Observing Hawking radiation in Bose-Einstein condensates via correlation measurements

A. Fabbri

Museo Storico della Fisica e Centro Studi e Ricerche "Enrico Fermi"
Piazza del Viminale 1, 00184 Roma, Italy and

Dipartimento di Fisica, Università di Bologna - Via Irnerio 46, 40126 Bologna, Italy and

Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC
C. Dr. Moliner 50, 46100 Burjassot, Spain

Summary. — Observing quantum particle creation by black holes (Hawking radiation) in the astrophysical context is, in ordinary situations, hopeless. Nevertheless the Hawking effect, which depends only on kinematical properties of wave propagation in the presence of horizons, is present also in the nongravitational contexts, for instance in stationary fluids undergoing supersonic flow. We present results on how to observe the analog Hawking radiation in Bose-Einstein condensates by a direct measurement of the density correlations due to the phonon pairs (Hawking-quanta–partner) created by the acoustic horizon.

PACS 04.62.+v – Quantum fields in curved spacetime.
PACS 04.70.Dy – Quantum aspects of black holes, evaporation, thermodynamics.
PACS 03.75.Kk – Dynamic properties of condensates; collective and hydrodynamic excitations, superfluid flow.

1. – Hawking radiation and experimental prospects

In 1974 Stephen Hawking [1] discovered that due to quantum mechanics black holes are not "black", but emit particles in the form of thermal radiation at a characteristic temperature

$$(1) \qquad\qquad T_H = \frac{\hbar\kappa}{2\pi k_B} \,,$$

where κ $(= c^3/4GM$ for a Schwarzschild black hole) is the horizon's surface gravity. This astonishing result was obtained by studying the vacuum of matter fields in the vicinity of the event horizon, the black hole's boundary marking the region from where nothing (not even light) can escape.
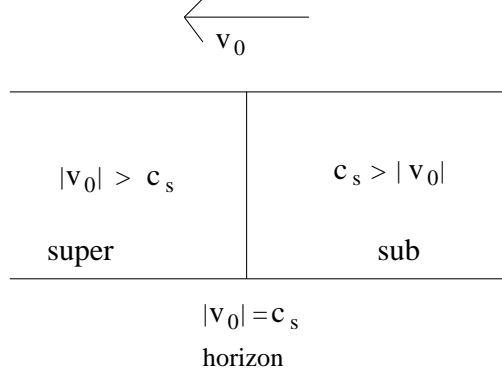
Fig. 1. – Schematic representation of an acoustic black hole in a fluid. $v_0 = v_0(x)$ and $c_s = c_s(x)$ are, respectively, the fluid and sound velocities. The left region has supersonic flow and the right one is subsonic; they are connected through an acoustic horizon.

Quantum vacuum fluctuactions are usually pictured in terms of pairs of virtual particles being continuously created out of the vacuum, but which almost instantaneously (after a time of the order of Planck time $t_P \sim 10^{-43} s$) annihilate and disappear leaving no observable effect. The presence of the event horizon can change dramatically this picture, since when one member of the pair is created just inside the horizon and the other just outside, the one in the interior (the partner) gets trapped and may leave the other free to propagate far away where it is detected as a real particle (Hawking quanta). This heuristic argument was given by Hawking in [2] to explain the basic mechanism responsible for the black hole's emission.

Because of its physical consequences (namely, the information loss problem [3]) the Hawking effect is nowadays considered as a milestone for the construction of a quantum theory of gravity. Nevertheless, it is practically impossible to observe it. Since we cannot, for obvious reasons, check at the same time the existence of the Hawking quanta and of their partners inside the horizon, an experimental verification of the Hawking effect requires the detection of the emitted thermal flux at the temperature (1) far from the black hole. The bad news is that, unfortunately, in ordinary situations where the black holes are created from gravitational collapse of massive stars the Hawking flux at $T_H \sim 10^{-7} \, \mathrm{K} \frac{M_{Sun}}{M} \leq 10^{-7} \, \mathrm{K}$ gets completely overwhelmed by the cosmic microwave background radiation at $T_{CMB} \sim 3 \, \mathrm{K}(^1)$.

