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The top-quark forward-backward asymmetry in the Standard Model

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Summary. — Standard Model calculations of the top-quark forward-backward asymmetry at the Tevatron are briefly reviewed.

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1. – Introduction

One of the most intriguing measurements to appear from the Tevatron in the last few years is that of the forward-backward (FB) asymmetry in top-quark pair production. By way of a quick introduction, recall that top-quark pair production at hadron colliders is completely dominated by QCD effects. The leading-order (LO) Feynman diagrams for the production process are shown in fig. 1. A short calculation of the differential partonic cross section with respect to the scattering angle θ_t of the top-quark shows that

$$\frac{\mathrm{d}\hat{\sigma}^{\mathrm{Born}}_{q\bar{q}}}{\mathrm{d}\cos\theta_t} = f_{q\bar{q}}(\cos^2\theta_t); \qquad \frac{\mathrm{d}\hat{\sigma}^{\mathrm{Born}}_{gg}}{\mathrm{d}\cos\theta_t} = f_{gg}(\cos^2\theta_t). \tag{1}$$

The functions f_i are unchanged under $\cos\theta_t \to -\cos\theta_t$, so the Born-level cross section is FB symmetric. A more detailed Standard Model (SM) calculation including higher-order QCD and electroweak effects shows a slight preference, on the order of 5–7%, to produce the top-quark in the same direction as the proton beam at the Tevatron, a prediction which is noticeably lower than both the CDF [1] and D0 [2] measurements. This discrepancy is a legitimate hint at new physics, but such an interpretation requires a good understanding of the SM calculation. Since compared to the production cross section the FB asymmetry appears first at next-to-leading order (NLO) in QCD, the calculation is non-trivial. The purpose of this talk is to review the basic elements that go into it. The focus is on total and differential FB asymmetries at the Tevatron. The same SM calculations are relevant for various differential charge asymmetries at the LHC but these will not be discussed in this short write-up.

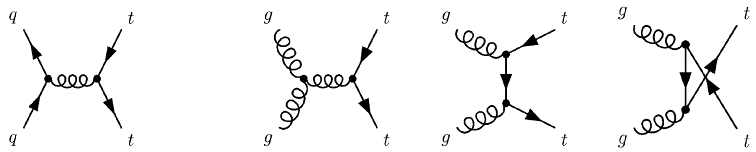

Fig. 1. – Born-level graphs for top-quark pair production.

2. – The forward-backward asymmetry at the Tevatron

There are two definitions of inclusive FB asymmetries used at the Tevatron. The first is

$$A^i_{\rm FB} = \frac{N_t(y^i_t > 0) - N_t(y^i_t < 0)}{N_t(y^i_t > 0) + N_t(y^i_t < 0)}\,, \tag{2}$$

where y^i_t is the top-quark rapidity in Lorentz frame i and N_t is the number of events. This frame-dependent definition is usually referred to as the top-quark FB asymmetry. Charge-conjugation invariance in QCD implies that $N_t(y) = N_{\bar t}(-y)$, so at the Tevatron this FB asymmetry is equivalent to a charge asymmetry. The second definition is the so-called "pair asymmetry"

$$A^{t\bar t}_{\rm FB} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}\,, \tag{3}$$

where $\Delta y = y_t - y_{\bar t}$. Unlike the first definition, the pair asymmetry depends on rapidity differences. It is therefore Lorentz invariant and can extracted in any reference frame. In the $t\bar t$ rest frame the definitions (2) and (3) coincide so it is common in the literature to simply use the first one and quote results for the top-quark asymmetry in the lab and $t\bar t$ frames.

