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Zero-point fluctuations in rotation: Perpetuum mobile of the fourth kind without energy transfer

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Summary. — In this paper we discuss a simple Casimir-type device for which the rotational energy reaches its global minimum when the device rotates about a certain axis rather than remaining static. This unusual property is a direct consequence of the fact that the moment of inertia of zero-point vacuum fluctuations is a negative quantity (the rotational vacuum effect). Moreover, the device does not produce any work despite the fact that its equilibrium ground state corresponds to a permanent rotation. Counterintuitively, the device has no internally moving mechanical parts while its very existence is consistent with the laws of thermodynamics. We point out that such devices may possibly be constructed using carbon nanotubes subjected to the strong—but experimentally feasible—magnetic field. The effect in the carbon nanotubes is enhanced due to the presence of massless charged excitations. We call this "zero-point–driven" device as the perpetuum mobile of the fourth kind.

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1. – Introduction

Perpetual motion: *Motion that continues indefinitely without any external source of energy; impossible in practice because of friction.* (Webster online dictionary [1]).

In the quoted Webster's definition, the moving object is assumed to be coupled to (or, in other words, interacting with) an external environment, otherwise the perpetual nature of the object's motion is a trivial and inevitable consequence of the energy conservation

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law. Due to the interaction with the environment, a moving (say, rotating) object will sooner or later release all its (rotational) kinetic energy to the surrounding media. The gradual loss of the rotational energy can be described by the precense of a friction force acting on the object from the environment. As a consequence, the object will ultimately stop its rotational motion coming to rest; hence the verdict "impossibility in practice" in Webster's definition.

In this paper, following refs. [2,3], we point out, that the perpetual motion may exist. The device, which we call "Perpetuum mobile of the fourth kind" [3], is suggested to have the following physical properties:

– It is a large object made of the Avogadro-scale ($N \sim 10^{23}$) constituent particles.

– It is a solid body which, counterintuitively, has no internally moving parts.

– It is made of a material, which possesses propagating (preferably, massless) elementary excitations.

– Its lowest-energy state corresponds to a uniform rotation of the device as a whole about a certain axis with a certain optimal angular frequency (a non-rotating device has a higher energy compared to its energy in the optimally rotating state).

– The unusual non-monotonic dependence of the rotational energy on the angular frequency are due to zero-point (vacuum) fluctuations of the elementary excitations.

Thus, we define the perpetuum mobile of the fourth kind as a solid macroscopic device for which the lowest-energy state corresponds to a uniform rotation driven by the zero-point fluctuations without any energy transfer.

The energy properties of our perpetuum mobile are very counterintuitive, because in order to stop the rotation of the perpetuum mobile we should invest (not gain!) some amount of work in this process because the cessation of rotation increases (not decreases) the rotational kinetic energy of the device. In an elementary act of interaction of the rotating device with an external environment (say, with a molecule of a gas), the device

– cannot loose energy and stop rotating because the rotating device is already residing in its lowest possible energy state;

– can gain energy and stop rotating.

Below, for the sake of brevity, we will sometimes call the perpetuum mobile of the fourth kind as "the device".

An alternative "time crystal" [4, 5] approach to the permanent motion was inspired by the persistency of the electric currents in superconducting rings (or, alternatively, one can think of the persistently flowing superfluid helium in a closed ring-shaped pipe). Due to specific properties of the relevant excitation spectra, the energy exchange between the superconducing (superfluid) and dissipative components of the corresponding systems is absent, so that the electric current (superfluid flow) is running forever. Mathematically, the "time crystal" is an exotic classical, semiclassical or quantum-mechanical system of moving particles which is described by either singular Hamiltonians involving multiple powers of time derivatives [4], or by Hamiltonians which involve time non-local terms [5]. This suggestion is both controversial [6] and inspiring. Recently, a very interesting "space-time crystal" system was suggested to be realised at cryogenic temperatures in a form a permanently flowing current of cold ions [7].

Contrary to the "time crystal" approach, our alternative approach—which we call "the perpetuum mobile of the fourth kind" [2, 3]—is based on standard nonsingular class of local free (!) Hamiltonians of relatively simple solid state systems (and may, presumably, be realised using doped torus-shaped carbon nanotubes [3]).

