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^{209}Bi level calculation

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Summary. — ^{209}Bi has recentely observed to α-decay with scintillating bolometer, to the ground state (GS) and to first the excited state (ES). although of the belief it was a stable nuclei. Even previous measurement observed the decay, but only the GS-GS transition. This decay belongs to a special class of decays occurring in odd-A nuclei with an extra nucleon outside a closed shell. ^{209}Bi alpha decay branching ratios (BR) to the ground and to two different excited states were numerically evaluated. While the BR for the ground-state decay is well known, the calculation for the excited states was performed for the first time. Even if the used model is a simplified theory (not strictly correct for deformed nuclei), it gives a good estimate of the decay half life.

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1. – Calculation

Although a great amount of research has been carried out on *favored* α-decays, occurring between the ground states of even-even nuclei, studies on *hindered* transitions are relatively few. Importantly there are rare investigations on a special class of α-decays, *i.e.*, those occurring in the ground states of odd-A nuclei with an extra nucleon outside a closed shell [1-3]. The α particle carries an odd angular momentum in this case, due to the different parity of parent and daughter nuclei. The newly discovered ^{209}Bi α-decay of is one of this class [4, 5]. For the calculation here presented the classical quantum-mechanical theory, developed in 1928 (Gamow and Gunrey and Condon), was used. The corrections applied to the bare formula (which works well only for spherical nuclei) are two:

– a centrifugal potential is added, which accounts for the different angular momentum variations ΔL between the decay to the ground and to the excited states, and which clearly modifies the Branching Ratios (BR);

– the not sharp nuclear dimensions are accounted considering the Wood-Saxon potential, assuming as input parameters $a = 0.75\,\text{fm}$ ans $V_0 \sim 25\,\text{MeV}$.

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TABLE I. – *Levels calculation for the* ^{209}Bi α*-decay.*

BR	J^P	L_α	$\tau_{1/2}[y]$	Q [MeV]
0.987	$1/2^+$	5	$2.06 \cdot 10^{19}$	3.14
0.011	$3/2^+$	3	$1.81 \cdot 10^{19}$	2.93
0.002	$3/2^+$	5	$9.52 \cdot 10^{21}$	2.93
$6.12 \cdot 10^{-9}$	$5/2^+$	7	$3.32 \cdot 10^{27}$	2.53
$1.14 \cdot 10^{-9}$	$5/2^+$	5	$1.79 \cdot 10^{27}$	2.53
$1.06 \cdot 10^{-10}$	$5/2^+$	3	$1.92 \cdot 10^{27}$	2.53

The nucleus shape has effects on several parameters, as the turning points, the effective nuclear radius R_N and the bouncing frequency. In order to reproduce the observed ^{209}Bi half life and to include the deformation with the simplest and direct approach, I performed a "fine-tuning" in the calculation, trying to find the correct R_N which reproduces the measurements. The final result is that, as input parameter, I found $R_N = 8.41\,\text{fm}$, while the value obtained with the classical formula $R_N = r_0 \cdot A^{1/3}$ would be 7.12 fm. In order to evaluate the barrier potential tunneling probabilities a Quantum Mechanics calculation is performed. The trasmission coeffcient G is first evaluated (I took the natural logarithm of the penetration factor P to be equal to twice the WKB, Wentzel-Kramers-Brillouin, integral)

$$G = -\int_{R_1}^{R_0} \sqrt{\frac{2m}{\hbar^2}} \cdot \left[\frac{2Ze^2}{2\pi r} + \sqrt{\frac{\hbar^2 l(l+1)}{r}}^{2} + \frac{V_0}{(1 + e^{\frac{r-R}{a}})} - Q \right] \mathrm{d}r \tag{1}$$

between the inner and outer classical turning points, where the integrand vanishes. Here m is the mass of the alpha particle, Ze is the charge on the daughter nucleus, l is the orbital angular momentum of the emitted alpha, and Q the Q-value for the transition. The integrations were carried out numerically. Then the probability trasmission trhough the barrier is $P = e^{-2G}$ and finally the frequency with which the α-particle presents itself at the barrier is $\nu = v/R_N = \hbar/m_\alpha \cdot R_N^2$. The most critical parameters are indeed the nuclear factors and the bouncing frequency ν. Even thought this oversimplified theory is not strictly correct, it gives a good estimate of the decay half life (see table I). The result from the calculation performed (GS-GS) is $\tau_{1/2} = 2.06 \cdot 10^{19}\,\text{y}$, while the measured half-life is $\tau_{1/2} = (2.01 \pm 0.08) \cdot 10^{19}\,\text{y}$.

From the comparison between calculated and measured data there is a good agreement for the half life and BR studied.

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