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Overview of two-boson exchange in electron-proton scattering

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Summary. — We review the role of two-boson exchange in elastic electron-proton scattering, focusing in particular on γZ interference in parity-violating reactions. We assess the impact of the two-boson corrections on the extraction of the strange form factors of the nucleon and the proton's weak charge.

PACS 12.15.Lk – Electroweak radiative corrections.
PACS 11.55.Fv – Dispersion relations.
PACS 13.60.Hb – Total and inclusive cross sections (including deep-inelastic processes).

1. – Introduction

The elastic *ep* polarization transfer experiments at Jefferson Lab revealed a significant discrepancy with the ratio of electric to magnetic form factors extracted from unpolarized cross sections [1]. Because essentially all electron scattering measurements are analyzed in the one-photon exchange approximation, this discrepancy led to a serious re-examination of the possible role played by two-photon exchange corrections (for a review see ref. [2]).

In addition to the exchange of one or more virtual photons between the electron and nucleon, the Standard Model allows the scattering to take place via the exchange of a neutral Z boson. Since the Z mass is some two orders of magnitude larger than the proton mass, the weak exchange process is strongly suppressed relative to the electromagnetic reaction. Nevertheless, asymmetries sensitive to the γZ interference amplitude, which are of order several parts per million, have been measured in modern accelerator facilities. These can be used to extract the strange electric and magnetic form factors of the nucleon, as well as the weak charge of the proton.

The γZ interference term is isolated by polarizing the incident electron and observing the difference between right- and left-handed electrons scattering from unpolarized protons. A parity-violating (PV) asymmetry can then be defined as $A_{\rm PV} = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$, where $\sigma_{R,L}$ are the cross sections for a right- and left-hand polarized electrons, respectively. The numerator in the asymmetry is sensitive to the interference of the vector and axial-vector currents, and hence violates parity. In view of the large

TPE effects found for electromagnetic form factors [3-5], the question naturally arises of what effect the exchange of two bosons may have on PV asymmetries. Because both the strange form factors and the proton weak charge are numerically small quantities, the two-boson exchange (TBE) contributions could affect their extraction significantly.

2. – Parity-violating electron scattering

For a nucleon target, the PV asymmetry can be written as

$$A^N_{\rm PV} = -\frac{K}{\sigma_R^{\gamma N}} \left\{ -2g_A^e \left(\varepsilon\, G_E^{\gamma N} G_E^{ZN} + \tau\, G_M^{\gamma N} G_M^{ZN} \right) + 2g_V^e\, \varepsilon'\, G_M^{\gamma N} G_A^{ZN} \right\}, \tag{1}$$

where $K = G_F Q^2/4\pi\alpha\sqrt{2}$, G_F is the Fermi constant, and α is the fine structure constant. The electromagnetic $G^{\gamma N}_{E,M}$ and weak $G^{ZN}_{E,M,A}$ form factors are functions of the exchanged four-momentum transfer squared, Q^2, with $\sigma_R^{\gamma N}$ the reduced γN cross section, and $g^e_{V,A}$ the vector and axial-vector couplings of the electron to the Z boson. The variable ε is related to the scattering angle θ by $\varepsilon = (1+2(1+\tau)\tan^2(\theta/2))^{-1}$, with $\tau = Q^2/4M^2$ and M the nucleon mass, and $\varepsilon' = \sqrt{\tau(1+\tau)(1-\varepsilon^2)}$. For a proton target, assuming isospin symmetry, the weak vector form factors $G^{Zp}_{E,M}$ can be related to the electromagnetic form factors of the proton (neutron) $G^{\gamma p(n)}_{E,M}$, so that

$$G_E^{Zp}(0) \equiv Q_W^p = 1 - 4\sin^2\theta_W \qquad \text{(Born approximation)} \tag{2}$$

is the weak charge of the proton. Because Q^p_W is numerically small, the overall contribution to $G^{Zp}_{E,M}$ from the proton electromagnetic form factors is suppressed.