The FB asymmetry in the SM is calculated by integrating the differential cross section over the appropriate phase space. It is then convenient to define asymmetric and symmetric cross sections corresponding to the numerator and denominator of the definitions above. Taking into account that the asymmetric part starts at one order higher than the symmetric part, the QCD contribution to the FB asymmetry has the form (suppressing for the moment the frame dependence)

$$A_{\rm FB} = \frac{\sigma_A}{\sigma_S} = \frac{\left[\int\limits_{y_t>0} \frac{{\rm d}\sigma}{{\rm d}y_t} - \int\limits_{y_t<0} \frac{{\rm d}\sigma}{{\rm d}y_t}\right]}{\left[\int\limits_{y_t>0} \frac{{\rm d}\sigma}{{\rm d}y_t} + \int\limits_{y_t<0} \frac{{\rm d}\sigma}{{\rm d}y_t}\right]} = \frac{\alpha_s^3\sigma_A^{(0)} + \alpha_s^4\sigma_A^{(1)} + \dots}{\alpha_s^2\sigma_S^{(0)} + \alpha_s^3\alpha_s\sigma_S^{(1)} + \dots}\,. \tag{4}$$

It is necessary to decide whether or not to expand the ratio of the symmetric and asymmetric pieces in orders of α_s. For a practitioner of perturbative QCD it is almost reflexive to do so, but for an experimentalist using an NLO event generator the tendency is to numerically evaluate the ratio with no further expansion. The differences in fixed-order perturbation theory are noticeable and should arguably added to the theory uncertainty, while those in resummed perturbation theory presented later on are not so large. In any case, this is a point to keep in mind when comparing results quoted by different authors.

With these definitions in place, we now turn to the status of the SM calculations. Generically, we can write the asymmetry including QCD and electroweak (EW) corrections as

$$\begin{aligned} A_{\mathrm{FB}} &= \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \dots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \dots} \\ &\equiv \alpha_s \left[A_{\mathrm{FB}}^{(0)} + \alpha_s A_{\mathrm{FB}}^{(1)} \right] + \alpha A_{\mathrm{FB}}^{\mathrm{EW+QCD},(0)} + \frac{\alpha^2}{\alpha_s^2} A_{\mathrm{FB}}^{\mathrm{EW},(0)} + \dots . \end{aligned}$$

At present, the SM calculations involve three components of the above expansion:

1) The leading QCD contribution $A_{\mathrm{FB}}^{(0)}$.

2) The leading EW corrections (both the EW+QCD interference and pure EW).

3) Higher-order QCD contributions as estimated by soft gluon resummation.

We now comment on these in turn.

The leading QCD contribution to the FB asymmetry arises from a subset of the NLO corrections to the differential pair production cross section and has been known for some time. An explicit analytic expression for the fully differential asymmetric cross section to this order can be found in [3]. This result was obtained by isolating the set of diagrams for the NLO cross section which are odd under the exchange of $t \leftrightarrow \bar{t}$ and calculating only that subset, which is much easier than calculating the whole NLO cross section and of course more efficient in a numeric code. These leading contributions to the FB asymmetry arise from the $q\bar{q}$ channel, which is numerically dominant, and from the qg and $\bar{q}g$ channels, which are numerically suppressed. The partonic cross section in the gluon channel is symmetric under $\cos\theta_t \to -\cos\theta_t$ to all orders in perturbation theory and so does not contribute to the asymmetric cross section.

Since the asymmetric cross section in QCD is suppressed by a power of α_s, EW corrections to the FB asymmetry are proportionally more important than for the total cross section. Certain subsets of EW corrections were originally identified in [3] and estimated to enhance the QCD asymmetry by roughly 8%. Recently, a more complete set of QCD+EW interference graphs (proportional to $\alpha_s^2\alpha$) and pure EW graphs (proportional to α^2) were calculated in [4] and confirmed in [5]. A numerical analysis from [4] and [5] indicates that the EW corrections are actually quite a bit larger than the above-mentioned 8%, a statement which is quantified below.