2. – A way out: analog Hawking radiation from acoustic black holes in fluids

To remedy this disappointing situation, in 1981 Bill Unruh [6] explored an alternative route. It was known that the propagation of sound waves in inhomogeneous Eulerian fluids and that of (massless) scalar fields in curved backgrounds are mathematically equivalent. Moreover, sound waves get trapped in regions of supersonic flow in the same way light is trapped inside a gravitational black hole. Therefore a stationary fluid configuration consisting in a region of subsonic flow and one of supersonic flow connected through an acoustic horizon represents an acoustic analog of a gravitational black hole, see fig. 1.

$(^1)$ Alternative possibilities to detect it are based on mini black holes (with a much higher T_H) production, by density fluctuatons in the early Universe [4] and in particle accelerators due to the existence of large extra dimensions [5].

Unruh pushed this analogy further at the quantum level. Quantized sound waves (phonons) in these acoustic black holes behave like quantized scalar fields in gravitational black holes. The same analysis Hawking used can then be applied here leading to the prediction that acoustic black holes will produce a thermal flux of phonons from their acoustic horizon at a temperature given by (for a $1D$ fluid for simplicity)

$$(2) \qquad T_H^{an} = \frac{1}{4\pi k_B c_s} \frac{\mathrm{d}(c_s^2 - v_0^2)}{\mathrm{d}x}|_{hor} \ ,$$

where c_s is the sound speed, v_0 the fluid velocity and the derivative is evaluated at the acoustic horizon (where $c_s = |v_0|$).

Although we limited here to the case of fluids, these same considerations apply also to other condensed-matter systems in the long-wavelength (hydrodynamic) approximation [7]. Besides offering new unexpected possibilities to test the Hawking effect in a nongravitational domain, this application also allows to address from first principles a delicate point in Hawking's derivation called the trans-Planckian problem.

It is well known that in the propagation from the horizon to infinity light signals undergo an infinite exponential redshift. Applied to Hawking's analysis, this implies that in order for the Hawking quanta to reach the asymptotic region (where they are detected) the relevant field modes in the near-horizon region must be characterized by an enormous initial frequency, well beyond the Planck scale. This poses a problem because in these regimes one should more appropriately use a quantum theory of gravity, the lack of which poses serious concerns both on the validity of Hawking's analysis and its final result.

One can address this problem in the fluid case, since the theory describing scales below the interatomic distance (the analogous of Planck scale in gravity) is in many cases well known.

Many directions are being explored, the most recent ones ranging from water tanks experiments (stimulated Hawking radiation from white hole flows [8]) to laser pulse filaments [9]([2]) and to Bose-Einstein condensates (BECs), the focus of this contribution.

3. – The gravitational analogy in BECs

Bose-Einstein condensates are ultracold bosonic systems characterized by the fact that (almost) all constituents share the same quantum state. Perfect condensates do not exist; due to quantum effects a small noncondensed fraction is always present, even at absolute zero temperature (quantum depletion). To describe them one writes down the Bose field operator $\hat{\Psi} = \Psi_0(1 + \hat{\phi})$, where the classical wave function Ψ_0 describes the macroscopic condensate and the quantum operator $\hat{\psi}$ the small noncondensed fraction. In the dilute gas approximation (see for instance [11]) Ψ_0 satisfies the Gross-Pitaevski equation

$$(3) \qquad i\hbar\partial_t\Psi_0 = \left(-\frac{\hbar^2\vec{\nabla}^2}{2m} + V_{ext} + g|\Psi_0|^2 \right)\Psi_0,$$

([2]) The interpetation of these results is still controversial, see for instance [10].

where m is the constituents mass, V_{ext} the trapping potential and g the nonlinear atom-atom interaction constant, while $\hat{\phi}$ treated as a linear perturbation follows the Bogoliubov-de Gennes equation

$$(4) \qquad i\hbar\partial_t\hat{\phi} = -\left(\frac{\hbar^2\vec{\nabla}^2}{2m} + \frac{\hbar^2}{m}\frac{\vec{\nabla}\Psi_0}{\Psi_0}\vec{\nabla}\right)\hat{\phi} + mc_s^2(\hat{\phi} + \hat{\phi}^\dagger),$$

with $c_s = \sqrt{\frac{gn}{m}}$ the speed of sound. Equation (3) gives the background while (4) describes the quantum fluctuations and is valid at all scales.

