^2 - 1}\left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}}\right], \\ H_{q\bar{q}\to\gamma g}^{\mathrm{Inc}} &= -H_{\bar{q}q\to\gamma g}^{\mathrm{Inc}} = \frac{e_q^2}{N_c^2} e_q^2 \left[\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right]. \end{aligned} \tag{11}$$

Here again we note that all these hard part functions have the same form in terms of Mandelstam variables $\hat{s}$, $\hat{t}$, $\hat{u}$, compared to those in the twist-3 collinear factorization approach [12]: $H_{ab\to c}^{\mathrm{Inc\text{-}I}}$ and $H_{ab\to c}^{\mathrm{Inc\text{-}F}}$ have the same functional form as the corresponding ones $H_{ab\to c}^{\mathrm{twist\text{-}3\text{-}I}}$ and $H_{ab\to c}^{\mathrm{twist\text{-}3\text{-}F}}$ in the twist-3 collinear factorization formalism, respectively.

However, there are two differences in the formalisms. First, in the twist-3 collinear approach, the hard part functions are given by

$$H_{ab\to c}^{\text{twist-3}}(\hat{s},\hat{t},\hat{u}) = H_{ab\to c}^{\text{twist-3}-\text{I}}(\hat{s},\hat{t},\hat{u}) + H_{ab\to c}^{\text{twist-3}-\text{F}}(\hat{s},\hat{t},\hat{u})\left(1+\frac{\hat{u}}{\hat{t}}\right), \tag{12}$$

i.e., there is an extra factor $(1+\hat{u}/\hat{t})$ accompanying the hard part functions $H_{ab\to c}^{\text{twist-3}-\text{F}}$ associated with final state interactions. However, in our modified GPM formalism as in eq. (10), there is no such factor. This difference can be traced back to the eikonal approximation we are using, see, *e.g.*, eq. (6), where we only keep the pole contribution $-k^+ + i\epsilon$ in the denominator under this approximation. However, there is an extra term linear in $k_\perp$ ($\propto p_c \cdot k_\perp$) which exists in the twist-3 collinear factorization formalism. This leads to the extra factor $(1+\hat{u}/\hat{t})$ for the final-state interaction contribution (for details, see ref. [12]). Second, in the twist-3 collinear factorization approach, all the parton momenta are collinear to the corresponding hadrons, thus $\hat{s}$, $\hat{t}$, $\hat{u}$ does not depend on the parton intrinsic transverse momentum. On the other hand, in the GPM approach the parton momenta involve intrinsic transverse momentum, thus $\hat{s}$, $\hat{t}$, $\hat{u}$ all depend on the the parton transverse momentum, k_{aT} and k_{bT}. In fact, because of the existence of the linear k_{aT}-dependence in $\epsilon^{k_{aT}S_A n\bar{n}}$, one has to keep another linear k_{aT}-dependence from the rest of the integrand in eq. (9), otherwise the integral over d^2k_{aT} vanishes. In other words, it is the linear in k_{aT} term in the hard part functions $H_{ab\to c}^{\text{Inc}}(\hat{s},\hat{t},\hat{u})$ and $\delta(\hat{s}+\hat{t}+\hat{t})$ that contributes to the asymmetry.

Even with these two issues, the similarities in terms of $\hat{s}$, $\hat{t}$, $\hat{u}$ suggest that there are close connections between our modified GPM formalism and the twist-3 collinear factorization approach.

3˙1. *Connections to twist-*3 *factorization.* – To explore this connection we make an expansion in k_{aT} and keeping only linear terms. We start by specifying the partonic kinematics. Keeping the linear in k_{aT} terms and dropping all the k_{bT}-dependence we have $p_a^\mu \approx x_a P_A^\mu + k_{aT}$ and $p_b^\mu \approx x_b P_B^\mu$, thus

$$\hat{s} \approx x_a x_b S, \qquad \hat{t} \approx \frac{x_a}{z_c}T - \frac{2P_{hT}\cdot k_{aT}}{z_c}, \qquad \hat{u} = \frac{x_b}{z_c}U. \tag{13}$$