It would be extremely desirable to have the full next-to-leading QCD contribution to the FB asymmetry (the $A_{\mathrm{FB}}^{(1)}$ in (5)). However, this involves next-to-next-to-leading order Feynman diagrams compared to the Born-level graphs shown in fig. 1 and calculating these corrections is quite challenging. In the absence of this calculation a more manageable way to improve on the leading-order calculation is to include what can be argued to be the dominant subset of higher-order corrections using soft-gluon resummation. Recently, two types of double differential cross sections were calculated using soft-gluon resummation to next-to-next-to-leading logarithmic (NNLL) order. The first was the two-particle inclusive cross section $\mathrm{d}^2\sigma/\mathrm{d}M_{t\bar{t}}\mathrm{d}\Delta y$ [6], which can be used to calculate the pair asymmetry $A_{\mathrm{FB}}^{t\bar{t}}$ (this was already done in [7] at NLL), the second was the single-particle inclusive cross section $\mathrm{d}^2\sigma/\mathrm{d}p_T\mathrm{d}y_t$ in the lab frame, which can be used to

TABLE I. – *Results for $A_{\rm FB}$ in the $t\bar{t}$ and lab frame at the Tevatron.*

	$A_{\rm FB}^{t\bar{t}}$ [%]	$A_{\rm FB}^{p\bar{p}}$ [%]
NLO	$7.32^{+0.69+0.18}_{-0.59-0.19}$	$4.81^{+0.45+0.13}_{-0.39-0.13}$
NLO+NNLL [10]	$7.24^{+1.04+0.20}_{-0.67-0.27}$	$4.88^{+0.20+0.17}_{-0.23-0.18}$
CDF [1]	15.0 ± 5.5	15.8 ± 7.4
D0 [2]	–	19.6 ± 6.5

determine $A_{\rm FB}^{p\bar{p}}$ [8,9]. It must be said that the most reliably way of estimating theoretical uncertainties in such resummed calculations is debatable, but these calculations still give insight into the structure of the higher-order calculations and are the currently most complete QCD results for the asymmetry. We focus below on soft-gluon resummation at NLO+NNLL order as implemented in [10]; a comparison with approximate NNLO results for the lab-frame asymmetry from [9] can be found in [11].

Numerical results summarizing current SM calculations of the FB asymmetry are shown in table I. (Here and below we commit a slight abuse of nomenclature and label the QCD results with the relative order at which the differential cross section is needed instead of the order at which the asymmetric cross section appears, which is one order lower in both fixed-order and resummed perturbation theory.) The NLO (NLO+NNLL) results use $\mu_f = m_t = 173.1\,\text{GeV}$ and MSTW2008NLO(NNLO) PDFs by default. The first uncertainty is the perturbative one as estimated by scale variations, and the second is the PDF uncertainty at 90% confidence level. Higher-order corrections as estimated by soft-gluon resummation are a very mild effect when evaluated at the scale $\mu_f = m_t$. Although not shown in the table, the EW corrections result in a significant enhancement by a factor of about 1.22 at $\mu_f = m_t$ [4] compared to the NLO calculation and it is thus mandatory to account for them in a comparison with experiment. In any event, the overall message is that the experimental errors are by far larger than the theory errors and that theory and experiment differ by about one (two) sigma for $A_{\rm FB}^{t\bar{t}}$ ($A_{\rm FB}^{p\bar{p}}$).

The measurements of the total FB asymmetries are rather consistent between CDF and D0. This is not the case for the FB asymmetry as a function of the invariant mass of the top-quark pair. Due to statistics, such a measurement was carried out in only two bins of invariant mass—one above and one below $M_{t\bar{t}} = 450\,\text{GeV}$. The SM calculations of the asymmetry in these two bins are shown in table II, along with the CDF measurement. The results show that the SM asymmetry grows as a function of $M_{t\bar{t}}$, and that the effect of resummation in each bin is roughly the same as for the total

TABLE II. – *Results for $A_{\rm FB}^{t\bar{t}}$ in two bins of pair invariant mass.*

$A_{\rm FB}^{t\bar{t}}$ [%]	$M_{t\bar{t}} < 450\,\text{GeV}$	$M_{t\bar{t}} > 450\,\text{GeV}$
NLO	$5.3^{+0.3+0.1}_{-0.4-0.1}$	$10.6^{+1.1+0.3}_{-0.8-0.1}$
NLO+NNLL [10]	$5.2^{+0.7+0.1}_{-0.5-0.0}$	$11.1^{+1.9+0.3}_{-1.0-0.0}$
CDF [1]	-11.6 ± 15.3	47.5 ± 11.2