2. – Negative moment of inertia of zero-point fluctuations

Let us assume for a moment that the perpetuum mobile of the fourth kind (described in the introduction) does exist: this large macroscopic device rotates permanently in its ground (lowest) state without producing any work. The necessary condition of the permanent rotation is that the non-rotating state should have a higher energy compared to the optimally rotating state. What should be a driving mechanism behind this unusual energy behaviour? In the present section we are sketching the answer to this question.

Let us consider a large macroscopic rigid body of an arbitrary shape which rotates about one of its a principal axis of inertia with the angular frequency Ω. For the sake of simplicity let us consider a slow, non-relativistic rotation. Then the classical kinetic energy of rotation is a quadratic function of the angular frequency:

$$E_{\rm cl}(\Omega) = \frac{I_{\rm cl}\Omega^2}{2}\,, \tag{1}$$

where $I^{\rm cl}$ is the corresponding classical principal moment of inertia. The energetically favourable state, $E^{\rm cl}_{\rm rot} = 0$, corresponds to the absence of rotation, $\Omega = 0$.

Thus, the classical formula (1) seems to imply that that the permanent rotation should not, in general, correspond to a lowest energy state. However, there is a hidden assumption which has led us to this mundane conclusion: We have implicitly assumed that the moment of inertia of the rigid body is a positive quantity:

$$I_{\rm cl} > 0. \tag{2}$$

Can we question the validity of eq. (2)? At the first sight it seems that the answer is "no" as the positiveness condition (2) is a very natural fact for any classical system. For example, consider a point mass m rotating about a certain axis at the fixed distance R (one can imagine that the point is attached to the axis by a massless rod). The rotational kinetic energy of the system is given by eq. (1), where the moment of inertia is

$$I_{\rm cl} = mR^2. \tag{3}$$

The classical mass m and the distance R are positive quantities so that the moment of inertia (3) is, naturally, a positive quantity too (2). Since any classical body can be considered as a superposition of point masses, the positivity condition (2) for all extended classical bodies is also an obvious fact.

Nevertheless, in refs. [2,3] we claim that there may exist certain systems which have a negative moment of inertia for, at least, small angular frequencies. These systems contain, generally, two interacting (classical and quantum) subsystems. The classical subsystem is a rigid body which has a positive moment of inertia. The quantum subsystem is described by zero-point fluctuations of quantum excitations which have, in general, a negative moment of inertia (the latter is the essence of the "rotational vacuum effect" [2]).

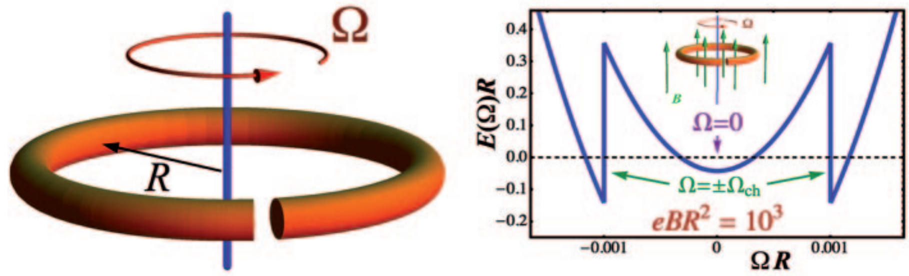

Fig. 1. – Left: The toy device which shows that zero-point vacuum fluctuations have a negative moment of inertia. A free massless scalar field may propagate along the infinitesimally thin ring, while the cut of the ring imposes a Dirichlet boundary condition on this field (from ref. [3]). Right: in the magnetic field background B the total (classical plus zero-point) rotational energy of the ring $E = E(\Omega)$ becomes a non-monotonic discontinuous function of the angular frequency Ω. In this example the minimum of the rotational energy (with $E(\Omega_{\rm ch}) < E(0)$) is reached at the nonzero characteristic frequency $\Omega_{\rm ch} \neq 0$, eq. (27); see the discussion in the text.

In order to illustrate our idea in general, and the rotational vacuum effect in particular, we consider a very simple device; see fig. 1(left). This toy device contains:

A) Classical subsystem:

- We consider an infinitely thin (but, generally, massive) ring of the radius R.
- The ring may rotate about its natural symmetry axis.
- The ring has a cut.

