Colloquia: LaThuile12

The time-dependent CP violation in charm

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Summary. — A model which describes the time-dependent CP formalism in D^0 decays has recently been proposed. There it has been highlighted a possible measurement of the angle β_c, in the charm unitarity triangle, using the decays $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$, and a measurement of the mixing phase ϕ_{MIX}. The same method can be used to measure the value of the parameter x, one of the two parameters defining charm mixing. We numerically evaluate the impact of a time-dependent analysis in terms of the possible outcomes from present and future experiments. We consider the scenarios of correlated D^0 mesons production at the center-of-mass energy of the $\Psi(3770)$ at Super$B$, uncorrelated production at the center-of-mass energy of the $\Upsilon(4S)$ at Super$B$ and Belle II, and LHCb. Recently a hint of direct CP violation in charm decays was reported by the LHCb Collaboration, we estimate the rate of time-dependent asymmetry that could be achieved using their available data, and we generalise the result for the full LHCb program. We conclude that LHCb is already able to perform a first measurement of $\beta_{c,eff}$, and slightly improve the present constraints on the parameters x and ϕ_{MIX}. A more precise determination of $\beta_{c,eff}$, ϕ_{MIX} and x will require a larger data sample, and most probably the cleaner environment of the new high-luminosities B-factories (both Super$B$ and Belle II) will be needed. We show that Super$B$ will be able to measure $\beta_{c,eff}$ and ϕ_{MIX} with a precision of $1.3°$ and improve the precision on x by a factor of two.

PACS 13.25.Hw – Decays of bottom mesons.
PACS 12.15.Hh – Determination of Cabibbo-Kobayashi & Maskawa (CKM) matrix elements.
PACS 11.30.Er – Charge conjugation, parity, time reversal, and other discrete symmetries.

1. – Introduction

Since the discovery in 1964 of CP violation in the kaon system [1], CP violation has been observed also in the B meson system [2,3]. In the charm sector, CP violation has long been expected to be too small to be observed at precision available until recently when, the LHCb Collaboration has reported a difference in direct CP asymmetries in $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ that is 3.5σ from the CP-conserving hypothesis [4]. In [5]

the standard model (SM) description of these decays using the same CKM paradigm that provides a rather satisfactory description of such decays of B^0 mesons is considered. Since the LHCb result, a broader view of this paradigm that might accomodate the large asymmetry is examined in [6]. It is clear that, in order to understand the nature of CPV in D^0 decays, measurements of weak phases in these decays are essential. In [5], it is proposed that, as with B^0 decays, time-dependent CP asymmetries in D^0 decays may provide the most direct way to measure these phases. In this paper, we further examine the precision that might be anticipated in four experimental scenarios that are likely to be available over the coming decade to evaluate the rate of time-dependent CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ in the three proposed environments (Super$B$, LHCb, Belle II).

2. – Time-dependent CP violation in the charm sector

In the standard model (SM), CP violation is described in terms of the complex phase appearing in Cabibbo-Kobayashi-Maskawa (CKM) matrix [7,8]. The matrix is a unitary 3×3 matrix which provides a description of quark mixing in terms of the coupling strengths for up-to-down quark type transitions, and it may be written as

$$(1) \qquad V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

Within this framework the probability to observe a transition between a quark q to a quark q' is proportional to $|V_{qq'}|^2$.

2˙1. *Buras parametrisations of the CKM matrix*. – In ref. [5] the CKM matrix has been written using the "*Buras*" parametrisation [9]:

$$(2) \quad V_{CKM} =$$
$$\begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta})+A\lambda^5(\bar{\rho}-i\bar{\eta})/2 \\ -\lambda+A^2\lambda^5[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-\lambda^2/2-\lambda^4(1+4A^2)/8 & A\lambda^2 \\ A\lambda^3[1-(\bar{\rho}+i\bar{\eta})] & -A\lambda^2+A\lambda^4[1-2(\bar{\rho}+i\bar{\eta})]/2 & 1-A^2\lambda^4/2 \end{pmatrix}$$
$$+\mathcal{O}(\lambda^6).$$