3. – Two-boson exchange corrections and strange form factors

Beyond the Born approximation, the PV asymmetry receives corrections from higher-order radiative effects, including TPE and corrections involving γZ loops. The full PV asymmetry, including all TBE corrections, is given by $A_{\rm PV} = (1+\delta)A^0_{\rm PV}$, where $A^0_{\rm PV}$ is the Born asymmetry in eq. (1), and $\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$. Here $\delta_{Z(\gamma\gamma)}$ and $\delta_{\gamma(Z\gamma)}$ denote the corrections from the interference between single Z boson and $\gamma\gamma$ exchange amplitudes, and between the one-photon exchange and γZ interference amplitudes, respectively, while $\delta_{\gamma(\gamma\gamma)}$ is the purely electromagnetic TPE correction.

The total TBE contributions from nucleon and Δ intermediate states are shown in fig. 1 as a function of ε for $Q^2 = 0.01$ and $0.1\,\text{GeV}^2$, relative to the Mo-Tsai infrared-divergent result, $\overline{\delta} \equiv \delta - \delta_{\rm IR}$. At low Q^2 the $\gamma(\gamma\gamma)$ and $Z(\gamma\gamma)$ contributions are very similar and largely cancel in the asymmetry, which is then determined mostly by the $\gamma(Z\gamma)$ component. The Δ correction is strongly suppressed at low ε, but grows with increasing ε, becoming as important as the nucleon elastic part near the forward limit.

The full effects of the TBE corrections on $A_{\rm PV}$ at kinematics corresponding to experiments designed to measure the strange quark form factors of the nucleon were considered in refs. [7-10]. For the forward angle HAPPEX [11] and G0 [12] measurements, the nucleon correction was found to be in the vicinity of $\sim 0.1\%$–0.2%, increasing to $\sim 1.0\%$–1.5% for the backward angle G0 [13] and the earlier SAMPLE [14] measurements. In contrast, the Δ contribution is almost negligible at backward angles, but becomes more important at forward angles.

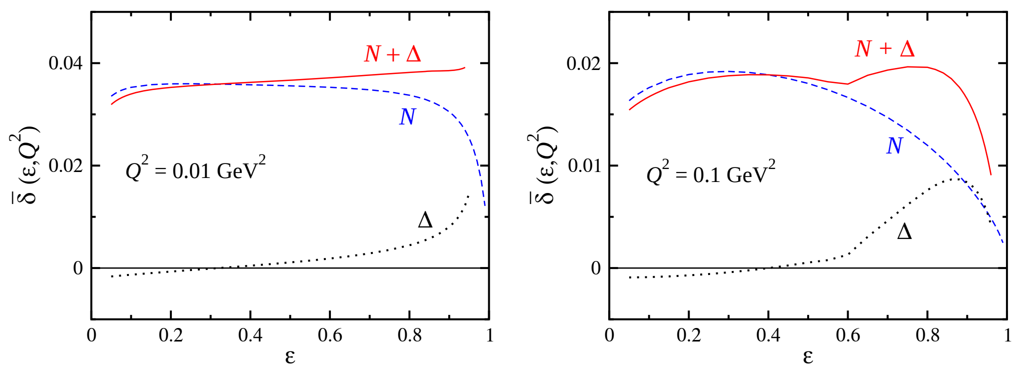

Fig. 1. – Total finite parts of the TBE corrections $\bar{\delta}$, relative to the Mo-Tsai contribution [6], with nucleon (dashed) and Δ (dotted) intermediate states, as well as the sum (solid), at $Q^2 = 0.01\,\mathrm{GeV}^2$ (left) and $0.1\,\mathrm{GeV}^2$ (right) [7].

The impact of these corrections on the strange form factors is difficult to gauge without performing a full reanalysis of the data, since in general different electroweak parameters and form factors are used in the various experiments. Using PV scattering data below $Q^2 = 0.3\,\mathrm{GeV}^2$, a preliminary analysis shows that the TBE corrections in the proton modify the strange electric and magnetic form factors [15] by $G_E^s = 0.0025(182) \longrightarrow 0.0023(182)$ and $G_M^s = -0.011(254) \longrightarrow -0.020(254)$ at a scale $Q^2 = 0.1\,\mathrm{GeV}^2$. The effect on the strange magnetic form factor is thus an almost factor of two increase in the magnitude, but is still well within the current experimental uncertainty. The shift in the strange electric form factor is somewhat smaller. Overall, the conclusion appears to be that TBE effects provide relatively mild corrections to strange quark form factors. On the other hand, a significantly larger effect from TBE has been found in near-forward PV electron scattering at very low Q^2, which we discuss next.