To make contact with the gravitational analogy we need to consider the hydrodynamical approximation of the above equations, and this is more easily achieved in the density-phase representation in which we write $\hat{\Psi} = \sqrt{\hat{n}}e^{i\hat{\theta}}$. Again we consider the splitting $\hat{n} = n_0 + \hat{n}_1$ and $\hat{\theta} = \theta_0 + \hat{\theta}_1$ with n_0, θ_0 the background and $\hat{n}_1, \hat{\theta}_1$ the linear fluctuations. Provided the background varies on scales bigger than the healing length $\xi = \frac{\hbar}{mc_s}$ (the microscopic scale, analogous to the Planck length in gravity), eq. (3) for n_0, θ_0 reduces to the continuity and Euler equations for irrotational inviscid fluids (the analogous of Einstein's field equations). Similar arguments applied to (4) give two operatorial equations for the density and phase fluctuations $\hat{n}_1 = n_0(\hat{\phi} + \hat{\phi}^\dagger)$, $\hat{\theta}_1 = \frac{\hat{\phi}-\hat{\phi}^\dagger}{2i}$ in the form

$$(5) \qquad \hat{n}_1 = -\frac{n}{mc_s^2}\left[v_0\partial_x\hat{\theta}_1 + \partial_t\hat{\theta}_1\right],$$

where $v_0 = \frac{\hbar\partial_x\theta_0}{m}$ is the condensate velocity (we consider a quasi-1D condensate for simplicity), while the equation for $\hat{\theta}_1$ decouples and is mathematically equivalent to the Klein-Gordon equation for a massless, minimally coupled scalar field

$$(6) \qquad \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\hat{\theta}_1) = 0$$

in the background metric

$$(7) \qquad \mathrm{d}s^2 = \frac{n}{mc_s}\left(-(c_s^2 - v_0^2)\mathrm{d}t^2 + 2v_0\mathrm{d}t\mathrm{d}x + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right).$$

Equation (6) is the equation Hawking used to quantize a scalar field in the Schwarzschild background. The acoustic metric (7) possesses an acoustic horizon located at $|v_0| = c_s$ with surface gravity $\kappa = \frac{1}{2c_s}\frac{\mathrm{d}(c_s^2-v_0^2)}{\mathrm{d}x}|_{hor}$, and unlike its gravitational analogue it does not need to have the interior (supersonic) region to terminate into a (nasty) spacelike singularity. Since Hawking's analysis only relies on the kinematical properties of the horizon, event or acoustic, the prediction that in the formation of an acoustic black hole a thermal flux of phonons at the temperature (2) will be produced is then straightforward.

The experimental realization of an acoustic black hole in a BEC was carried out for the first time in [12]. Because of their extremely low temperatures (of the order of $T_C \sim 100\,\mathrm{nK}$, much below T_{CMB}) they offer much more favourable experimental conditions for the detection of the Hawking effect with respect to gravity. Indeed, acoustic black-hole configurations can be constructed for which $T_H^{an} \sim 10\,\mathrm{nK}$, only one order of magnitude below T_C. Nevertheless, the signal is tiny and competing effects (the bigger
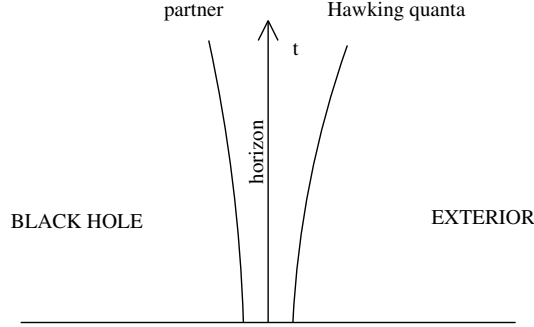
Fig. 2. – Pair creation (Hawking-quanta–partner) from the black-hole horizon.

background temperature, quantum noise, etc. (^{3})) still easily overcome the Hawking flux of the created phonons making it practically unobservable.