We adopt the convention of writing the CKM matrix in terms of $\bar{\rho}$ and $\bar{\eta}$ because these represent the coordinates of the apex of the well known bd unitarity triangle. Since unitarity triangles are mathematically exact, it is very important to measure their angles and sides to verify unitarity. One of the six unitarity relationships of the CKM matrix may be written as

$$(3) \qquad\qquad V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0,$$

which represents the cu triangle that we will call the *charm* unitarity triangle or simply *charm* triangle. The internal angles of this triangle are given by

$$(4) \qquad\qquad \alpha_c = \arg\left[-V_{ub}^* V_{cb}/V_{us}^* V_{cs}\right],$$
$$(5) \qquad\qquad \beta_c = \arg\left[-V_{ud}^* V_{cd}/V_{us}^* V_{cs}\right],$$
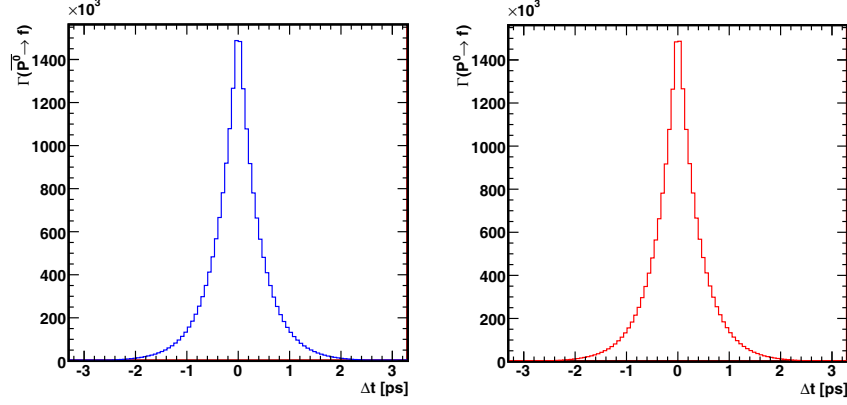$$(6) \qquad\qquad \gamma_c = \arg\left[-V_{ub}^* V_{cb}/V_{ud}^* V_{cd}\right].$$

Fig. 1. – Generated distributions according to our formulas for $\overline{D^0} \to f$(left) and for $D^0 \to f$(right) produced at the center-of-mass energy of the $\Psi(3770)$.

In ref. [5] we proposed the measurement of $\beta_{c,eff}$ using time-dependent CP asymmetries in charm decays and using the results of Global CKM fits, predicted that

$$\beta_c = (0.0350 \pm 0.0001)°. \tag{7}$$

On comparing eq. (5) with eq. (2), one can see that $V_{cd} = V_{cd}e^{i(\beta_c - \pi)}$ in this convention.

2'2. Time-dependent formalism. – We consider two different cases of D^0 meson production: uncorrelated and correlated D^0 production. Uncorrelated D^0's are produced from the decays of B mesons in electron-positron colliders when particles are collided at a center-of-mass energy corresponding to the $\Upsilon(4S)$ resonance, from $c\bar{c}$ continuum, or in hadrons collider. The correlated D^0 mesons are produced in an electron-positron machine running at a center-of-mass energy corresponding to the $\Psi(3770)$ resonance. The time evolution for both situations, shown in fig. 1, is given by [5]

uncorrelated case

$$\Gamma_\pm \propto e^{-\Gamma_1 t} \left[\frac{(1 + e^{\Delta\Gamma t})}{2} + \frac{\text{Re}(\lambda_f)}{1 + |\lambda_f|^2} \left(1 - e^{\Delta\Gamma t}\right) \right. \tag{8}$$
$$\left. \pm e^{\Delta\Gamma t/2} \left(\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos\Delta M t - \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin\Delta M t \right) \right],$$

correlated case

$$\Gamma_\pm \propto e^{-\Gamma_1 |\Delta t|} \left[\frac{h_+}{2} + \frac{\text{Re}(\lambda_f)}{1 + |\lambda_f|^2} h_- \right. \tag{9}$$
$$\left. \pm e^{\Delta\Gamma \Delta t/2} \left(\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos\Delta M \Delta t - \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin\Delta M \Delta t \right) \right],$$

where Γ_+ refers to $D^0(q_c = +2/3)$ decays and Γ_- to $\overline{D^0}(q_c = -2/3)$ decays, $h_\pm = 1 \pm e^{\Delta\Gamma \Delta t}$ and $\lambda_f = \frac{q}{p}\frac{\overline{A}}{A}$. Here q and p are the parameters defining the mixing and A $(\overline{A})$