4. – γZ corrections to the proton weak charge

At forward scattering angles, the PV asymmetry in the low-energy limit is related to the weak charge of the proton, $A^{\mathrm{PV}} \to K\,t\,Q_W^p$, where $t \equiv -Q^2$. The absolute γZ box correction to Q_W^p at zero energy is denoted by $\Box_{\gamma Z}(0) \equiv Q_W^p\,\delta_{\gamma Z}$, and has contributions from both the *vector electron–axial vector hadron* and *axial vector electron–vector hadron* couplings of the Z boson. The vector hadron contribution $\Box_{\gamma Z}^V$ vanishes in the limit of zero energy E, but is finite at $E > 0$. The axial hadron correction $\Box_{\gamma Z}^A$, which is dominant at the very low E relevant to atomic parity-violation experiments, was estimated some time ago by Marciano and Sirlin [16] in terms of a free quark model-inspired loop calculation.

In the limit $t \to 0$, the correction $\Box_{\gamma Z}$ can be computed from its imaginary part using forward dispersion relations [17]. The imaginary part of γZ exchange amplitude can be written in terms of the cross section for all possible hadronic final states with mass W, parametrized through the γZ structure functions $F_{1,2,3}^{\gamma Z}$. The contributions to the structure functions can be split into elastic, resonance, and deep-inelastic scattering (DIS) regions. For the latter, the contributions to $\Box_{\gamma Z}$ can be written in terms of moments

$M^{(n)}_{1,2,3} = \int \mathrm{d}x\, x^{n-2}\{xF_1^{\gamma Z}, F_2^{\gamma Z}, xF_3^{\gamma Z}\}$ of the structure functions,

$$\text{(3)}\quad \Box^{V\,(\mathrm{DIS})}_{\gamma Z}(E) = \frac{\alpha}{\pi} 2ME \int_{Q_0^2}^{\infty} \frac{\mathrm{d}Q_1^2}{Q_1^4(1+Q_1^2/M_Z^2)}\Bigg[M_2^{(2)} + \frac{2}{3}M_1^{(2)} + \frac{2M^2}{3Q_1^4}(E^2 - Q_1^2)M_2^{(4)} + \frac{2M^2}{5Q_1^4}(4E^2 - 5Q_1^2)M_1^{(4)} + \dots\Bigg],$$

$$\text{(4)}\quad \Box^{A\,(\mathrm{DIS})}_{\gamma Z}(E) = \hat{v}_e \frac{3\alpha}{2\pi} \int_{Q_0^2}^{\infty} \frac{\mathrm{d}Q_1^2}{Q_1^2(1+Q_1^2/M_Z^2)}\Bigg[M_3^{(1)} + \frac{2M^2}{9Q_1^4}(5E^2 - 3Q_1^2)M_3^{(3)} + \dots\Bigg],$$

where Q_1^2 is the virtuality of the exchanged boson and M_Z is the Z boson mass. The large-x contributions to $M_i^{(n)}(Q_1^2)$ become more important for large n; however, the higher moments are suppressed by increasing powers of $1/Q_1^2$. In practice, the integrals are dominated by the lowest moments, with the $1/Q_1^2$ corrections being relatively small in DIS kinematics. For the axial-vector hadron part, the lowest moment $M_3^{(1)}(Q_1^2)$ is the γZ analog of the Gross-Llewellyn Smith sum rule for νN DIS. The corresponding quantity for γZ is $\sum_q 2e_q\, g_A^q = 5/3$, so that at next-to-leading order in the $\overline{\mathrm{MS}}$ scheme $M_3^{(1)}(Q_1^2) = 5/3(1 - \alpha_s(Q_1^2)/\pi)$.

