

## Strangeness in the nucleon: What have we learned?

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**Summary.** — We review the state of our knowledge concerning the contribution of strange quarks to various nucleon properties. In the case of the electric and magnetic form factors, the level of agreement between theory and experiment is very satisfactory and gives us considerable confidence in our capacity to make reliable calculations within non-perturbative QCD. In view of the importance of the scalar form factors to the detection of dark matter candidates such as neutralinos, we place a particular emphasis on the determination of the  $\pi N$  and strange quark sigma commutators.

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### 1. – Introduction

As the only serious candidate for a fundamental theory of the strong interaction, QCD presents some remarkable challenges. The quantisation of such a highly non-linear quantum field theory is extremely difficult and at present only lattice QCD has a claim of significant success. In assessing our progress towards a complete understanding of QCD it is critical to check that key calculations actually agree with experiment. For QCD the strange form factors of the nucleon occupy a position of comparable importance to that of the Lamb shift in the history of QED. While lattice QCD has accurately described a number of valence quark dominated hadronic properties, the strange form factors of the nucleon can only arise through quantum fluctuations, in which a strange–anti-strange pair briefly bubble into and out of existence. Thus the calculation of the strange form factors and their verification by experiment is of fundamental importance.

In the next section we briefly review the state of play with respect to the strange electric and magnetic form factors of the nucleon, which have been the focus of intensive experimental effort for the past two decades. We then turn to the strange sigma commutator (the strange scalar form factor,  $\sigma_s$ ), for which the best value has undergone a major

shift in the last two or three years. After summarising recent work on the determination of  $\sigma_s$ , including a parallel analysis of data on the mass of the  $H$ -dibaryon, we explain the relevance to dark matter searches. The final section is devoted to some concluding remarks.

## 2. – Vector form factors

It is now more than 20 years since it was realized that parity violating electron scattering (PVES) could provide a third, independent constraint on the vector form factors of the nucleon, thus allowing one to solve for the strange vector matrix elements [1]. Following a series of state-of-the-art measurements at MIT-Bates, Mainz and JLab (for a recent review see ref. [2]), as well as a systematic study of the relevant radiative corrections [3, 4] and a careful global analysis [5], we now know that the strange magnetic form factor is at most a few percent of the proton magnetic form factor at low- $Q^2$  and the strange electric radius is also at most a few percent of the proton charge radius.

Remarkably, it is now twenty years since the initial studies of nucleon vector form factors in lattice QCD, by Leinweber and collaborators [6]. In combination with chiral extrapolation of lattice data using finite range regularisation, indirect techniques developed by Leinweber and Thomas [7] led to a very precise determination of both the strange magnetic moment of the proton and its strange charge radius [8] some 15 years later. Although it initially seemed as though these calculations disagreed with the PVES data, it is now clear that the agreement is excellent. Furthermore, precise, direct calculations of these form factors from the Kentucky group in just the last two years [9] agree very well with the results of the earlier indirect work and with the experimental results.

At present, the accuracy of the theoretical calculations exceeds that of the best experiments by almost an order of magnitude – a remarkable exception in strong interaction physics. Clearly the challenge is there for a clever new idea to take us beyond the current experimental limitations. Nevertheless, this future challenge should not blind us to the tremendous achievements thus far and especially to the fact that QCD has passed its equivalent of the ‘‘Lamb shift test’’ with flying colours.

## 3. – Sigma commutators

The so-called  $\pi N$  sigma commutator

$$(1) \quad \sigma_{\pi N} = m_l \langle p | (\bar{u}u + \bar{d}d) | p \rangle \equiv m_l \frac{\partial M_N}{\partial m_l},$$

with  $m_l = (m_u + m_d)/2$  and the strange sigma commutator

$$(2) \quad \sigma_s = m_s \langle p | \bar{s}s | p \rangle \equiv m_s \frac{\partial M_N}{\partial m_s},$$

not only tell us directly how much the corresponding quark masses contribute to the mass of the nucleon but they also constitute direct measures of chiral symmetry breaking in QCD. They have therefore been of great interest ever since the phenomenological importance of chiral symmetry was realized.