B) Quantum subsystem:

- The ring is made of a material which possesses massless (scalar) excitations.
- The excitations are free (they do not interact with each other).
- The excitations may propagate along the ring.
- The excitations cannot propagate through the cut of the ring.

C) The classical and quantum subsystems are coupled to each other thanks to the cut.

For the sake of simplicity, we assume that the cut of the ring is infinitesimally thin while the tunnelling of excitations from one side of the cut to another side is absent.

Our next logical steps are as follows:

1) It is easy to show that the zero-point (vacuum) fluctuations of the excitations in the ring have a negative energy (called, in general, the Casimir energy [8]).

2) It turns out that zero-point fluctuations do indeed have a negative moment of inertia ("the rotational vacuum effect") as was rigorously demonstrated in refs. [2, 3] and confirmed in ref. [9].

3) The negative moment of inertia implies that the rotational energy of the zero-point vacuum fluctuations decreases as the angular frequency Ω of the device increases.

4) However, the rotational vacuum effect—in its simplest formulation—is very small [2]: the (negative) zero-point moment of inertia constitutes a small fraction of the (positive) classical moment of inertia of the device itself.

5) Nevertheless, the rotational vacuum effect may be enhanced if

   a) the scalar excitations in the ring are electrically charged; and

   b) the ring is pierced by an external magnetic flux.

6) Due to the enhancement, the negative zero-point rotational energy may overcome the positive classical rotational energy for certain angular frequencies. Then, the lowest-energy state should correspond to a permanently rotating device [3].

Below we will briefly discuss the rotational vacuum effect as well as the possible existence of the perpetuum mobile of the fourth kind following refs. [2, 3].

3. – The rotational vacuum effect in the vacuum of neutral scalar fields

Consider a neutral massless scalar field $\phi = \phi(t, \varphi)$ which is defined on a circle (infinitesimally thin ring) of radius R; see fig. 1(left). The corresponding Lagrangian is

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi \equiv \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{R^2}\left(\frac{\partial \phi}{\partial \varphi}\right)^2\right]. \tag{4}$$

The scalar field is a periodic function $[\phi(t, \varphi{+}2\pi) \equiv \phi(t, \varphi)]$ of the angular coordinate $\varphi \in [0, 2\pi)$. The cut of the ring—see fig. 1(left)—imposes the simplest, Dirichlet boundary condition on the neutral field ϕ (we call the cut as the "Dirichlet cut").

First, let us consider a static, non-rotating circle. The boundary condition at the Dirichlet cut is as follows (the points $\varphi = 0$ and $\varphi = 2\pi$ are identified):

$$\phi(t, \varphi)\Big|_{\varphi=0} \equiv \phi(t, \varphi)\Big|_{\varphi=2\pi} = 0. \tag{5}$$

It is pretty straightforward to evaluate the zero-point (Casimir) energy associated with the vacuum fluctuations of the scalar field ϕ. The local energy density,

$$\bar{\mathcal{E}}(R) = \frac{1}{2\pi R} \sum_{\text{modes}\, m} \varepsilon_m(R), \tag{6}$$

is given by the infinite sum over energies $\varepsilon_m(R)$ of all individual fluctuation modes m. This sum is a divergent quantity both for the circle with a finite radius R and in a free space ($R \to \infty$). However, the difference between these energy densities, $\bar{\mathcal{E}}^{\text{phys}}(R) = \bar{\mathcal{E}}(R) - \bar{\mathcal{E}}(\infty)$, is a finite, experimentally measurable observable which is often called the Casimir energy density [8]. The total energy of the quantum fluctuations is as follows:

$$E^{\text{phys}}(R) = 2\pi R\, \bar{\mathcal{E}}^{\text{phys}}(R) \equiv 2\pi R \left[\bar{\mathcal{E}}(R) - \bar{\mathcal{E}}(\infty)\right]. \tag{7}$$

The local energy density of the quantum fluctuations $\mathcal{E}$ is given by the vacuum expectation value of a time-time component of the stress-energy tensor, $T^{\mu\nu}$:

$$\mathcal{E}(t,\varphi) \equiv \langle T^{00}(t,\varphi)\rangle = \lim_{t'\to t}\lim_{\varphi'\to\varphi}\frac{1}{2i}\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'} + \frac{1}{R^2}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial\varphi'}\right) G(t,t';\varphi,\varphi'), \tag{8}$$

where we have expressed the expectation value of the stress-energy tensor $T^{\mu\nu}$,

$$\langle T^{\mu\nu}(x)\rangle = \left(\partial^\mu\partial'^\nu - \frac{1}{2}g^{\mu\nu}\partial^\lambda\partial'_\lambda\right)\frac{1}{i}G(x,x')\Big|_{x\to x'}, \tag{9}$$

via a Feynman-type Green function [10]: $G(x,x') = i\langle \mathrm{T}\phi(x)\phi(x')\rangle$. Here the symbol "T" stands for the time-ordering operator and $x = (t,\varphi)$.

