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Asymptotia for total and elastic pp and $\bar{p}p$ cross-sections

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Summary. — We discuss recent LHC data for total and elastic pp scattering collected at $\sqrt{s} = 7\,\text{TeV}$ through the asymptotic properties of the scattering amplitude, such as saturation of the Froissart bound and the black-disk limit. A simple model with two exponentials and a phase is used to describe the elastic differential cross-section for both pp and $\bar{p}p$ and test two asymptotic rules derived from the hypothesis of total absorption.

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PACS 13.85.Dz – Elastic scattering.
PACS 13.85.Lg – Total cross sections.

1. – What is asymptotia in hadronic collisions?

LHC has recently provided new information about hadron-hadron scattering through the $\sqrt{s} = 7\,\text{TeV}$ (LHC7) measurement of the total and elastic differential cross-section by the TOTEM experiment [1, 2]. Thus, the question has been posed whether we have now observed the asymptotic regime in hadronic collisions [3], predicted for the elastic amplitude $F(s,t)$ at very high energies. The main asymptotic predictions discussed here concern:

- The Froissart-Martin bound [4, 5], *i.e.* $\sigma_{total} \lesssim \log^2 s$
- The black-disk limit, $\mathcal{R} = \sigma_{elastic}/\sigma_{total} = 1/2$, for the ratio of the elastic to the total cross-section [6]
- The asymptotic vanishing of $\rho(s,0) = \Re e F(s,0)/\Im m F(s,0)$ [7]
- Total absorption for the elastic amplitude in impact parameter space [8, 9].

2. – Tests of asymptotia

We start by discussing the energy-behaviour of the total cross-section. While ref. [3] would assert that the present TOTEM measurement indicates a saturation of the Froissart bound, the phenomenology resulting from our QCD model with soft-gluon resummation [10] leaves some space for further rise.

A mini-jet model for the total cross-section. Mini-jet models for the total cross-section derive the rise with energy through QCD phenomenological quantities such as low-p_t jet cross-sections, aka mini-jets, *i.e.* jets with $p_t > p_{tmin}$, with $p_{tmin} \gtrsim 1\,\text{GeV}$. Unitarity is then insured by embedding the mini-jet cross-sections into the eikonal formalism. Such formalism requires a model for the impact parameter dependence. In the soft gluon resummation model we have developed through the years [10], the impact parameter dependence is obtained as the Fourier transform of the resummed soft gluon spectrum, as

$$\sigma_{total} = 2\int \mathrm{d}^2\mathbf{b}[1-\exp(-1/2(\bar{n}_{soft}(b,s)+A_{hard}(b,s)\sigma_{jet}(s,p_{tmin})))], \tag{1}$$

$$A_{hard}(b,s) = \mathcal{N}\exp[-\frac{16}{3\pi}\int \frac{\mathrm{d}k_t}{k_t}\alpha_{eff}(k_t)\log\frac{2q_{max}}{k_t}(1-J_0(k_t b))], \tag{2}$$

where $\mathcal{N}$ is a normalization factor and we propose a phenomenological ansatz for the single soft gluon spectrum which allows the extension of the k_t integration below the usual cut-off of Λ_{QCD} such as

$$\alpha_{eff}(k_t) = \frac{12\pi}{11N_c - 2N_f}\frac{p}{\log[1+p(k_t/\Lambda_{QCD})^{2p}]} \tag{3}$$

with $1/2 < p < 1$. Using LO parton density functions (PDFs) with LO parton-parton cross-sections, and parametrizing the low-energy part of the total cross-section through $\bar{n}_{soft}$, we find that the TOTEM value is obtained at the upper edge of a band, corresponding to MRST72 densities, $p_{tmin} = 1.25\,\text{GeV}$ and $p = 0.66$ [11]. Considering that the parameter p in our model is related to the asymptotic behaviour [12] as $\sigma_{total} \sim [\log s]^{1/p}$ we conclude that, at LO in our QCD description, data up to the TOTEM measurement are consistent with $\sigma_{total} \sim \log^{3/2} s$ [11]. Within this model then, and unlike the analysis of [3], the Froissart limit is not yet reached. Higher order corrections to the model, may of course modify this result.