One often used measure of the relative importance of the strange quarks in nucleon structure is the ratio  $y$ :

$$(3) \quad y = \frac{2 \langle p | \bar{s}s | p \rangle}{\langle p | (\bar{u}u + \bar{d}d) | p \rangle} = \frac{m_l}{m_s} \frac{2 \sigma_s}{\sigma_{\pi N}}.$$

Early calculations of quantities such as this relied on the naive application of formulas for the octet baryon masses based upon first order breaking of flavour  $SU(3)$  symmetry. For example, the *non-singlet* combination  $\sigma_0$ :

$$(4) \quad \sigma_0 = m_l \langle p | \bar{u}u + \bar{d}d - 2\bar{s}s | p \rangle,$$

was found from such an analysis to be  $\sigma_0 = 36 \pm 7 \text{ MeV}$  [10]. Then, after the  $\pi N$  sigma commutator is deduced from the historical method of pion-nucleon dispersion relations to get the value of the scalar form factor at the unphysical Cheng-Dashen point ( $t = 2m_\pi^2$ ) and then corrected for form factor effects to the point  $t = 0$ , the ratio  $y$  may be found as  $(\sigma_{\pi N} - \sigma_0)/\sigma_{\pi N}$ .

This method has a number of practical problems. First there is some controversy over the value at the Cheng-Dashen point extracted from dispersion relations. Second, the form factor correction is sizeable and not model independent. Finally, the numerical difference between  $\sigma_0$  and  $\sigma_{\pi N}$  is small compared with the errors in both of them. As a result, values quoted for  $y$  have typically ranged between 0 and 0.4, with  $\sigma_s$  itself being taken to be of order 300 MeV for many years. Indeed, this large estimate, combined with early overestimates of the strange vector form factors played a crucial role in motivating the parity violation experiments discussed above.

Since the strange sigma commutator may be interpreted as the contribution to the mass of the nucleon from the strange quark, a value as large as 300 MeV would indeed be remarkable. It would mean that almost a third of the nucleon mass arises from quarks that are non-valence. This appears incompatible with the widely used constituent quark models, for example. It is not surprising that this has motivated an enormous amount of theoretical interest in pinning down the empirical value of  $\sigma_s$  as accurately as possible. However, it is in the search for dark matter that the value of  $\sigma_s$  has had its most immediate impact [11].

**3.1. Importance in the search for dark matter.** – In the minimal supersymmetric extension of the Standard Model, constrained by all particle physics and WMAP data, the so-called CMSSM, the favoured candidate for dark matter is the neutralino [12], a weakly interacting fermion with mass of order of a hundred GeV or more. For the old values of  $\sigma_s \sim 300 \text{ MeV}$ , its dominant interaction with the nucleon was through the strange quark. The old method of determining  $\sigma_s$ , through the difference between  $\sigma_{\pi N}$  and  $\sigma_0$ , led Ellis, Olive and collaborators to call desperately for a more accurate determination of  $\sigma_{\pi N}$ , not for its own sake but in order to pin down  $\sigma_s$  [13].

We return to this topic below, after reviewing more modern methods to determine  $\sigma_s$ , based upon lattice QCD and in particular the careful chiral analysis of lattice data.

**3.2. Lattice QCD.** – Over the past 15 years there have been extensive studies of the sigma commutators within lattice QCD, with the initial calculations tending to support the large values for  $\sigma_s$ . On the other hand, in the absence of sufficient computing power to directly calculate hadron properties at the physical quark masses, we have been fortunate

to be given data that has opened new methods to determine  $\sigma_s$ . In particular, a wealth of data has been generated for a variety of hadronic properties *within QCD* as a function of the masses of the quarks. This is of course information that Nature cannot give us but which is nevertheless an invaluable guide to how QCD actually works [14]. From those studies it is clear that for whatever reason (and we suggest one below) the properties of baryons and non-Goldstone mesons made of light quarks behave exactly as one would expect in a constituent quark model once the pion mass is above about 0.4 GeV. That is, all the famous, rapid, non-analytic variation associated with Goldstone boson loops is seen to disappear in this region [15, 16].