Thus we can write the δ-function as

$$\delta(\hat{s}+\hat{t}+\hat{u}) = \frac{1}{x_b S + T/z_c}\delta\left(x_a - x - \frac{2P_{hT}\cdot k_{aT}}{z_c x_b S + T}\right), \tag{14}$$

where, $x_a = x + \frac{2P_{hT}\cdot k_{aT}}{z_c x_b S+T}$, and $x = -x_b U/(z_c x_b S + T)$ is independent of k_{aT}. Now performing the integrate over x_a in eq. (9) and using the δ-function we obtain,

$$E_h\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int \mathrm{d}^2k_{aT}\frac{\epsilon^{k_{aT}S_A n\bar{n}}}{M}\frac{1}{x_a}f_{1T}^{\perp a,\text{SIDIS}}(x_a,k_{aT}^2)\int\frac{\mathrm{d}x_b}{x_b}f_{b/B}(x_b) \cdot \int\frac{\mathrm{d}z_c}{z_c^2}D_{h/c}(z_c)H_{ab\to c}^{\text{Inc}}(\hat{s},\hat{t},\hat{u})\,\frac{1}{x_b S + T/z_c}\bigg|_{x_a = x + \frac{2P_{hT}\cdot k_{aT}}{z_c x_b S+T}}. \tag{15}$$

After replacing x_a as above, one has $\hat{s} = \tilde{s} - \frac{\tilde{s}}{\tilde{u}} 2P_{hT} \cdot k_{aT}/z_c$, $\hat{t} = \tilde{t} + \frac{\tilde{s}}{\tilde{u}} 2P_{hT} \cdot k_{aT}/z_c$, and $\hat{u} = \tilde{u}$, where $\tilde{s} = x x_b S$, $\tilde{t} = xT/z_c$, $\tilde{u} = x_b U/z_c$ and they are all independent of k_{aT}. Now besides the $\epsilon^{k_{aT} S_A n \bar{n}}$, the linear in k_{aT} contributions in eq. (15) can come from, either (a) x_a-dependence in $f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a, k_{aT}^2)$, or (b) the $\hat{s}$- and $\hat{t}$-dependence in $H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u})$. This is because x_a, $\hat{s}$, and $\hat{t}$ are the only terms in eq. (15) which depend linearly in k_{aT}. We now make k_{aT} expansion one by one. First for contribution (a): since

$$\frac{\partial x_a}{\partial k_{aT}^{\alpha}} = \frac{2P_{hT\alpha}}{z_c x_b S + T}\,, \tag{16}$$

to the linear term in k_{aT}, we have

$$\begin{aligned} E_h \frac{\mathrm{d}\Delta\sigma^{(a)}}{\mathrm{d}^3 P_h} = {} & \\ & \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \mathrm{d}^2 k_{aT} \frac{\epsilon^{k_{aT} S_A n \bar{n}}}{M} k_{aT}^{\alpha} \frac{2P_{hT\alpha}}{z_c x_b S + T} \frac{\mathrm{d}}{\mathrm{d}x_a} \left[\frac{f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a, k_{aT}^2)}{x_a} \right]_{x_a \to x} \\ & \cdot \int \frac{\mathrm{d}x_b}{x_b} f_{b/B}(x_b) \int \frac{\mathrm{d}z_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\mathrm{Inc}}(\tilde{s}, \tilde{t}, \tilde{u}) \frac{1}{x_b S + T/z_c}\,, \end{aligned} \tag{17}$$

where we have dropped all k_{aT} dependence in $H_{ab\to c}^{\mathrm{Inc}}$, thus replacing the k_{aT}-dependent $\hat{s}$, $\hat{t}$, $\hat{u}$ by the k_{aT}-independent $\tilde{s}$, $\tilde{t}$, $\tilde{u}$ in $H_{ab\to c}^{\mathrm{Inc}}$. Then using

$$\int \mathrm{d}^2 k_{aT} k_{aT}^{\beta} k_{aT}^{\alpha} f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a, k_{aT}^2) = -\frac{1}{2} \int \mathrm{d}^2 k_{aT}\, g^{\beta\alpha}\, |\vec{k}_{aT}|^2 f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a, k_{aT}^2), \tag{18}$$

and the relation between the Sivers function and the Efremov-Teryaev-Qiu-Sterman function $T_{a,F}(x,x)$ [35],

$$T_{a,F}(x,x) = -\frac{1}{M} \int \mathrm{d}^2 k_{aT} |\vec{k}_{aT}|^2 f_{1T}^{\perp a,\mathrm{SIDIS}}(x, k_{aT}^2), \tag{19}$$

one can rewrite eq. (17) as

$$\begin{aligned} E_h \frac{\mathrm{d}\Delta\sigma^{(a)}}{\mathrm{d}^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} & \int \frac{\mathrm{d}z_c}{z_c^2} D_{h/c}(z_c) \frac{\epsilon^{P_{hT} S_A n \bar{n}}}{z_c \tilde{u}} \frac{1}{x} \left[T_{a,F}(x,x) - x \frac{\mathrm{d}}{\mathrm{d}x} T_{a,F}(x,x) \right] \\ & \cdot \int \frac{\mathrm{d}x_b}{x_b} f_{b/B}(x_b) H_{ab\to c}^{\mathrm{Inc}}(\tilde{s}, \tilde{t}, \tilde{u}) \frac{1}{x_b S + T/z_c}\,. \end{aligned} \tag{20}$$