The Green function $G(t,t';\varphi,\varphi')$ is defined by the following equation:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right) G(t,\varphi;t',\varphi') = \frac{1}{R}\delta(\varphi-\varphi')\delta(t-t'). \tag{10}$$

The Green's function G should vanish at the position of the Dirichlet cut, $\varphi,\varphi' = 0, 2\pi$.

The Green function $G(t,t';\varphi,\varphi')$ can be expressed via the eigenvalues $\lambda_{\omega,m}$ and eigenfunctions $\phi_{\omega,m}$ of the second-order differential operator in the right-hand side of eq. (10):

$$\left(\frac{\partial^2}{\partial t^2} - \frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right)\phi_{\omega,m}(t,\varphi) = \lambda_{\omega,m}\,\phi_{\omega,m}(t,\varphi), \tag{11}$$

where $m = 1,2,3,\dots$ and $\omega \in \mathbb{R}$, and the orthonormal and complete eigensystem is

$$\lambda_{\omega,m} = \frac{m^2}{4R^2} - \omega^2, \qquad \phi_{\omega,m}(t,\varphi) = e^{-i\omega t}\chi_m(\varphi), \qquad \chi_m(\varphi) = \frac{1}{\sqrt{\pi R}}\sin\frac{m\varphi}{2}\,. \tag{12}$$

For the sake of convenience, the real-valued eigenfunctions of eq. (11) with the same eigenvalues are combined into one complex function $\phi \equiv \phi^{(C)}_{\omega,m} - i\phi^{(S)}_{\omega,m}$.

The Green's function is given by the following formula:

$$G(t,t';\varphi,\varphi') = \int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}\sum_{m=1}^{\infty}\frac{\phi_{\omega,m}(t,\varphi)\phi^\dagger_{\omega,m}(t',\varphi')}{\lambda_{\omega,m} - i\epsilon} = \frac{i}{\pi}\mathcal{G}\left(\frac{\varphi}{2},\frac{\varphi'}{2},\frac{|(t-t')|}{2R}\right), \tag{13}$$

where function $\mathcal{G}$ is defined as follows:

$$\mathcal{G}(x,y,z) = \sum_{m=1}^{\infty}\frac{\sin(mx)\sin(my)}{m}e^{-imz} = \frac{1}{4}\ln\frac{\left[1-e^{i(x+y-z)}\right]\left[1-e^{-i(x+y+z)}\right]}{\left[1-e^{i(x-y-z)}\right]\left[1-e^{i(-x+y-z)}\right]}. \tag{14}$$

The infinitesimal term $i\epsilon$ ensures that the Green function (13) is of the Feynman type [10].

Using the explicit representation of the Green's function (13) the (non-regularised and divergent) local energy density (8) can be computed straightforwardly:

$$\langle T^{00}(t,\varphi)\rangle = -\frac{1}{2\pi}\lim_{t'\to t}\frac{1}{(t'-t)^2} - \frac{1}{96\pi R^2}\,. \tag{15}$$

Notice that the energy density (15) depends on neither the angular variable φ nor time variable t. As the divergent piece in eq. (15) does not depend on the radius of the ring, it is easy to get the regularised expression for the zero-point energy $E^{\rm phys}$ and the mean energy density $\bar{\mathcal{E}}^{\rm phys}$ of the scalar field at the non-rotating circle with the Dirichlet cut:

$$E^{\rm phys} \equiv 2\pi R\, \bar{\mathcal{E}}^{\rm phys} = -\frac{1}{48R}\,, \qquad \bar{\mathcal{E}}^{\rm phys} = -\frac{1}{96\pi R^2}\,. \tag{16}$$

The energy vanishes in the limit of the infinitely large circle, $R \to \infty$, as it should be. The result (16) is known as the Casimir energy of the string of the length $l = 2\pi R$ [10,11].