There has been some speculation that this scale (which corresponds to a quark mass around 40 MeV) may have something to do with the size of an instanton. However, it is clear that such behaviour does emerge naturally if one takes into account the finite size of the hadrons, which necessarily suppresses meson loops at large momentum transfer and at large mass for the Goldstone bosons. Put simply, meson loops are suppressed when the corresponding Compton wavelength is smaller than the size of the hadron emitting or absorbing the meson. Given that a typical hadron size is 1 fm, and the Compton wavelength of a meson of mass 0.4 GeV only 0.5 fm, one has a natural explanation.

This qualitative lesson from QCD itself, leads us to anticipate that the contribution of strange quarks to nucleon properties should be suppressed, because the mass of the kaon is 0.5 GeV — *i.e.*, it is above the critical scale.

This idea has also been developed into a method of analysis which has enabled quantitative advances in the calculation of hadron properties using finite range regularization (FRR) [17]. This technique allows one to effectively resum the chiral expansion of hadronic properties and to accurately describe the variation of properties such as the mass of the baryon octet over a much larger range of quark masses than expected within naive chiral perturbation theory.

**3.3. Formal expansion of the sigma commutator.** — The literature is dominated with suggestions that the sigma commutators are dominated by the leading terms in the chiral expansion of the nucleon mass. For example, the chiral coefficient  $c_1$ , which yields the term proportional to  $m_\pi^2$  in the standard expansion of  $m_N$ , is often referred to as the term that gives  $\sigma_{\pi N}$ . Of course, it has been known for decades that the leading non-analytic (LNA) term, proportional to  $m_\pi^3$  is almost as big (and opposite in sign) but the discussion usually stops there. While higher order terms in the chiral expansion are not model independent, within FRR with the formulae fit to lattice data for  $m_N$ , it has been shown that the value of  $\sigma_{\pi N}$  extracted is independent of the model used for the regulator [18, 19]. It therefore seems reasonable to use FRR chiral perturbation theory to serve as a guide to the importance of higher order terms in the expansion of  $\sigma_{\pi N}$  in powers of  $m_\pi$ .

As an example, using a dipole form factor one finds for the expansion of the  $\pi N$  loop which yields the leading non-analytic (LNA) behaviour of  $m_N$ :

$$(5) \quad \delta M_N = c_{\text{LNA}} \left( \frac{\Lambda^3}{16} - \frac{5\Lambda}{16} m_\pi^2 + m_\pi^3 - \frac{35}{16\Lambda} m_\pi^4 + \dots \right),$$

where  $c_{\text{LNA}} = -3g_A^2/32\pi f_\pi^2$  and  $\Lambda$  is the dipole mass parameter. From this one may easily show that the corresponding contribution to  $\sigma_{\pi N}$ , at the physical pion mass, is

$$(6) \quad \delta\sigma_{\pi N} = 35\Lambda - 23 + \frac{9.6}{\Lambda} - \frac{3}{\Lambda^2} + \dots,$$

where  $\Lambda$  is in GeV and the result in MeV. Thus, for a typical value of  $\Lambda \sim 1$  GeV, we see that the  $m_\pi^4$  term contributes almost 10 MeV and so, even for  $\sigma_{\pi N}$  the series expansion is very slowly convergent.

However, for  $\sigma_s$ , where the LNA behaviour in  $m_K$  has the same form, with a very similar value of  $c_{\text{LNA}}$ , the much larger mass ( $m_K/m_\pi \sim 3.5$ ) means that the cubic term is of order  $-1$  GeV and the quartic term around  $+1.4$  GeV. Clearly, the expansion of  $\sigma_s$  in powers of  $m_s$  about zero is badly divergent and therefore useless. On the other hand, using a FRR to resum the series in a fit to lattice data for  $m_N$  versus  $m_\pi$  over a large range does lead to results that are independent of the model used. This is a powerful technique.