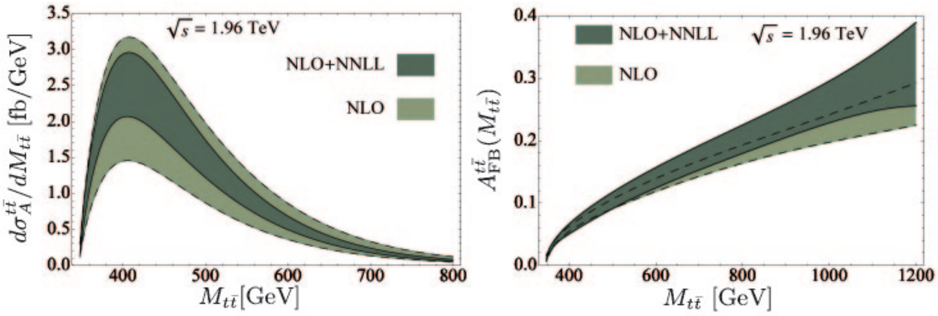

Fig. 2. – Invariant-mass dependence of the asymmetric cross section and pair asymmetry (taken from [10]).

asymmetry. The same is true of the EW corrections, which at $\mu = m_t$ enhance the NLO QCD result by a factor of roughly 1.22 in each bin [4]. The CDF measurement in the higher invariant-mass bin differs from the SM prediction by roughly 3 sigma (slightly less once EW effects are included). Such production level results were not given by D0 in [2], but the reconstruction level results show no sharp increase in the high invariant-mass bin compared to the low invariant-mass one, in contrast to the CDF result.

It is also interesting to study the asymmetry as a continuous function of the pair invariant mass. To do so, we define the differential FB asymmetry and asymmetric cross section as

$$A_{\rm FB}^{t\bar{t}}(M_{t\bar{t}}) = \frac{\left(\frac{{\rm d}\sigma}{{\rm d}M_{t\bar{t}}}\right)_{\Delta y>0} - \left(\frac{{\rm d}\sigma}{{\rm d}M_{t\bar{t}}}\right)_{\Delta y<0}}{\left(\frac{{\rm d}\sigma}{{\rm d}M_{t\bar{t}}}\right)_{\Delta y>0} + \left(\frac{{\rm d}\sigma}{{\rm d}M_{t\bar{t}}}\right)_{\Delta y<0}} \equiv \frac{\frac{{\rm d}\sigma_A^{t\bar{t}}}{{\rm d}M_{t\bar{t}}}}{\frac{{\rm d}\sigma}{{\rm d}M_{t\bar{t}}}}. \tag{5}$$

Results at NLO and NLO+NNLL order using the same input as for the total asymmetry are shown in fig. 2, where the bands refer to perturbative uncertainties only. The result for the asymmetric cross section in the left-hand side of the figure shows a decrease in scale uncertainties in the resummed calculation compared to the fixed-order one. For the asymmetry itself, shown in the right-hand side of the figure, one actually observes a slight increase in scale uncertainties due to large cancellations in the fixed-order calculation upon calculating the ratio. This shows that simply varying the scales in the NLO result can easily lead to an underestimate of the perturbative uncertainties, a comment which also applies to the total FB asymmetry in table I. Notice also that while the asymmetry as a function of the invariant mass grows roughly linearly up to a TeV or so, the asymmetric cross section is peaked slightly above 400 GeV. In fact, the integral of the asymmetric cross section up to $M_{t\bar{t}} \sim 450$ GeV is roughly equal to the integral above 450 GeV. Therefore, from the viewpoint of the SM, the experimental border of 450 GeV is a natural one for a two-bin analysis, as it roughly divides the asymmetric sample in two.