4. – One step forward: the Hawking effect in density correlations in BECs

Another experimental advantage of having to do with acoustic black holes instead of gravitational ones is that acoustic horizons, unlike event horizons, allow measurements to be performed on both sides, the exterior (subsonic) and the interior (supersonic) regions. This is very important because we can then try to characterize experimentally the Hawking effect through the pair production process responsible for it. This was done in [14], to which we refer for more details.

In [14] the phase fluctuation $\hat{\theta}_1$ was treated as a 2D conformal scalar field propagating in the (t, x) section of the acoustic metric (7), for which the two-point function $\langle \hat{\theta}_1 \hat{\theta}_1 \rangle$ is known analytically(^{4}). This powerful result allowed us to construct in closed form, using (5), the equal time normalized density-density correlation function in the Unruh state (the state appropriate to describe black hole evaporation). Focussing in particular on points situated on either side with respect to the acoustic horizon, we got the characteristic stationary behaviour

$$(8) \qquad G_{BH}^{(2)} \equiv \frac{1}{n_0^2} \langle \hat{n}_1 \hat{n}_1 \rangle \sim \kappa^2 \cosh^{-2} \left[\frac{\kappa}{2} \left(\frac{x}{v + c_l} - \frac{x'}{v + c_r} \right) \right].$$

Here $v < 0$, $i.e.$ the flow is from right to left while $x = 0$ is the acoustic horizon dividing the right ($x > 0$) subsonic and left ($x < 0$) supersonic regions. The formation of the horizon triggers the spontaneous creation of pairs of phonons (of equal and opposite frequencies, as required by energy conservation) on both sides of the horizon. In the lab they propagate along the lines $x = (v + c_l)t$ (< 0) (partner), $x' = (v + c_r)t$ (> 0) (Hawking quanta), see fig. 2(^{5}). This happens for all t and leads to a stationary signal

---

(^{3}) For a recent experiment disentangling these contributions see [13].
(^{4}) This approximation only slightly overestimates the flux of the emitted phonons.
(^{5}) Although we are considering its hydrodynamic/relativisistic approximation, the system we started with is nonrelavistic and velocities add up following Galileo's law.
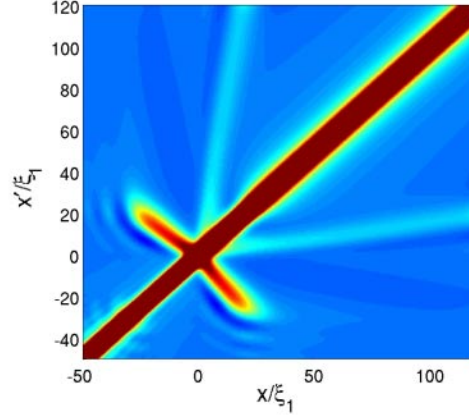
Fig. 3. – Numerical results for the density correlator confirming the existence of the Hawking peak (red tongue perpendicular to the main diagonal) in the microscopic BEC theory.

correlating points x, x' such that

$$(9) \qquad \frac{x}{v + c_l} = \frac{x'}{v + c_r} \, .$$

The correlator (8) has a peak exactly along this line: this signal is a clear manifestation of the pair-production process at the base of the Hawking effect. Quantitative estimates based on existing experiments suggest the the magnitude of this peak, of the order of 10^{-3}, is small but not negligible.