In order to find the moment of inertia of the zero-point vacuum fluctuations let us repeat the above calculations for the case of the rotating ring, fig. 1(left), which rotates uniformly about its axis with the angular velocity Ω. Mathematically, the rotation leads to the following time-dependent boundary condition imposed by the Dirichlet cut(^{1}):

$$\phi(t,\varphi)\Big|_{\varphi=[\Omega t]_{2\pi}} = 0, \tag{17}$$

where $[x]_{2\pi} = x + 2\pi n$, $n \in \mathbb{Z}$, $0 \leqslant [x]_{2\pi} < 2\pi$ denotes the modulo operation with the base of 2π. In the static limit, $\Omega = 0$, the boundary condition (17) reduces to eq. (5).

The Green function in the rotating case should satisfy the general equation (10) and respect the time-dependent boundary conditions (17). Similarly to eq. (13) the Green function can be expressed via the orthonormal and complete eigensystem of the problem (10) and (17) in the laboratory frame:

$$\phi_{m,\omega}(t,\varphi) = \sqrt{\frac{1}{\pi R}} \sin\left[\frac{m}{2}[\varphi - t\Omega]_{2\pi}\right] \exp\left\{-i\omega\left(t - \frac{\Omega R^2\,[\varphi - t\Omega]_{2\pi}}{1-\Omega^2R^2}\right)\right\}, \tag{18}$$

$$\lambda_{\omega,m} = \frac{1-\Omega^2R^2}{4R^2} m^2 - \frac{\omega^2}{1-\Omega^2R^2}\,. \tag{19}$$

The explicit form of the Green function of the rotating ring is as follows [3]:

$$G_\Omega(t,t;\varphi,\varphi') = \frac{i}{\pi}\mathcal{G}\left(\frac{\theta(t,\varphi)}{2}, \frac{\theta(t',\varphi')}{2}, \frac{\big|(1-\Omega^2R^2)(t-t') - \Omega R^2\,[\theta(t,\varphi) - \theta(t',\varphi')]\big|}{2R}\right),$$

where $\theta(t,\varphi) = [\varphi - \Omega t]_{2\pi}$ and the function $\mathcal{G}(x,y,z)$ is given in eq. (14). The corresponding zero-point energy density (8) is given by the following formula:

$$\big\langle T^{00}(t,\varphi)\big\rangle = -\frac{1}{2\pi}\lim_{t'\to t}\frac{1}{(t'-t)^2} - \frac{1+\Omega^2R^2}{96\pi R^2}\,. \tag{20}$$

The energy density (20) of the rotating ring is space-time independent similarly to the energy density of the static ring (15). The divergent piece does not depend on neither

(^{1}) Another characteristics of the zero-point energy, the associated centripetal force, was considered in ref. [12]. We do not calculate this quantity in our approach since the dynamics of the rotating body depends on other quantities, which are the rotational energy, moment of inertia and angular momentum.

the radius of the ring R nor the angular frequency Ω so that it does not contribute to the physical part of the rotational energy density of zero-point (ZP) fluctuations:

$$\mathcal{E}^{\mathrm{ZP}}_{\Omega}(t,\varphi) \equiv \left\langle T^{00}(t,\varphi)\right\rangle^{\mathrm{phys}} = -\frac{1+\Omega^2 R^2}{96\pi R^2}\,. \tag{21}$$

For a static device, $\Omega = 0$, the energy density of the zero-point vacuum fluctuations (21) reduces to the known result (16).

Finally, we arrive at the exact *relativistic* expression for the total zero-point energy of the rotating ring with the cut [2]

$$E^{\mathrm{ZP}}_{\Omega} \equiv R\int_0^{2\pi} \mathrm{d}\varphi\, \mathcal{E}^{\mathrm{ZP}}_{\Omega}(t,\varphi) = -\frac{1+R^2\Omega^2}{48R}\,. \tag{22}$$

Curiously, the Ω-dependent part of the rotational energy of the relativistic zero-point fluctuations (22) has a very simple form resembling the non-relativistic classical expression (1). Result (22) was confirmed in ref. [9] by using a different method.