**3.4. Modern values of the sigma commutators.** – Once one has an accurate parametrization of the mass of the nucleon as a function of pion and kaon mass, based on a fit to modern lattice data for the nucleon octet using FRR, it is trivial to extract the light quark sigma commutators by differentiation, using the Feynman-Hellmann theorem. It is this approach which has recently shown that  $\sigma_s$  is almost an order of magnitude smaller than had been generally believed for 20 years [20]. Indeed, in a systematic study of the masses of the octet baryons, using chiral perturbation theory with FRR, Young and Thomas found the value  $\sigma_s = 31 \pm 15 \pm 4 \pm 2$  MeV [20]. Combining this with the value they obtained for  $\sigma_{\pi N} = 47 \pm 8 \pm 1 \pm 3$  MeV, this yields a value for  $y \sim 0.05$ .

It is interesting to note that a small value for  $\sigma_s$  had been published by the Adelaide group some years earlier, in the context of the search for variations in fundamental constants [21]. It is well known that in many grand unified theories a change in  $\alpha$  with time leads to a much larger change with time of quark masses. That this may have observable consequences in modern precision measurements of atomic spectroscopy means that one needs to know the variation of parameters such as the nucleon mass and magnetic moment with both light and strange quark masses. Based solely on the contribution from  $\pi$ ,  $K$  and  $\eta$  loops, Flambaum *et al.* reported a value of  $\sigma_s \sim 10$  MeV. The value reported above, in which the meson loops are calibrated by fitting accurate octet mass data, represents a natural improvement in the earlier estimate. However, the fact that  $\sigma_s$  is *much* smaller than had been believed is a feature of both calculations.

In direct contrast with the early lattice simulations, which tended to reinforce the large values of  $\sigma_s$ , very similar conclusions have been reached by several other modern lattice simulations [22-24], with most agreeing that the value is between 20 and 50 MeV. Although at first sight it is shocking that there can be such a large shift in a fundamental property of the nucleon, it is quite a common phenomenon when it comes to fundamental parameters that apparent convergence can be followed by a shift by far more than the quoted uncertainties when a new technique becomes available.

Recent work from UKQCD supports earlier suggestions that the underlying reason for erroneous large values of  $\sigma_s$  in the early lattice studies is operator mixing. Indeed, after a careful analysis of the operator mixing under renormalization, Bali *et al.* recently reported  $\sigma_s = 11 \pm 13 \overset{+9}{-3}$  MeV [25]. We note that this was at a light quark mass somewhat larger than the physical value and correcting for this should raise the final value by several MeV.

The European Twisted Mass Collaboration also reported a new result for  $y$  recently [26], namely  $y = 0.066 \pm 0.011 \pm 0.002$ . This agrees very well with the determination of Young and Thomas quoted above.

**3'5.  $H$ -dibaryon mass.** – In the context of recent new lattice results which suggested that the  $H$ -dibaryon might indeed be bound with respect to the  $\Lambda$ - $\Lambda$  threshold [27, 28], Shanahan *et al.* recently built on the work of Mulders and Thomas [29] to make a detailed FRR analysis of both the  $H$  data and the full octet baryon data-set [30]. Their result, namely that at the physical quark masses the  $H$  is most likely slightly unbound (by  $13 \pm 14$  MeV), is of great interest both because of its potential implications for the equation of state of dense matter and in connection with the enhancement above  $\Lambda$ - $\Lambda$  threshold already reported experimentally [31]. It is clearly of great importance that the latter be pursued in experiments at the new J-PARC facility.

In the present context that study is of direct interest because the analysis of the full octet data-set actually led to a much improved error band on the values of  $\sigma_{\pi N}$  and  $\sigma_s$ . For details of the systematic study of potential sources of error in the analysis of Shanahan *et al.* we refer to ref. [32]. Suffice it to say that under variations of the various chiral parameters and the form of the UV regulator, the values of  $\sigma_{\pi N}$  varied between 42 and 51 MeV, with a statistical error of order 5 MeV. In fact, the only major shift was for the sharp cut-off which is somewhat unphysical. Omitting that and combining the errors in quadrature the result is  $\sigma_{\pi N} = 44 \pm 5$  MeV. For  $\sigma_s$  the variation with regulator was negligible and the value deduced was  $21 \pm 7$  MeV. This is arguably the most reliable current determination. For these values of  $\sigma_{\pi N}$  and  $\sigma_s$  and using eq. (3) we find  $y = 0.04 \pm 0.02$ .