One can object that this result and the thermal flux prediction, being based on the gravitational analogy, are subject to a "trans-Planckian" problem analogous to that in gravity. Indeed, sound waves propagating from the near horizon region get exponentially redshifted, and this implies that the physics of the Hawking effect is in principle sensitive to microscopic scales, in particular those smaller than ξ. In BECs we can study the "trans-Planckian" problem by going back to the basic equations (3) and (4), valid at all scales (we do not have the analogous equations in gravity).

The result (8) served both as a motivation and a guide for a subsequent numerical analysis performed within the full theory using Montecarlo techniques [15]. The results, shown in fig. 3, confirmed the existence of the Hawking peak in the microscopic theory. A quantitative comparison of the height and width of the peak with (8) showed excellent agreement with the hydrodynamical prediction in the regime $\kappa \ll \frac{1}{\xi}$. Outside this regime (which in the gravity context would require knowledge of a quantum theory of gravity) acoustic black holes still emit at an approximately thermal rate (up to a certain cutoff frequency $w_{max} \sim \frac{1}{\xi}$ [16]) and the location of the Hawking peak is still given by (9) (for analytical results in this regime, see [17] and references therein).

A nice way to explain why there is no trans-Planckian problem in BECs is depicted in fig. 4. By tracing backwards in time the trajectories of the Hawking quanta and partners, once the near horizon region is reached microscopic physics effects change the phonon's dispersion relation which becomes 'superluminal'. This causes the two trajectories to stop hovering close to the horizon (where, instead, they would remain forever in the

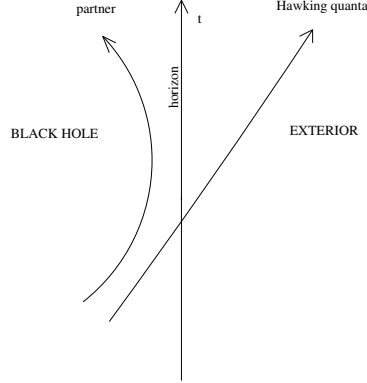Fig. 4. – Because of short-distance effects (scales smaller than ξ) Hawking quanta and partners appear to be produced in the interior of the acoustic black hole, thus resolving the trans-Planckian problem in BECs.

hydrodynamical/relativistic approximation) and enter the interior region. The resulting blueshift along the trajectories is now finite, of the order of $\frac{1}{\xi}$ (we recover the infinite blueshift in the hydrodynamcal $\xi \to 0$ limit).

Another important result of the numerical analysis in [15] is that the Hawking peak of fig. 3 is still visible even when a nonzero thermal background (with a temperature bigger than T_H) is present. Thus, unlike the emitted thermal flux (which is easily masked by the background) the correlator signal (8) is the looked for 'smoking gun' of the Hawking effect in the experimental search.

5. – Acoustic white holes in BECs: another experimental signature of the Hawking effect

White holes, time reversal of black holes, have not captured much interest in gravity. They expel to the exterior everything that lies inside their horizon, exactly the opposite behaviour of black holes. They "emerge" from an initial singularity, a kind of big-bang singularity, which prevents physical predictions about them to be made until we shall find a way to deal with singularities in the quantum theory.
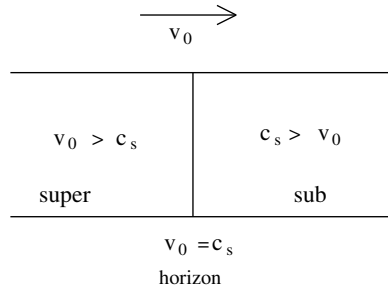

Fig. 5. – Schematic representation of an acoustic white hole in a fluid, obtained from fig. 1 by reversing the sign of the fluid velocity v_0.
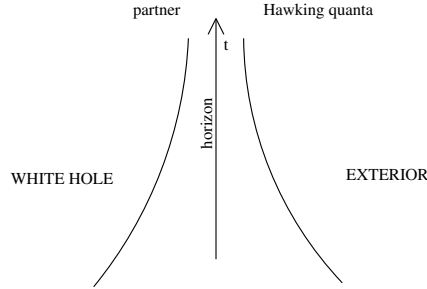
Fig. 6. – Trajectories of Hawking quanta and partners in acoustic white holes.