is the amplitude for the D ($\overline{D}$) decay to a final state f. If $|A|^2 \neq |\overline{A}|^2$ there is direct CP violation (in the decay) and $|q/p| \neq 1$ would signify CP violation in mixing. The study of λ_f (which should not be confused with the term λ appearing in the CKM matrix) is able to probe the combination of CP violation due to mixing and decay, and this form of CP violation is referred to as CP violation in the interference between mixing and decay. Considering eqs. (8) and (9) the time-dependent asymmetries associated with the time evolution of the D^0 mesons can be written in terms of the physical decay rate including the mistag probability, $\omega(\bar{\omega})$, for incorrect tagging of the D^0 ($\overline{D}^0$) flavour as follows:

$$(10) \qquad \Gamma^{Phys}(t) = (1 - \omega)\Gamma_+(t) + \overline{\omega}\,\Gamma_-(t),$$

$$(11) \qquad \overline{\Gamma}^{Phys}(t) = \omega\Gamma_+(t) + (1 - \overline{\omega})\Gamma_-(t),$$

where $\Gamma_+(t)$ and $\Gamma_-(t)$ are from eqs. (8) and (9) and ω ($\overline{\omega}$) represents the mistag probability for the particle (antiparticle) apparent decay rates for D^0 and $\overline{D}^0$, respectively. Hence for uncorrelated mesons the time dependent CP asymmetry accounting for mistag probability is

$$(12) \quad \mathcal{A}^{Phys}(t) = \frac{\overline{\Gamma}^{Phys}(t) - \Gamma^{Phys}(t)}{\overline{\Gamma}^{Phys}(t) + \Gamma^{Phys}(t)}$$
$$= \Delta\omega + \frac{(D - \Delta\omega)e^{\Delta\Gamma t/2}[(|\lambda_f|^2 - 1)\cos\Delta M t + 2\mathrm{Im}\lambda_f \sin\Delta M t]}{h_+(1 + |\lambda_f|)^2/2 + \mathrm{Re}(\lambda_f)h_-},$$

where $\Delta\omega = \omega - \overline{\omega}$ and $D = 1 - 2\omega$.

Similarly the asymmetry for correlated mesons is

$$(13) \, \mathcal{A}^{Phys}(\Delta t) = \frac{\overline{\Gamma}^{Phys}(\Delta t) - \Gamma^{Phys}(\Delta t)}{\overline{\Gamma}^{Phys}(\Delta t) + \Gamma^{Phys}(\Delta t)}$$
$$= -\Delta\omega + \frac{(D + \Delta\omega)e^{\Delta\Gamma\Delta t/2}[(|\lambda_f|^2 - 1)\cos\Delta M\Delta t + 2\mathrm{Im}\lambda_f \sin\Delta M\Delta t]}{h_+(1 + |\lambda_f|)^2/2 + \mathrm{Re}(\lambda_f)h_-}.$$

The above equations may be written in terms of x and y allowing for the measurement of the mixing phase. We report here the time-dependent asymmetry equation for correlated mesons (similar results may be obtained in the uncorrelated case):

$$(14) \quad \mathcal{A}^{Phys}_{x,y}(\Delta t) = -\Delta\omega + \frac{(D + \Delta\omega)e^{y\Gamma\Delta t}[(|\lambda_f|^2 - 1)\cos x\Gamma\Delta t + 2\mathrm{Im}\lambda_f \sin x\Gamma\Delta t]}{h_+(1 + |\lambda_f|)^2/2 + \mathrm{Re}(\lambda_f)h_-}.$$

3. – MC test of time-dependent numerical analysis

One of the issues raised in [5] is the possibility to use different decay channels of the D^0 mesons to constrain the value of the angle β_c of the *charm* triangle. The decay $D^0 \to K^+K^-$ will be used to measure the mixing phase, the decay $D^0 \to \pi^+\pi^-$ will be used to measure $\phi_{MIX} - 2\beta_c$ and the difference between the two channels will provide a first measurement of the angle β_c. In this framework, long distance contributions to decay are not considered. The latter together with the different contribution to decay $D^0 \to \pi^+\pi^-$ from *penguin* topologies will introduce theoretical uncertainties, and for