We observe that this *form* is the same as that in the twist-3 collinear factorization approach. In particular, note that there is no k_{aT}-dependence in the hard part functions $H_{ab\to c}^{\mathrm{Inc}}$. The difference to the twist-3 collinear factorization formalism [12] (as mentioned above) is the extra factor $(1 + \hat{u}/\hat{t})$ accompanying the hard part functions associated with final-state interactions, see eqs. (10) and (12).

Moreover, in our modified GPM formalism, we have another contribution due to the k_{aT}-dependence from $H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u})$ in eq. (15). As is noted above, $\hat{u}$ is independent of

k_{aT} while both $\hat{s}$ and $\hat{t}$ depend on k_{aT}. Since $\hat{s}+\hat{t}+\hat{u}=0$, one could then set $\hat{t}=-\hat{s}-\hat{u}$ in $H^{\rm Inc}_{ab\to c}$ and then expand only $\hat{s}$ in k_{aT}. That is,

$$\frac{\partial}{\partial k^{\alpha}_{aT}} H^{\rm Inc}_{ab\to c}(\hat{s},\hat{t},\hat{u})\bigg|_{k_{aT}\to 0} = -\frac{2\tilde{s}}{\tilde{u}}\frac{P_{hT\alpha}}{z_c}\frac{\partial}{\partial\tilde{s}}H^{\rm Inc}(\tilde{s},-\tilde{s}-\tilde{u},\tilde{u}). \tag{21}$$

Then we have the contribution denoted as (b),

$$E_h\frac{{\rm d}\Delta\sigma^{(b)}}{{\rm d}^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{{\rm d}z_c}{z_c^2}D_{h/c}(z_c)\frac{\epsilon^{P_{hT}S_A n\bar{n}}}{z_c\tilde{u}}\frac{1}{x}T_{a,F}(x,x)\int\frac{{\rm d}x_b}{x_b}f_{b/B}(x_b) \cdot\left[-\tilde{s}\frac{\partial}{\partial\tilde{s}}H^{\rm Inc}_{ab\to c}(\tilde{s},-\tilde{s}-\tilde{u},\tilde{u})\right]\frac{1}{x_bS+T/z_c}. \tag{22}$$

Thus to the leading order (linear in k_{aT} terms), the spin-dependent cross section in our modified GPM formalism can be written as

$$E_h\frac{{\rm d}\Delta\sigma}{{\rm d}^3P_h} = E_h\frac{{\rm d}\Delta\sigma^{(a)}}{{\rm d}^3P_h} + E_h\frac{{\rm d}\Delta\sigma^{(b)}}{{\rm d}^3P_h}\,, \tag{23}$$

with the contributions (a) and (b) given by eqs. (20) and (22), respectively. The term (a) *almost* reproduces the twist-3 collinear factorization formalism in ref. [12] mod the extra factor $(1+\hat{u}/\hat{t})$ associated with final state interactions, for which the origin of the difference is understood.

On the other hand, the extra term (b), does not appear in the usual twist-3 collinear factorization formalism. This deserves further investigation [36]. Here it is important to note, from the phenomenological perspective, as already shown in [12], the derivative of the correlation function $T_{a,F}(x,x)$ is the dominant contribution to the SSAs, thus we expect the term (b), which contains no derivative, to play a less important role in generating the SSAs compared with term (a). That is, even though this modified GPM has an extra piece compared with the well-known twist-3 collinear factorization formalism, phenomenologically (numerically) this formalism could give a good approximation to the SSAs. This remains to be confirmed [36]. If this were the case, it will provide further support to the modified GPM approach to the SSAs.