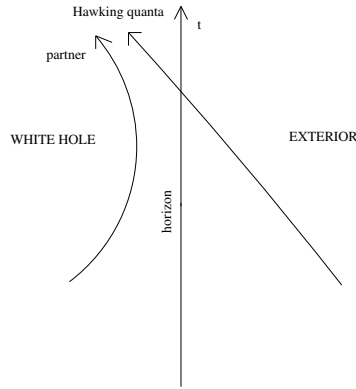
Fig. 7. – Short-distance physics (scales smaller than ξ) cause Hawking quanta and partner to enter the white-hole horizon instead of accumulating along the horizon on both sides (fig. 6).

This problem is not present in the fluid case, and acoustic black and white holes can be studied on the same footing. A schematic representation of an acoustic white hole is given in fig. 5, and is simply obtained by reversing the sign of the fluid velocity v_0 in fig. 1 (and also in the acoustic metric (7)). An acoustic white hole is represented by a supersonic fluid decelerating until it becomes subsonic, whereas the acoustic black hole is the other way around, namely an accelerating fluid turning from subonic to supersonic. In both cases, the acoustic horizon lies where the fluid velocity equals the sound speed.

Spontaneous pair production in stationary backgrounds requires the existence of negative energy states. These exist in supersonic regions of both acoustic black holes and white holes, indicating that an analog Hawking effect exists in both configurations and not just in black holes([6]). Its features in white holes are however totally different. Because of the time-reversal property with respect to the black hole case (fig. 2), the trajectories in fig. 6 show that Hawking quanta and partners now accumulate along the horizon, and they do this with a diverging blueshifted frequency. This implies that hydrodynamical white-hole horizons (as well as relativistic ones) are unstable: the Hawking effect in white holes crucially depends on the details of the short-distance physics.

([6]) Perhaps surprisingly, such an effect exists also for an everywhere inhomogeneous supersonic fluid with no acoustic horizon, see [17]. What seems to depend on the existence of an acoustic horizon is the thermal character of the created flux.

The "superluminal" modification of phonon's dispersion relation in BECs causes (as in the black hole case) departure from the hydrodynamical picture in fig. 6 when a maximum frequency $\sim \frac{1}{\xi}$ is reached, allowing both trajectories to enter the horizon (fig. 7).

These arguments indicate that the main Hawking signal in BEC acoustic white holes takes place inside the horizon. Analytical results derived within the linear Bogoliubov theory show that for $x, x' < 0$, *i.e.* inside the horizon, the density-density correlator has the typical oscillating behaviour [18]

$$(10) \qquad G_{WH}^{(2)}(x, x') \sim [A \cos p_0(x + x') + B \sin p_0(x + x') + C \cos p_0(x - x')] \, I_\epsilon,$$

with p_0 the nontrivial zero mode ($\sim \frac{1}{\xi}$) of the "superluminal" Bogoliubov dispersion (present only in the supersonic region), and the overall amplitude $I_\epsilon = \int_\epsilon \frac{\mathrm{d}w}{w}$, formally infrared divergent, is regulated by introducing the low-frequency cutoff $\epsilon = \frac{1}{t}$ where t measures time elapsed from horizon formation. A closer inspection shows [19] that the two-point function in the relevant low-frequency regime factorizes as the product of the same function ϕ_0 evaluated at the points x and x', typical of an emerging classical behaviour. It describes the emission of a zero-frequency undulating wave away from the horizon with a macroscopically growing amplitude (*i.e.* the $\frac{1}{w}$ term in I_ϵ) fixed by the low-frequency spectrum of the spontaneously produced Hawking phonons. It is worth mentioning that undulations were observed in water tanks experiments in [8], but their connection with the Hawking effect was not pointed out.