The total vector and axial hadron corrections $\Box^{V,A}_{\gamma Z}(E)$ are shown in fig. 2 as a function of the incident electron energy E. For the vector hadron correction $\Box^V_{\gamma Z}$, fig. 2 (left), most of the strength ($\sim 80\%$) comes from relatively low energies, below 4 GeV, where the Q_1^2 range extends to $\sim 6\,\mathrm{GeV}^2$, and W to ~ 3 GeV. The nonresonant contribution to $\Box^V_{\gamma Z}$ is small at low E, rising linearly with E in this region. The resonant part increases steeply to a maximum at $E \sim 1$ GeV, before falling off like $1/E$ [17, 18]. Sibirtsev *et al.* find the resonant and nonresonant contributions to $\Box^V_{\gamma Z}$ to be 0.0026 and 0.0021, respectively, at the energy relevant for the Q_{weak} experiment, $E = 1.165$ GeV.

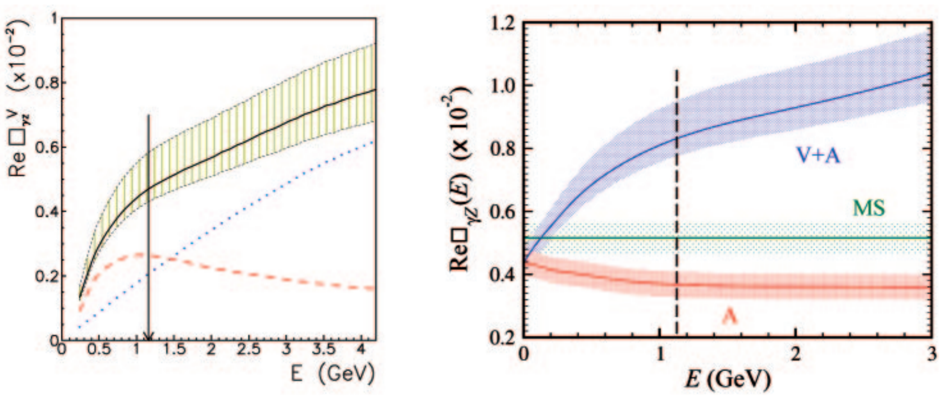

Fig. 2. – γZ box corrections to Q_W^p for the vector hadron $\Box^V_{\gamma Z}$ (left) showing the resonant (dashed) and nonresonant (dotted) components, and the sum (solid, and shaded) [18]; and axial hadron $\Box^A_{\gamma Z}$ (right), together with the V+A sum, and the $E = 0$ result from refs. [16,19] ("MS", extended to finite E) [20]. The vertical lines at $E = 1.165$ GeV indicate the energy of the Q_{weak} experiment [21].

The axial hadron correction $\Box^A_{\gamma Z}$ in fig. 2 (right) is dominated by the DIS contribution, which has negligible E dependence. On the other hand, the resonance and low-Q^2 DIS contributions dominate the uncertainties [20]. The total axial hadron correction $\Box^A_{\gamma Z}(E)$ is 0.0044(4) at $E = 0$, and 0.0037(4) at $E = 1.165\,\text{GeV}$. This should be compared to the value 0.0052(5) used in ref. [19], which is assumed to be energy independent. Combined with the correction to $\Box^V_{\gamma Z}$, this shifts the theoretical estimate for Q^p_W from 0.0713(8) to 0.0705(8), with a total correction of $0.0040^{+0.0011}_{-0.0004}$ at $E = 1.165\,\text{GeV}$.

The corrections $\Box^{V,A}_{\gamma Z}$ are important for the interpretation of the Q_{weak} experiment, given its projected uncertainty of ± 0.003 [21], which is expected to constrain possible sources of parity violation from beyond the Standard Model at a mass scale of $> 2\,\text{TeV}$ [22]. The uncertainties in the corrections can be reduced with future parity-violating structure function measurements at low Q^2, such as those planned at Jefferson Lab. The high-precision determination of Q^p_W would then allow more robust extraction of signals for new physics beyond the Standard Model.

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