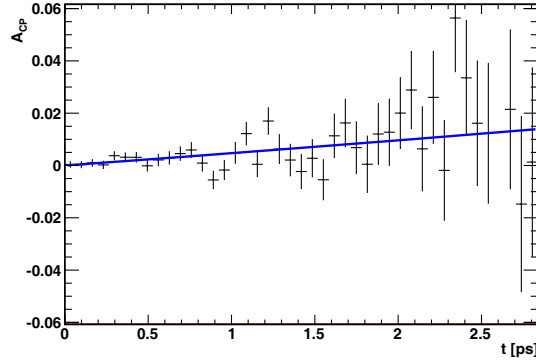Fig. 2. – The time-dependent CP asymmetry expected for $D^0 \to \pi^+\pi^-$ decays in a $75\,\mathrm{ab}^{-1}$ sample of data at the $\Upsilon(4S)$.

this reason we refer to the angle β_c as $\beta_{c,eff}$ where *effective* indicates that there are theoretical uncertainties that need to be evaluated. To evaluate the asymmetry, and estimate the precision on $\beta_{c,eff}$ that one might achieve in the different experimental environments described in the previous section, we generate a set of one hundred Monte Carlo data samples. Each one based on the expected number of tagged D^0 decays in the corresponding experimental setup, and we generate data according to the distributions given in eqs. (8) and (9), where the parameters involved are evaluated as in ref. [10]. We evaluate the asymmetry given in eqs. (12) and (13) including the expected mistag probabilities, and perform a binned fit to the simulated data. The distributions that we are considering here have been expressed as functions of $|\lambda_f|$ and $\arg(\lambda_f) \equiv \phi = \phi_{MIX} - 2\phi_{CP}$, and the fit is performed keeping $|\lambda_f| = 1$ and allowing $\arg(\lambda_f)$ to vary. The same results are obtained when also $|\lambda_f|$ is also allowed to vary in the fit. It is important to mention that a measurement of $\lambda_f \neq 1$ in an experiment would be a signature of direct CP violation [5].

3˙1. *SuperB at the* $\Upsilon(4S)$. – The Super$B$ Collaboration is expected to start taking data in 2017 [11-14], and the integrated luminosity which will be achieved with the full program is expected to be $75\,\mathrm{ab}^{-1}$. With this luminosity one would expect to reconstruct 6.6×10^6 tagged $D^0 \to \pi^+\pi^-$ events in a data sample of $75\,\mathrm{ab}^{-1}$ with a purity of 98% [5]. The results of the numerical analysis are shown in fig. 2.

The asymmetry parameters determined here have a precision of $\sigma_{arg(\lambda_{\pi\pi})} = \sigma_{\phi_{\pi\pi}} = 2.2°$. The same procedure when applied to the $D^0 \to K^+K^-$ channel to measure $\sigma_{arg(\lambda_{KK})} = \sigma_{\phi_{KK}}$, for which one would expect to reconstruct 1.8×10^7 events, leads to precision of $\sigma_{\phi_{KK}} = 1.6°$. When the results from $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ are combined one obtains a precision in $\beta_{c,eff}$ of $\sigma_{\beta_{c,eff}} = 1.4°$.

3˙2. *SuperB at the* $\Psi(3770)$. – The Super$B$ Collaboration is planning to have a dedicated run at the center-of-mass energy of the $\Psi(3770)$ resonance, to collect an integrated luminosity of $1.0\,\mathrm{ab}^{-1}$. With this luminosity one would expect to record 979000 $D^0 \to \pi^+\pi^-$ reconstructed events, when the full set of semi-leptonic decays $K^{(*)}\ell\nu_\ell$ $\ell = e, \mu$ is used to tag the flavor of D^0 mesons (with negligible mistag probability). The results of the numerical analysis are shown in fig. 3.
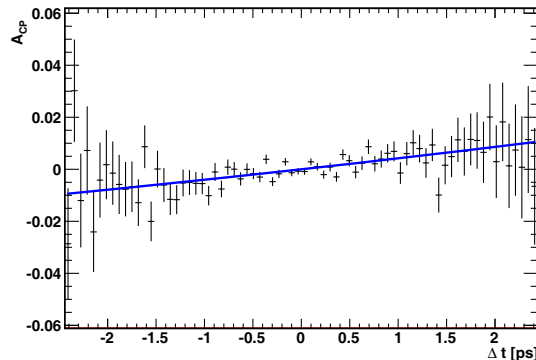
Fig. 3. – The time-dependent CP asymmetry expected for $D^0 \to \pi^+\pi^-$ decays in $1\,\mathrm{ab}^{-1}$ sample of data at the $\Psi(3770)$.