Finally, we also emphasize that the contribution calculated in ref. [12] only comes from the so-called soft-gluon-pole (SGP) in the twist-3 collinear factorization approach. However, there are also contributions from so-called soft-fermion-pole (SFP) [37, 38]. Even though our modified GPM formalism might capture the main feature of SGP contributions, it seems unlikely to reproduce the SFP contributions. In this respect the twist-3 formalism is "internally complete" in the sense that the collinear factorization is expected to hold for this formalism [39]. Finally, while TMD factorization is assumed in both GPM and our modified GPM formalisms, it is likely not to hold in these processes [20, 33, 34]. However, the extent to which it is broken is not known numerically. Thus, calculations within (modified) GPM formalisms should bear this in mind and thus be used with extra care.

4. – Numerical estimate of the SSAs

Here we present an estimation of the SSAs for single inclusive hadron and direct photon production in *pp* collisions at RHIC energy by using our modified GPM formalism

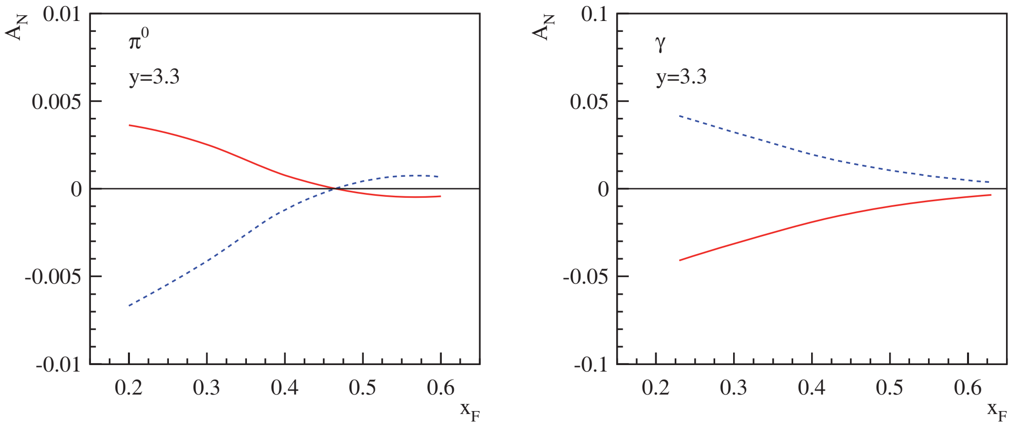

Fig. 3. – A_N for inclusive particle production as a function of x_F at RHIC energy $\sqrt{s} = 200\,\text{GeV}$: $p^\uparrow p \to \pi^0 + X$ (left) and $p^\uparrow p \to \gamma + X$ (right). The dashed curves are for the conventional GPM calculation, and the solid curves are for our modified GPM calculation. We have used the latest Sivers function from [41], and DSS fragmentation function [45].

in eq. (9). We will compare our results with those calculated from the conventional GPM formalism as in eq. (4).

To calculate the spin-averaged cross section, we use GRV98 LO parton distribution functions [40] along with a Gaussian-type k_T-dependence [41, 42]. The hard part functions for different partonic scattering channels are available in the literature [12, 43, 44]. For the spin-dependent cross section, we use the latest Sivers functions from [41] which are extracted from the recent SIDIS experiments. To consistently use this set of Sivers function, we will use DSS fragmentation function [45]. For the numerical predictions below, we work in a frame in which the polarized hadron moves in the $+z$-direction, choosing $S_\perp, P_{h\perp}$ along y- and x-directions, respectively, where all the relevant distribution functions and fragmentation functions evaluated at the scale $P_{h\perp}$ [22]. In fig. 3, we plot the A_N as a function of x_F for inclusive π^0 (left) and direct photon (right) production at rapidity $y = 3.3$ for RHIC energy $\sqrt{s} = 200\,\text{GeV}$. The estimates using the conventional GPM formalism in eq. (4) are shown as dashed lines, while those using our modified GPM formalism in eq. (9) are shown as solid lines. One immediately see that for both inclusive π^0 and direct photon, A_N change signs compare to the conventional GPM formalism. For π^0, the conventional GPM predicts a negative asymmetry (though very small from this set of Sivers functions), while the modified GPM formalism predicts a positive asymmetry. On the other hand, for direct photon, conventional GPM formalism predicts a positive asymmetry, while modified GPM formalism predicts that the asymmetry is negative, which is consistent with the predictions from twist-3 collinear factorization approach [12]. This can also be easily understood as follows. In the conventional GPM approach, one use H^U in the calculation of the spin-dependent cross section. For direct photon production, the dominant channel comes from $qg \to \gamma q$, with [12, 43]

$$H^U_{qg\to\gamma q} = \frac{1}{N_c} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right], \tag{24}$$

while the hard part in the modified GPM formalism is given by

$$H^{\text{Inc}}_{qg\to\gamma q} = -\frac{N_c}{N_c^2-1} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right] . \tag{25}$$