Also worth mentioning is the FB asymmetry in the $t\bar{t}X$ + jet channel. In QCD the differential cross section with an additional jet involves an extra parton compared to the Born graphs in fig. 1, so the FB asymmetry starts at the same order in α_s as the fully differential cross section. Moreover, the NLO corrections to $t\bar{t}X$+jet were obtained in [12] and [13]. An interesting result from those works is that the LO FB asymmetry of roughly 8% at $\mu_f = m_t$ is reduced to about 2% by the NLO corrections, an effect that is in no way reflected by the scale uncertainties in the LO result, which are in fact quite small. This result raises the question of whether similarly large corrections may occur in the fully inclusive FB asymmetry, even though uncertainties estimated through scale variation and the results from soft-gluon resummation give no indication that it should. The authors of [13] give arguments as to why these large corrections should be unique to $t\bar{t}X$ + jet and not affect the fully inclusive asymmetry, but it is worth pointing out the obvious fact that the only way to know for sure it to actually do the calculation.

So far we have discussed asymmetries in $t\bar{t}$ production as if the top quarks were stable particles. Needless to say, they in fact decay almost as soon as they are produced, and if theory predictions stop at the level of $t\bar{t}$ final states then experimentalists must correct their measurements of the decay products back to that level. An especially simple way to minimize this mismatch between theory and experiment is to instead define FB asymmetries in terms of the decay products. For instance, in the di-lepton channel, one can define lepton and lepton-pair asymmetries in exact analogy to (2) and (3). Very little is lost in using leptonic variables, since NLO calculations of the differential cross sections are known in the narrow-width approximation including electroweak corrections [14] (the narrow width approximation works quite well, as confirmed by the calculation with off-shell top quarks in [15, 16]). Recent measurements by the D0 collaboration [2] are considerably higher than the SM predictions, consistent with the findings for production-level measurements using $t\bar{t}$ final states.

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REFERENCES

[1] Aaltonen T. *et al.* (CDF Collaboration), *Phys. Rev. D*, **83** (2011) 112003 [arXiv:1101.0034 [hep-ex]].
[2] Abazov V. M. *et al.* (D0 Collaboration), arXiv:1107.4995 [hep-ex].
[3] Kuhn J. H. and Rodrigo G., *Phys. Rev. D*, **59** (1999) 054017. [hep-ph/9807420].
[4] Hollik W. and Pagani D., *Phys. Rev. D*, **84** (2011) 093003 [arXiv:1107.2606 [hep-ph]].
[5] Kuhn J. H. and Rodrigo G., arXiv:1109.6830 [hep-ph].
[6] Ahrens V. *et al.*, *JHEP*, **09** (2010) 097 [arXiv:1003.5827 [hep-ph]].
[7] Almeida L. G. *et al.*, *Phys. Rev. D*, **78** (2008) 014008 [arXiv:0805.1885 [hep-ph]].
[8] Ahrens V. *et al.*, *JHEP*, **09** (2011) 070 [arXiv:1103.0550 [hep-ph]].
[9] Kidonakis N., *Phys. Rev. D*, **84** (2011) 011504 [arXiv:1105.5167 [hep-ph]].
[10] Ahrens V. *et al.*, *Phys. Rev. D*, **84** (2011) 074004 [arXiv:1106.6051 [hep-ph]].
[11] Kidonakis N. and Pecjak B. D., arXiv:1108.6063 [hep-ph].
[12] Dittmaier S., Uwer P. and Weinzierl S., *Phys. Rev. Lett.*, **98** (2007) 262002.
[13] Melnikov K. and Schulze M., *Nucl. Phys. B*, **840** (2010) 129 [arXiv:1004.3284 [hep-ph]].
[14] Bernreuther W. and Si Z.-G., *Nucl. Phys. B*, **837** (2010) 90 [arXiv:1003.3926 [hep-ph]].
[15] Denner A. *et al.*, *Phys. Rev. Lett.*, **106** (2011) 052001 [arXiv:1012.3975 [hep-ph]].
[16] Bevilacqua G. *et al.*, *JHEP*, **02** (2011) 083 [arXiv:1012.4230 [hep-ph]].