The black disk and the Pumplin limit. To estimate the value of $\mathcal{R}_{el} = \sigma_{elastic}/\sigma_{total}$ at $\sqrt{s} = 57\,\text{TeV}$, we use the Auger Collaboration value for σ_{inel} extracted from their recent measurement for the p-air cross-section [3,13] and the total cross-section estimate by Block and Halzen [3] at $\sigma^{\text{BH}}_{\text{total}}(57\,\text{TeV}) = (134.8 \pm 1.5)\,\text{mb}$. We then obtain [14] $\sigma_{\text{elastic}}(57\,\text{TeV}) = \sigma^{BH}_{\text{total}} - \sigma^{\text{Auger}}_{\text{inelastic}} = (44.8 \pm 11.6)\,\text{mb}$ and $\mathcal{R}_{el}(57 \pm 6\,\text{TeV}) = 0.33 \pm 0.09$. Comparing $\mathcal{R}_{el}$ with the black disk limit $\mathcal{R}_{el} = 1/2$ and with accelerator data up to and including LHC and the Auger result, we find that, even at such ultra high energies, the black disk limit has not been reached yet. Notice that the Pumplin limit [15] predicts $(\sigma_{elastic} + \sigma_{diffractive})/\sigma_{total} \rightarrow 1/2$, as also discussed in [16].

The ρ parameter. According to the Khuri-Kinoshita theorem [17] asymptotically $\rho(s,0) = \Re e F(s,0)/\Im m F(s,0) \sim \pi/\log(s/s_0)$, *i.e.* $\rho \to 0$. Our mini-jet model provides an expression for $\rho(s,0)$ in agreement with the asymptotic theorems, and a value, at LHC7, consistent with estimates used by the TOTEM collaboration. At high energies *and very small t values*, crossing symmetry implies that the leading C-even amplitude is a function of the complex variable $se^{-i\pi/2}$. To include both the case of saturation of the Froissart bound and a slower rise compatible with $\ln s$ behaviour, we can asymptotically write

$$F(s,0) \sim i\left[\ln\left(\frac{s}{s_0}e^{-i\frac{\pi}{2}}\right)\right]^{1/p} = i\left[\ln\left(\frac{s}{s_0}\right) - i\frac{\pi}{2}\right]^{1/p} \to i\left[\ln\left(\frac{s}{s_0}\right)\right]^{1/p}\left[1 - \frac{i\pi}{2p\ln(s/s_0)}\right] \tag{4}$$

with $1/2 \le p \le 1$. The above equation gives an estimate for the leading contribution to the parameter $\rho(s,0)$, namely $\rho(s,0) = \frac{\Re e F(s,0)}{\Im m F(s,0)} \approx \frac{\pi}{2p\ln(s/s_0)}$, where the approximate sign refers to the fact that the real part of the amplitude can receive also (non-leading) contributions from other terms. The Khuri-Kinoshita result, valid when the Froissart bound is saturated, is obtained for $p = 1/2$. Using the results from our phenomenology of the total cross-section, we have $p = 0.66$ and hence $\rho = 0.134$ at LHC7.

Asymptotic sum rules in b-space. Consider the elastic scattering amplitude $F(s,t)$ normalized so that

$$\sigma_{\text{total}}(s) = 4\pi\, \Im m\, F(s,0), \tag{5}$$

$$F(s,t) = i\int_0^\infty (b\mathrm{d}b) J_0\left(b\sqrt{-t}\right)\left[1 - e^{2i\delta_R(b,s)}e^{-2\delta_I(b,s)}\right]. \tag{6}$$

From the Froissart bound, one can see [8] that there must exist a finite angular momentum value, *below* which all partial waves are absorbed. Under the stronger hypothesis of total absorption as $b \to 0$ in the ultra high energy limit, namely all "low-b" waves are completely absorbed, we have two Asymptotic Sum Rules, *i.e.*

$$SR_1 = \frac{1}{2}\int_{-\infty}^0 (\mathrm{d}t)\Im m F(s,t) \to 1;\ \text{as } s \to \infty, \tag{7}$$

$$SR_0 = \frac{1}{2}\int_{-\infty}^0 (\mathrm{d}t)\Re e F(s,t) \to 0;\ \text{as } s \to \infty. \tag{8}$$

Satisfaction of these rules is a good measure of whether the asymptotic limit has been reached, and would reinforce the statement [3] based on the TOTEM data for total cross section, that we may have reached asymptotia. Notice that the Froissart-Martin bound is obtained under a hypothesis weaker than total absorption [8] and it would lead to $SR_1 \to 2$; $SR_0 \to 0$. As we shall see, asymptotically eq. (7) is phenomenologically favoured.