The phase $\phi_{\pi\pi}$ could be measured with a precision of $\sigma_{\phi_{\pi\pi}} = 5.7°$. One may also consider using hadronically tagged events, for example, $D^0 \to K^- X$ $(K^+ X)$, where X is anything, which corresponds to 54% (3%) of all D^0 meson decays from which one would expect $\omega \simeq 0.03$, and that the asymmetry in particle identification of K^+ and K^- in the detector will naturally lead to a small, but non-zero value of $\Delta\omega$. We expect that there would be approximately 4.8 million kaon tagged $D^0 \to \pi^+\pi^-$ events in $1.0\,\mathrm{ab}^{-1}$ at charm threshold. Using these data alone, one would be able to measure $\phi_{\pi\pi}$ to a precision of 2.7°. Hence if one combines the results from semi-leptonic and kaon tagged events, a precision of $\sigma_{\phi_{\pi\pi}} \sim 2.4°$ is achievable.

3$^.$**3.** *LHCb*. – Another possible scenario is that of measuring time-dependent asymmetries from uncorrelated D mesons in a hadronic environment, in particular the LHCb experiment. Here dilution and background effects will be larger than those at an e^+e^- machine, but the data are already available and it would be interesting to perform the time-dependent analysis, especially after the recent results on time integrated CP violation in ref. [4]. As already mentioned a measurement of $|\lambda_f| \neq 1$ will signify direct CP violation. Given that the measurement of λ_f is likely expected to be dominated by uncertainties, especially in ω and $\Delta\omega$, it is not clear what the ultimate precision obtained from LHCb will be. The best way to ascertain this would be to perform the measurement on the existing data set. We have estimated that LHCb will collect 4.9×10^6 D^* tagged $D^0 \to \pi^+\pi^-$ decays in $5\,\mathrm{fb}^{-1}$ of data, based on the $0.62\,\mathrm{fb}^{-1}$ of data shown in [4], and we consider also the outcome of a measurement for $1.1\,\mathrm{fb}^{-1}$ (equivalent to 0.7×10^6 D^* tagged $D^0 \to \pi^+\pi^-$ decays) already available after the 2011 LHC run. In [5] we estimate a purity of $\simeq 90\%$ and $\omega \simeq 6\%$ which results in the asymmetry obtained in fig. 4 for $5\,\mathrm{fb}^{-1}$ of data.

This fit is translated into a potential measurement of the phase $\phi_{\pi\pi}$ with a precision of 3.0°. With $1.1\,\mathrm{fb}^{-1}$ of data we estimate that LHCb may be able to reach a precision of 8° on $\phi_{\pi\pi}$.

3$^.$**4.** *Belle II*. – The last scenario considered here is that of Belle II with $50\,\mathrm{ab}^{-1}$ of data collected at the center-of-mass energy of the $\Upsilon(4S)$ [15]. We have considered the same efficiency and mistag probability as for Super$B$ and we expect that 4.4×10^6 D^* tagged $D^0 \to \pi^+\pi^-$ will be collected. The resulting asymmetry is shown in fig. 5. The precision on $\phi_{\pi\pi}$ obtained for this scenario is estimated to be 2.8°.
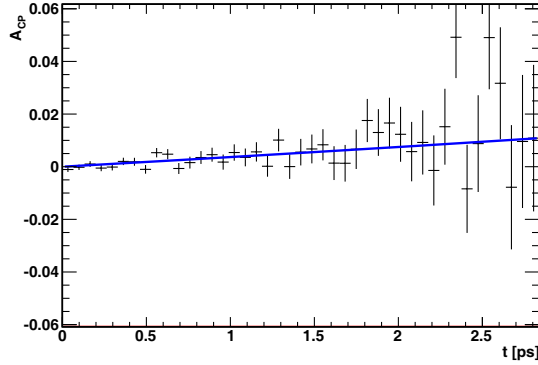
Fig. 4. – The time-dependent CP asymmetry expected for $D^0 \to \pi^+\pi^-$ decays in a $5\,\mathrm{fb}^{-1}$ sample of data at LCH*b*.