6. – Conclusions and future prospects

We have described how the analog Hawking radiation can be measured in Bose-Einstein condensates by a direct measurement of the density correlations between the spontaneously produced pairs of phonons (Hawking quanta and partners) on both sides of the acoustic horizon (such a measurement would be impossible to perform in gravity). This provides the looked for "smoking gun" of the Hawking effect and makes these type of measurements very promising for an experimental verification of the Hawking effect in the near future. Methods to amplify this signal have been proposed [20] by letting the condensate to expand freely after the acoustic black hole is formed.

Finally, it is worth to mention that density correlation measurements between atoms velocities were performed recently in [21] to observe the creation of correlated excitations with equal and opposite momenta in homogeneous condensates undergoing a sudden variation of their trapping potential. While this effect can be interpreted as the analog of cosmological particle creation, this technique will prove useful to test the Hawking effect in stationary supersonic flows.

* * *

REFERENCES

[1] HAWKING S. W., *Nature*, **248** (1974) 30; *Commun. Math. Phys.*, **43** (1975) 199.
[2] HAWKING S. W., *Sci. Am.*, **236** (1977) 34.
[3] HAWKING S. W., *Phys. Rev. D*, **14** (1976) 2460.

[4] CARR B., *Astrophys. J*, **206** (1975) 1.

[5] ARKANI-HAMED N., DIMOPOULOS S. and DVALI G. R., *Phys. Lett. B*, **429** (1998) 263; RANDALL L. and SUNDRUM R., *Phys. Rev. Lett.*, **83** (1999) 3370.

[6] UNRUH W. G., *Phys. Rev. Lett.*, **46** (1981) 1351.

[7] BARCELÒ C., LIBERATI S. and VISSER M., *Living Rev. Rel.*, **8** (2005) 12; *Living Rev. Rel.*, **14** (2011) 3.

[8] WEINFURTNER S., TEDFORD E. W., PENRICE M. C. J., UNRUH W. G. and LAWRENCE G. A., *Phys. Rev. Lett.*, **106** (2011) 021302.

[9] BELGIORNO F., CACCIATORI S. L., CLERICI M., GORINI V., ORTENZI G., RIZZI L., RUBINO E., SALA V. G. and FACCIO D., *Phys. Rev. Lett.*, **105** (2010) 203901.

[10] SCHÜTZHOLD R. and UNRUH W. G., *Phys. Rev. Lett.*, **107** (2011) 149401.

[11] PITAEVSKI L. and STRINGARI S., *Bose Einstein condensation* (Oxford University Press) 2003.

[12] LAHAV O., ITAH A., BLUMKIN A., GORDON C., RINOTT S., ZAYATS A. and STEINHAUER J., *Phys. Rev. Lett.*, **105** (2010) 240401.

[13] ARMIJO J., *Phys. Rev. Lett.*, **108** (2012) 225306.

[14] BALBINOT R., FABBRI A., FAGNOCCHI S., RECATI A. and CARUSOTTO I., *Phys. Rev. A*, **78** (2008) 021603.

[15] CARUSOTTO I., FAGNOCCHI S., RECATI A., BALBINOT R. and FABBRI A., *New J. Phys.*, **10** (2008) 103001.

[16] MACHER J. and PARENTANI R., *Phys. Rev. A*, **80** (2009) 043601.

[17] BALBINOT R., CARUSOTTO I., FABBRI A., MAYORAL C. and RECATI A., *Understanding Hawking radiation from simple models of atomic Bose-Einstein condensates*, arXiv:1207.2660 [gr-qc].

[18] MAYORAL C., RECATI A., FABBRI A., PARENTANI R., BALBINOT R. and CARUSOTTO I., *New J. Phys.*, **13** (2011) 025007.

[19] COUTANT A., PARENTANI R. and FINAZZI S., *Phys. Rev. D*, **85** (2012) 024021.

[20] CORNELL E., talk given at the workshop *Towards the observation of Hawking radiation in condensed matter systems, Valencia 2009*, `www.uv.es/workshopEHR`.

[21] JASKULA J. C., PARTRIDGE G. N., BONNEAU M., LOPES R., RUAUDEL J., BOIRON D. and WESTBROOK C. I., *Phys. Rev. Lett.*, **109** (2012) 220401.