This introduces an extra color factor $-N_c^2/(N_c^2-1)$, thus opposite to the conventional GPM formalism. This prediction comes from the process-dependence of the Sivers functions, and has the same origin as in the photon+jet calculation [18]. On the other hand, for the inclusive π^0 production, the dominant channel comes from $qg \to qg$, particularly in the forward direction, one has

$$H^{\text{Inc}}_{qg\to qg} = H^{\text{Inc-I}}_{qg\to qg} + H^{\text{Inc-F}}_{qg\to qg} \to -\frac{N_c^2}{2(N_c^2-1)}\frac{2\hat{s}^2}{\hat{t}^2} - \frac{1}{N_c^2-1}\frac{2\hat{s}^2}{\hat{t}^2} = -\frac{N_c^2+2}{N_c^2-1}\frac{\hat{s}^2}{\hat{t}^2} ,$$

where we have used that in the forward direction, $\hat{t}$ is small, while $\hat{u} \sim -\hat{s}$, whereas [12,43]

$$H^{U}_{qg\to qg} = \frac{N_c^2-1}{2N_c^2} \left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}} \right] + \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \to \frac{2\hat{s}^2}{\hat{t}^2} . \tag{26}$$

We thus also see the sign is reversed in our modified GPM formalism compared with the conventional GPM approach.

We observe that the x_F-dependence in both modified and conventional GPM formalisms are different from those observed in the RHIC experiments where larger asymmetries have been observed in the forward direction (large x_F) [4,5,46,47]. Of course, in order to have a comparison with the experimental data for inclusive hadron production at RHIC experiments, one must include both Sivers (as studied in this paper) and Collins effects [14]. The latter describes a transversely polarized quark jet fragmenting into an unpolarized hadron, whose transverse momentum relative to the jet axis correlates with the transverse polarization vector of the fragmenting quark. This latter correlation can also generate the transverse-spin asymmetry (which is not studied here). Currently attempts at global fitting with both SIDIS and *pp* experimental data are ongoing [25]. We encourage the use of the modified GPM formalism in such a global analysis, to study the effect of the associated ISIs and FSIs (process-dependence of the Sivers functions). We also emphasize [18] that there is only Sivers contribution in direct photon production. Since the modified and conventional GPM predict opposite asymmetries, direct photon production presents a favorable opportunity to test the process dependence of the Sivers function, or the effect of the associated ISIs.

5. – Summary

We have presented a study of the single transverse-spin asymmetries in the single inclusive particle production in hadronic collisions. We find [19, 23] the Sivers functions in such processes are generally different from those probed in the SIDIS process because of different initial- and final-state interactions. By carefully taking into account the process-dependence in the Sivers functions (under one-gluon exchange approximation), we derive a new formalism within the framework of GPM approach. We find this formalism has close connections with the collinear twist-3 approach. Within this formalism, we make predictions for the inclusive π^0 and direct photon production in *pp* collisions at RHIC energies and find that the asymmetries predicted from the modified GPM formalism are opposite to those in the conventional GPM approach. This sign difference comes from the color gauge interaction, which has the same origin as the sign change for Sivers functions between SIDIS and DY processes. Our predictions about the sign are consistent with those from the twist-3 collinear factorization approach. We encourage a global analysis of both SIDIS and *pp* experimental data using this modified GPM formalism.