4. – Time-dependent sensitivity studies

4˙1. *Sensitivity to* x. – We consider the same data sample discussed in the previous sections for $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$. While we find that results from the time-dependent analysis are not sensitive to the parameter y, and that with $1.0\,\mathrm{ab}^{-1}$ of data collected at charm threshold at Super$B$ it will be possible to improve the currently known precision on x by a factor of two with respect to the most recent HFAG values [16]. The precision that could be reached is shown in table I.

4˙2. *Sensitivity to* $\beta_{c,eff}$, ϕ_{MIX} *and* ϕ_{CP}. – We show in table II a summary of the possible sensitivities that the different experiments could achieve when measuring the mixing and the weak phase.

At first order the decay $D^0 \to K^+K^-$ measure the mixing phase, therefore one can consider $\phi_{KK} = \arg(\lambda_{KK}) = \phi_{MIX}$ and use the time dependent analysis to measure it to a precision of $\approx 1.4°$–$1.6°$.

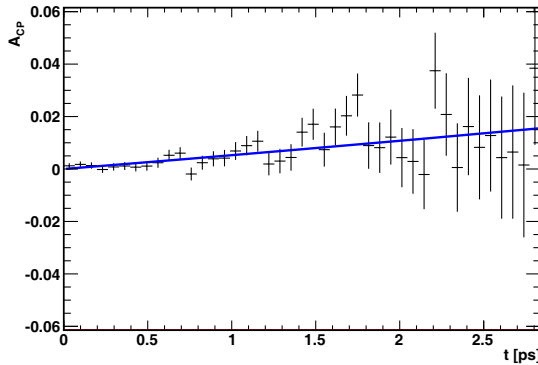Fig. 5. – The time-dependent CP asymmetry expected for $D^0 \to \pi^+\pi^-$ decays in a $50\,\mathrm{ab}^{-1}$ sample of data at the $\Upsilon(4S)$ at Belle II.

TABLE I. – *Estimates of the sensitivity on x for all the experimental scenarios and their projected luminosities for the decays $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ and $\phi = \phi_{MIX} - 2\beta_{c,eff}$.*

Experiment/HFAG	$\sigma_x(\phi = \pm 10°)$	$\sigma_x(\phi = \pm 20°)$
Super$B$ [$\Upsilon(4S)$]		
$D^0 \to \pi^+\pi^-$	0.12%	0.06%
$D^0 \to K^+K^-$	0.08%	0.04%
Super$B$ [$\Psi(3770)$]		
$D^0 \to \pi^+\pi^- (SL)$	0.30%	0.15%
$D^0 \to \pi^+\pi^- (SL+K)$	0.13%	0.06%
$D^0 \to K^+K^- (SL)$	0.19%	0.10%
$D^0 \to K^+K^- (SL+K)$	0.08%	0.04%
LHCb		
$D^0 \to \pi^+\pi^-$ (1.1 fb^{-1})	0.40%	0.20%
$D^0 \to K^+K^-$ (1.1 fb^{-1})	0.22%	0.11%
$D^0 \to \pi^+\pi^-$ (5.0 fb^{-1})	0.15%	0.08%
$D^0 \to K^+K^-$ (5.0 fb^{-1})	0.09%	0.04%
Belle II		
$D^0 \to \pi^+\pi^-$	0.14%	0.07%
$D^0 \to K^+K^-$	0.10%	0.04%
HFAG		0.20%

4˙3. *Systematic uncertainties*. – The knowledge of the parameters x and y which define the mixing is limited by their relative uncertainties. Since our analysis is not sensitive to the parameter y, we considered the most recent results from the HFAG [16] and we evaluated the effect of varying the parameter $\Delta\Gamma = 2y\Gamma$ considering plus-and-minus one standard deviation. This is the systematic uncertainty due to the limited precision in y.