**3'6. Implications for dark matter searches.** – It is in the search for dark matter that the new value of  $\sigma_s$  has had its most immediate impact [11]. In the context of the CMSSM, with the neutralino as the candidate for dark matter, the large values of  $\sigma_s$  meant that strange quarks dominated the dark matter cross section. On the other hand, the new value reported above changes the situation dramatically. No longer do the strange quarks dominate the cross section. Not only is the expected cross section reduced by an order of magnitude compared with earlier optimistic expectations but the values found are far more accurate. We refer to the work of Giedt *et al.* for more details [11].

#### 4. – Conclusion

We have seen that the role played by strange quarks in the structure of the nucleon is important for a multitude of reasons. In any analysis of the electromagnetic form factors of the nucleon it is critical to know the strange quark contribution. After two decades of work it is now clear that this contribution is rather small but most important that it is in excellent agreement with the values given by QCD.

In contrast with the suggestion that strange quarks might contribute as much as a third of the mass of the nucleon, sophisticated modern calculations based on lattice QCD and FRR chiral perturbation theory have shown that the contribution is nearer to a few percent. This is a remarkable shift in such a fundamental quantity and it has profound consequences for the search for dark matter as well as for possible changes in fundamental “constants” of Nature.

We close with a brief remark on a serious conundrum. The classic analysis of the nucleon mass by Shifman *et al.* [33] suggests that each heavy flavour contributes of the order of 70 MeV to the mass of the nucleon. In contrast, as we have shown, the three active flavours ( $u$ ,  $d$  and  $s$ ) each contribute around 20–30 MeV to the mass of the nucleon. Understanding how it is possible that the contributions from  $c$ ,  $b$  and  $t$  quarks could be

two or three times larger, if this is indeed correct, would be a fundamental contribution to our knowledge of QCD and hadron structure.