3. – A model to check asymptotia

To study whether the two sum rules are satisfied at LHC7, we have used a simple and quite old model, proposed in 1973 by Barger and Phillips [18] (BP model). This model,

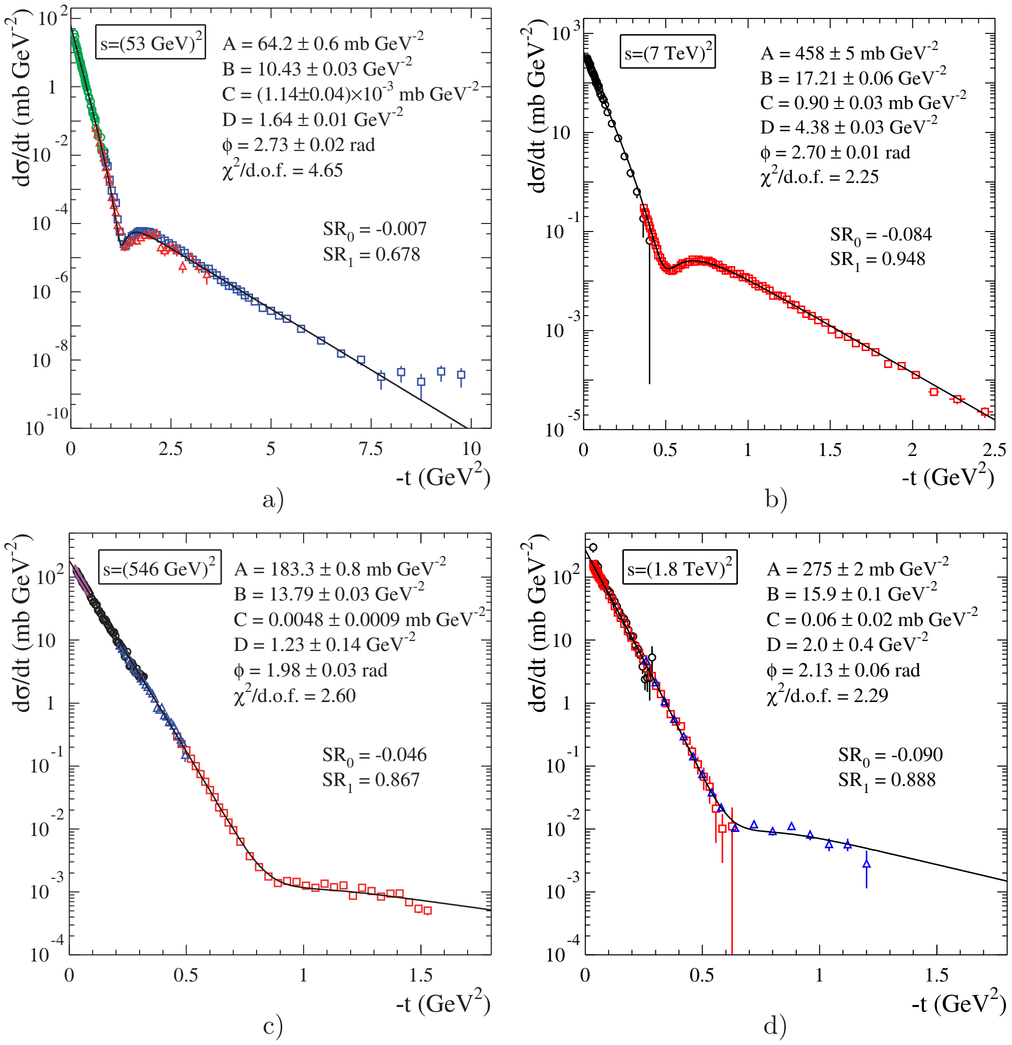

Fig. 1. – a) ISR data [19] *vs.* the BP model [18]. b) LHC7 data read from [2] and BP model. c) $S\bar{p}pS$ data [20] and BP model. d) TeVatron data [21] and the BP model.

consisting of two terms, with a relative phase, is the simplest one can use to describe data in the range before, through and after the dip, or shoulder, in the elastic cross-section. With the elastic scattering amplitude written as

$$\mathcal{A}(s,t) = i\left[\sqrt{A(s)}e^{\frac{1}{2}B(s)t} + \sqrt{C(s)}e^{i\phi(s)}e^{\frac{1}{2}D(s)t}\right], \tag{9}$$

a five-parameter fit to pp data at $\sqrt{s} = 53\,\text{GeV}$, $7\,\text{TeV}$ and to $\bar{p}p$ at $\sqrt{s} = 546$ and $1800\,\text{GeV}$ gave the results shown, respectively, in figs. 1a), b), and c), d). In the approximation of this 5-real-parameter fit, we find SR_0 being always negative since $\pi/2 < \phi < \pi$, while SR_1 is seen to approach the asymptotic value, $SR_1 = 1$, at LHC7.

4. – Conclusions

We have presented a study of pp and $\bar{p}p$ scattering focused on predictions about asymptotic behaviour. A simple model, with two exponentials and a phase, is used to check two asymptotic sum rules based on the hypothesis of total absorption at impact parameter zero. Such a simple model describes well the TOTEM data and we propose to apply such parametrization to the elastic differential cross-section data at the higher energies to be reached at LHC.

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