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REFERENCES

[1] D'Alesio U. and Murgia F., *Prog. Part. Nucl. Phys.*, **61** (2008) 394.
[2] Bunce G. *et al.*, *Phys. Rev. Lett.*, **36** (1976) 1113.
[3] Kang Z.-B., Qiu J.-W., Vogelsang W. and Yuan F., *Phys. Rev. D*, **83** (2011) 094001.
[4] Adams J. *et al.*, *Phys. Rev. Lett.*, **92** (2004) 171801.
[5] Adler S. S. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 202001.
[6] Zhu L. Y. *et al.*, *Phys. Rev. Lett.*, **99** (2007) 082301.
[7] Airapetian A. *et al.*, *Phys. Rev. Lett.*, **103** (2009) 152002.
[8] Alekseev M. *et al.*, *Phys. Lett. B*, **673** (2009) 127.
[9] Abe K. *et al.*, *Phys. Rev. Lett.*, **96** (2006) 232002.
[10] Efremov A. V. and Teryaev O. V., *Phys. Lett. B*, **150** (1985) 383.
[11] Qiu J.-W. and Sterman G., *Phys. Rev. Lett.*, **67** (1991) 2264.
[12] Kouvaris C., Qiu J.-W., Vogelsang W. and Yuan F., *Phys. Rev. D*, **74** (2006) 114013.
[13] Sivers D. W., *Phys. Rev. D*, **41** (1990) 83.
[14] Collins J. C., *Nucl. Phys. B*, **396** (1993) 161.
[15] Bacchetta A., Bomhof C. J., Mulders P. J. and Pijlman F., *Phys. Rev. D*, **72** (2005) 034030.
[16] Collins J. C., *Phys. Lett. B*, **536** (2002) 43.
[17] Brodsky S. J., Hwang D. S. and Schmidt I., *Nucl. Phys. B*, **642** (2002) 344.
[18] Bacchetta A., Bomhof C., D'Alesio U., Mulders P. J. and Murgia F., *Phys. Rev. Lett.*, **99** (2007) 212002.
[19] Gamberg L. and Kang Z.-B., *Phys. Lett. B*, **696** (2011) 109.
[20] Collins J. and Qiu J.-W., *Phys. Rev. D*, **75** (2007) 114014.
[21] Field R. D. and Feynman R. P., *Phys. Rev. D*, **15** (1977) 2590.
[22] Anselmino M., Boglione M. and Murgia F., *Phys. Lett. B*, **362** (1995) 164.
[23] D'Alesio U., Gamberg L., Kang Z.-B., Murgia F. and Pisano C., *Phys. Lett. B*, **704** (2011) 637.
[24] Boglione M., D'Alesio U. and Murgia F., *Phys. Rev. D*, **77** (2008) 051502.
[25] Anselmino M., Boglione M., D'Alesio U., Leader E., Melis S. *et al.*, arXiv:0809.3743 [hep-ph] (2008).
[26] Bacchetta A., D'Alesio U., Diehl M. and Miller C. A., *Phys. Rev. D*, **70** (2004) 117504.
[27] Qiu J.-W. and Sterman G. F., *Nucl. Phys. B*, **378** (1992) 52.
[28] Qiu J.-W. and Sterman G., *Phys. Rev. D*, **59** (1999) 014004.
[29] Collins J. C. and Soper D. E., *Nucl. Phys. B*, **193** (1981) 381.
[30] Ji X.-D., Ma J.-P. and Yuan F., *Phys. Rev. D*, **71** (2005) 034005.
[31] Collins J. C., *Foundations of Perturbative QCD* (Cambridge University Press, Cambridge) 2011.
[32] Aybat S. M., Collins J. C., Qiu J.-W. and Rogers T. C., (2011).

[33] Vogelsang W. and Yuan F., *Phys. Rev. D*, **76** (2007) 094013.
[34] Rogers T. C. and Mulders P. J., *Phys. Rev. D*, **81** (2010) 094006.
[35] Boer D. and Mulders P. J., *Phys. Rev. D*, **57** (1998) 5780.
[36] Gamberg L. and Kang Z., in preparation (2011).
[37] Koike Y. and Tomita T., *Phys. Lett. B*, **675** (2009) 181.
[38] Kanazawa K. and Koike Y., *Phys. Rev. D*, **82** (2010) 034009.
[39] Qiu J.-W. and Sterman G. F., *AIP Conf. Proc.*, **223** (1991) 249.
[40] Gluck M., Reya E. and Vogt A., *Eur. Phys. J. C*, **5** (1998) 461.
[41] Anselmino M., Boglione M., D'Alesio U., Kotzinian A., Melis S. *et al.*, *Eur. Phys. J. A*, **39** (2009) 89.
[42] Anselmino M. *et al.*, *Phys. Rev. D*, **72** (2005) 094007.
[43] Owens J., *Rev. Mod. Phys.*, **59** (1987) 465.
[44] Kang Z.-B. and Yuan F., *Phys. Rev. D*, **81** (2010) 054007.
[45] de Florian D., Sassot R. and Stratmann M., *Phys. Rev. D*, **75** (2007) 114010.
[46] Abelev B. I. *et al.*, *Phys. Rev. Lett.*, **99** (2007) 142003.
[47] Arsene I. *et al.*, *Phys. Rev. Lett.*, **101** (2008) 042001.