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## REFERENCES

- [1] MCKEOWN R. D., *Phys. Lett. B*, **219** (1989) 140.
- [2] PASCHKE K. *et al.*, *J. Phys.: Conf. Ser.*, **299** (2011) 012003.
- [3] BLUNDEN P. G., MELNITCHOUK W. and THOMAS A. W., *Phys. Rev. Lett.*, **107** (2011) 081801 [arXiv:1102.5334 [hep-ph]]; SIBIRTSEV A. *et al.*, *Phys. Rev. D*, **82** (2010) 013011 [arXiv:1002.0740 [hep-ph]].
- [4] GORCHTEIN M. and HOROWITZ C. J., *Phys. Rev. Lett.*, **102** (2009) 091806 [arXiv:0811.0614 [hep-ph]]; GORCHTEIN M., HOROWITZ C. J. and RAMSEY-MUSOLF M. J., *Phys. Rev. C*, **84** (2011) 015502 [arXiv:1102.3910 [nucl-th]].
- [5] YOUNG R. D., ROCHE J., CARLINI R. D. and THOMAS A. W., *Phys. Rev. Lett.*, **97** (2006) 102002 [nucl-ex/0604010].
- [6] LEINWEBER D. B., WOLOSHYN R. M. and DRAPER T., *Phys. Rev. D*, **43** (1991) 1659-1678.
- [7] LEINWEBER D. B. and THOMAS A. W., *Phys. Rev. D*, **62** (2000) 074505 [hep-lat/9912052].
- [8] LEINWEBER D. B. *et al.*, *Phys. Rev. Lett.*, **94** (2005) 212001 [hep-lat/0406002]; LEINWEBER D. B. *et al.*, *Phys. Rev. Lett.*, **97** (2006) 022001 [hep-lat/0601025].
- [9] DOI T. *et al.*, *Phys. Rev. D*, **80** (2009) 094503 [arXiv:0903.3232 [hep-ph]].
- [10] BORASOY B. and MEISSNER U.-G., *Ann. Phys. (N.Y.)*, **254** (1997) 192 [hep-ph/9607432].
- [11] GIEDT J., THOMAS A. W. and YOUNG R. D., *Phys. Rev. Lett.*, **103** (2009) 201802 [arXiv:0907.4177 [hep-ph]].
- [12] JUNGMAN G., KAMIONKOWSKI M. and GRIEST K., *Phys. Rep.*, **267** (1996) 195-373 [hep-ph/9506380].
- [13] ELLIS J. R., OLIVE K. A. and SAVAGE C., *Phys. Rev. D*, **77** (2008) 065026 [arXiv:0801.3656 [hep-ph]].
- [14] DETMOLD W. *et al.*, *Pramana*, **57** (2001) 251 [nucl-th/0104043].
- [15] LEINWEBER D. B., LU D.-H. and THOMAS A. W., *Phys. Rev. D*, **60** (1999) 034014 [hep-lat/9810005].
- [16] LEINWEBER D. B., THOMAS A. W., TSUSHIMA K. and WRIGHT S. V., *Phys. Rev. D*, **61** (2000) 074502 [hep-lat/9906027].
- [17] YOUNG R. D., LEINWEBER D. B. and THOMAS A. W., *Prog. Part. Nucl. Phys.*, **50** (2003) 399 [arXiv:hep-lat/0212031].
- [18] LEINWEBER D. B., THOMAS A. W. and YOUNG R. D., *Phys. Rev. Lett.*, **92** (2004) 242002 [hep-lat/0302020].
- [19] LEINWEBER D. B., THOMAS A. W. and WRIGHT S. V., *Phys. Lett. B*, **482** (2000) 109-113 [hep-lat/0001007].
- [20] YOUNG R. D. and THOMAS A. W., *Phys. Rev. D*, **81** (2010) 014503 [arXiv:0901.3310 [hep-lat]].
- [21] FLAMBAUM V. V., LEINWEBER D. B., THOMAS A. W. and YOUNG R. D., *Phys. Rev. D*, **69** (2004) 115006 [hep-ph/0402098].
- [22] OHKI H. *et al.*, *Phys. Rev. D*, **78** (2008) 054502.
- [23] TOUSSAINT D. and FREEMAN W. (MILC COLLABORATION), *Phys. Rev. Lett.*, **103** (2009) 122002.
- [24] OHKI H. *et al.*, *PoS, LAT2009* (2009) 124 [arXiv:0910.3271 [hep-lat]].
- [25] BALI G. S. *et al.* (QCDSF COLLABORATION), [arXiv:1111.1600 [hep-lat]].

- [26] DINTER S., DRACH V. and JANSEN K., *PoS, Lattice2011* (2011) 152.
- [27] BEANE S. R. *et al.* (NPLQCD COLLABORATION), *Phys. Rev. Lett.*, **106** (2011) 162001 [arXiv:1012.3812 [hep-lat]].
- [28] INOUE T. *et al.* (HAL QCD COLLABORATION), *Phys. Rev. Lett.*, **106** (2011) 162002.
- [29] MULDER P. J. and THOMAS A. W., *J. Phys. G*, **G9** (1983) 1159.
- [30] SHANAHAN P. E., THOMAS A. W. and YOUNG R. D., *Phys. Rev. Lett.*, **107** (2011) 092004 [arXiv:1106.2851 [nucl-th]].
- [31] YOON C. J. *et al.*, *Phys. Rev. C*, **75** (2007) 022201.
- [32] THOMAS A. W., SHANAHAN P. E. and YOUNG R. D. *Few-Body Syst.*, DOI: 10.1007/s00601-012-0372-8 [arXiv:1111.0114 [nucl-th]].
- [33] SHIFMAN M. A., VAINSHTEIN A. I. and ZAKHAROV V. I., *Phys. Lett. B*, **78** (1978) 443.