The moment of inertia of the zero-point vacuum fluctuations is a negative quantity:

$$I^{\mathrm{ZP}} \equiv \frac{\partial^2}{\partial\Omega^2} E^{\mathrm{ZP}}_{\Omega} = -\frac{\hbar R}{24c}\,, \tag{23}$$

where we have restored the Planck constant $\hbar$ and the speed of light c. The energy of the zero-point vacuum fluctuations decreases, as the angular frequency increases. Thus, the classical positivity condition (2) is not valid for the zero-point vacuum fluctuations (23).

Notice that despite the fact that $I^{\mathrm{ZP}} < 0$, an *isolated* device cannot self-accelerate because of the conservation of the angular momentum (the device should exchanges the angular momentum with environment by, say, emitting or absorbing a photon).

Equation (23) provides us with the lowest bound on the moment of inertia for the ring-type devices (fig. 1(left)) driven by the neutral scalar vacuum [9]. The zero-point moment of inertia (23) is an very small quantity, corresponding to a fractional (in terms of $\hbar$) angular momentum [9]. Thus, the positive moment of inertia of a real massive device (3) is expected to be always greater than the negative moment of inertia of the zero-point fluctuations (23).

However, the negative moment of inertia can be drastically enhanced by an external magnetic field B if the massless particles are electrically charged [3]. The corresponding zero-point energy $E^{\mathrm{ZP}}_{\Omega,B}$ and the zero-point angular momentum $L^{\mathrm{ZP}}_{\Omega,B}$,

$$E^{\mathrm{ZP}}_{\Omega,B} \equiv R\int_0^{2\pi} \mathrm{d}\varphi\, \left\langle T^{00}(t,\varphi)\right\rangle = -\bigl[1+6M_{\Omega,B}(M_{\Omega,B}+1)\bigr]\frac{1+\Omega^2R^2}{24R}\,, \tag{24}$$

$$L^{\mathrm{ZP}}_{\Omega,B} \equiv R\int_0^{2\pi} \mathrm{d}\varphi\, \left[x^2\left\langle T^{10}\right\rangle - x^1\left\langle T^{20}\right\rangle\right] = -\bigl[1+6M_{\Omega,B}(M_{\Omega,B}+1)\bigr]\frac{\Omega R}{12}\,, \tag{25}$$

are related to each other by a "classical" relation:

$$L^{\mathrm{ZP}}_{\Omega,B} = \left.\frac{\partial E^{\mathrm{ZP}}_{\Omega,B}}{\partial\Omega}\right|_{\frac{eB\Omega R^3}{1-\Omega^2R^2}\notin\mathbb{Z}}\,, \qquad \text{where} \qquad M_{\Omega,B} = \left\lfloor \frac{eB\Omega R^3}{1-\Omega^2R^2}\right\rfloor \in \mathbb{Z}, \tag{26}$$

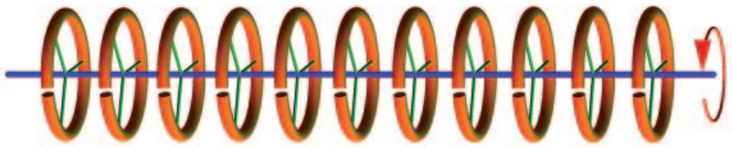

Fig. 2. – A macroscopic permanently rotating device is visually similar to the very first proposal of a metamaterial [16] made of the C-shaped split-ring resonators (from ref. [3]).

provided the angular frequency Ω and the magnetic field B do not correspond to the discontinuities in the integer-valued quantity $M_{\Omega,B}$ (here $\lfloor x \rfloor$ is a largest integer which is smaller than x). In the limit of vanishing magnetic field the charged-field result (24) turns into the neutral-field expression (22) multiplied by a factor of two because the charged (complex) field has two degrees of freedom. In the non-relativistic limit, $\Omega R \ll c$, the first discontinuity of the zero-point energy (24) happens at the characteristic frequency,

$$\Omega_{\rm ch}(B) = \frac{\hbar c}{eBR^3}\,. \tag{27}$$

A global minimum of the total (classical plus zero-point) rotational energy is reached, generally, at a nonzero optimal frequency, $\Omega = n\,\Omega_{\rm ch}$ with $n \in \mathbb{Z}$ (up to small relativistic corrections). The illustrative example of the total rotational energy, shown in fig. 1(right), corresponds to $\Omega = \pm\Omega_{\rm ch} \neq 0$; see ref. [3] for further details.