TABLE II. – *Summary of expected uncertainties from 1 ab^{-1} of data at charm threshold, 75 ab^{-1} of data at the $\Upsilon(4S)$, 5 fb^{-1} of data from LHCb, and 50 ab^{-1} of data at the $\Upsilon(4S)$ at Belle II for $D^0 \to \pi^+\pi^-$ decays. The column marked SL corresponds to semi-leptonic tagged events, and the column SL+K corresponds to semi-leptonic and kaon tagged events at charm threshold. The last row shows the precision in $\beta_{c,eff}$ expected from a simultaneous fit to $\pi\pi$ and KK where we assume that, for KK, the decay is dominated by a tree amplitude.*

Parameter	Super$B$ $\Psi(3770)$ SL	Super$B$ $\Psi(3770)$ SL+K	$\Upsilon(4S)$ $\pi_s^\pm$	LHCb $\pi_s^\pm$	Belle II $\pi_s^\pm$
$\sigma_{\phi_{\pi\pi}} = \sigma_{arg(\lambda_{\pi\pi})}$	5.7°	2.4°	2.2°	3.0°	2.8°
$\sigma_{\phi_{KK}} = \sigma_{arg(\lambda_{KK})}$	3.5°	1.4°	1.6°	1.8°	1.8°
$\sigma_{\beta_{c,eff}}$	3.3°	1.4°	1.4°	1.9°	1.7°

TABLE III. – *Summary of expected systematic uncertainties due to the limited knowledge of the parameter y from* $1\,\mathrm{ab}^{-1}$ *of data at charm threshold and* $75\,\mathrm{ab}^{-1}$ *of data at the* $\Upsilon(4S)$. *The column marked SL corresponds to semi-leptonic tagged events, and the column SL+K corresponds to semi-leptonic and kaon tagged events at charm threshold while* $\pi_s^{\pm}$ *refers to the slow pion tag at the* $\Upsilon(4S)$.

Parameter	$\Psi(3770)$ SL	$\Psi(3770)$ SL+K	$\Upsilon(4S)$ $\pi_s^{\pm}$
$\sigma_{\phi_{\pi\pi}}$ (sys.)	$0.5°$	$0.2°$	$0.05°$
$\sigma_{\phi_{KK}}$ (sys.)	$0.2°$	$0.1°$	$0.02°$
$\sigma_{\beta_{c,eff}}$ (sys.)	$0.27°$	$0.11°$	$0.03°$

The value of the uncertainty in the parameter y is 0.013% and it is given in [16]. The results are shown in table III.

4˙4. *Combined results for SuperB*. – We evaluated the combination of the results obtained for the different centre-of-mass energy at Super$B$. The final results are made on the assumption that $\phi = \phi_{MIX} - 2\beta_c = \pm 10°$ and they are shown in table IV.

5. – Conclusions

This paper elucidates the time-dependent analysis of the D^0 mesons discussed in ref. [5]. We concentrated on the possible measurement of the $\beta_{c,eff}$ angle of the charm unitarity triangle, on the mixing phase ϕ_{MIX} and on the mixing parameters. We estimate our results and compare them for the experimental environments that we think could and should perform this analysis: Super$B$, LHCb and Belle II. We found that Super$B$ may perform better this analysis, but time is required before the collaboration will start data taking. LHCb will have to control the background levels to perform this measurement resulting then in a challenging analysis. However as referred to in the article the LHCb Collaboration has already available an amount of data to analyse. This same amount of data has already shown a first hint of direct CP violation in charm, we think it would be worth going through the time-dependent formalism. The Belle II Collaboration will start data taking in few years, and the background-clean environment will allow to perform a time-dependent analysis and an evaluation of the mixing phase and of the $\beta_{c,eff}$ at high precision.

TABLE IV. – *Combined sensitivities at SuperB.*

Parameter	Statistical sensitivity	Systematic sensitivity
σ_x ($D^0 \to \pi^+\pi^-$)	0.09%	–
σ_x ($D^0 \to K^+K^-$)	0.05%	–
$\sigma_{\phi_{\pi\pi}}$	$1.62°$	$0.14°$
$\sigma_{\phi_{KK}}$	$1.05°$	$0.02°$
$\sigma_{\beta_{c,eff}}$	$0.92°$	$0.03°$

* * *

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