The magnetic field strongly enhances the zero-point energy (24) because the integer number $M_{\Omega,B}$ may become very large. In ref. [3] a possible application of this approach to torus-shaped carbon nanotubes was discussed and it was shown that the enhancement factor may reach astronomically high values (10^{15} and higher). To this end, it is enough to compute the enhanced rotational vacuum effect assuming the bosonic, and not fermionic, nature of the massless excitations, because in one spatial dimension the zero-point (Casimir) energies for a free massless scalar field and a free massless fermion field for certain boundary conditions are identical [13]. For a carbon nanotube of a finite diameter the fermionic nature of the massless excitations should affect certain features of the zero-point energy [14]. Indeed, the nanotube is a spatially two-dimensional system because it could be considered as a (two-dimensional) graphene sheet—which possesses massless fermionic excitations [15]—rolled into a cylinder.

Notice, however, that in computing of eq. (24) we have assumed that the charged particles are free, so that they may interact with the external magnetic field while the photon exchange between them is absent. The importance of the photon-mediated interaction is unclear for a moment.

Finally, we notice that the device may be made macroscopically large by assembling the open rings along their common axis in a metamaterial-like [16] fashion; see fig. 2.

4. – Perpetuum mobile of the fourth kind and thermodynamics

In this section we briefly notice that the perpetuum mobile of the fourth kind does not violate the laws of thermodynamics (ref. [3] contains a more detailed discussion):

I) The very existence of this device is obviously consistent with the first law of thermodynamics because no work is produced by the object which rotates in its lowest-energy ground state (otherwise it would be the perpetuum mobile of the *first kind* which is obviously forbidden by the energy conservation law).

II) Due to the same reason our device does not spontaneously convert thermal energy into mechanical work so that the second law is not violated as well (otherwise it would be the perpetuum mobile of the *second kind* which is forbidden too).

III) Again, due to the same reason this device cannot serve as a perpetual energy storage (thus our device is not the perpetuum mobile of the *third kind*).

IV) Our device is the perpetuum mobile of the *fourth kind*: it does not produce work and it cannot serve as a perpetual energy storage. Instead, it is a rigid (one-piece) macroscopic device which exhibits perpetual motion due to zero-point fluctuations.

Can a permanently rotating macroscopic body be in thermodynamic equilibrium with the external environment(2)? In the equilibrium, the angular velocity $\boldsymbol{\Omega}$, angular momentum $\boldsymbol{L}$, energy E and entropy S of the body are related to each other as follows [17]:

$$\boldsymbol{\Omega} = \left(\frac{\partial E}{\partial \boldsymbol{L}}\right)_S . \tag{28}$$

If the energy of the system is a smooth function of the angular momentum, $E = E(\boldsymbol{L})$, then in the equilibrium the angular velocity of the body should always be zero, $\boldsymbol{\Omega} = 0$ (this statement works, obviously, at any temperature including the absolute zero).

Thus, we arrive at the important conclusion [3];

"the rotational energy of a permanently rotating device must be a (single-valued) discontinuous function of its angular momentum".

Remarkably, this property is always satisfied for our device due to the presence of the discontinuities in the zero-point contributions to the energy (24) and to the angular momentum (25); see fig. 1(right) for illustration. Moreover, in ref. [3] we have explicitly demonstrated that for our device the thermodynamic relation (28) does indeed give rise to non-zero angular velocity at the equilibrium, $\boldsymbol{\Omega} \neq 0$. Thus, the existence of the perpetuum mobile of the fourth kind is consistent with the laws of thermodynamics.

5. – Conclusions

We have demonstrated that zero-point vacuum fluctuations may possess a negative moment of inertia so that the rotational energy of the zero-point fluctuations decreases with the increase of the angular frequency ("the rotational vacuum effect" [2]). The presence of a magnetic-field background may drastically enhance the negative moment of inertia of zero-point fluctuations so that the negative rotational energy of the zero-point fluctuations may compensate the positive classical rotational energy of the device itself [3]. In this case the device—which does not produce any work—becomes a perpetuum mobile of the fourth kind driven by the zero-point fluctuations. Technologically, such devices may be realised in carbon nanotube systems subjected to the strong, experimentally reachable, static time-independent magnetic field.

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