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HOM-free accelerating dielectric cavities with metallic inclusions

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Summary. — Previous investigations demonstrated that point-defected photonic bandgap cavities based on periodic and aperiodic dielectric arrangements can be successfully employed as single cell in particle accelerators. In this paper, we present a study aimed at highlighting the possible advantages of using *hybrid* structures based on the above dielectric configurations, but featuring *metallic* rods in the outermost regions, for the design of accelerating resonators of extremely high-quality factor. In this framework, we consider diverse configurations, with different (periodic and aperiodic) lattice geometries, sizes, and dielectric/metal fractions. The use of superconducting plates to further increase the cavity performances is taken into account. Results from our comparative studies, based on numerical full-wave simulations backed by experimental validations (at room and cryogenic temperatures) in the microwave region, identify the candidate parametric configurations capable of yielding the highest quality factor.

PACS 42.70.Qs – Photonic bandgap materials.
PACS 42.60.Da – Resonators, cavities, amplifiers, arrays, and rings.
PACS 29.20.-c – Accelerators.

1. – Introduction and background

The next-generation colliders (which require large current and high-energy beams) and the needs of medical and industrial applications of accelerators (which ask for compact and easy-to-fabricate structures) constitute a pressing demand for the development of resonators based on novel, unconventional concepts. Suppression of higher-order modes (HOMs) is presently a real challenge in designing and building compact accelerating structures efficiently coupled to the particle beam.

When a bunched beam travels through a cavity, the amplitude and the distribution of the electromagnetic (EM) field inside the cavity can be altered by the particles beam transit, due to its harmonic content. The use of high-intensity beams, for instance, requires usually very short bunches, which produce consequently relevant higher-order harmonics. An energy transfer process between the beam and the cavity can take place only if the high-order harmonics are synchronous with the cavity modes. A fraction of the excited EM field in this case will sustain unwanted modes in the resonant cavity until it is naturally damped.

Such phenomenon, called "beam loading" of the accelerating cavity, gives rise to both transverse and longitudinal coupled-bunch instabilities, and increases linearly with the beam intensity. As a consequence, the particle current is severely limited if the instability growth rate is larger than the natural damping. Even if the fundamental mode can be compensated by varying the amplitude and phase of the feeding voltage, the detrimental HOMs need to be suppressed. Standard solutions to this problem (at no expense to the accelerator performance) are based on connecting to the cavity a number of waveguides with various cut-off frequencies. Such HOM-removal mechanism is really effective when one operates at low frequency, but becomes unfeasible as the frequency increases.

It is well know that periodic photonic crystals (PCs) can be successfully employed for accelerating cells in microwave or laser-driven particle accelerators [1]. The key concept underlying these structures is the presence of a photonic bandgap (PBG), a frequency region where the EM propagation along the specific directions of the PC is forbidden. In such structures, introducing lattice defects, light localization nearby can be easily achieved and employed to realize PBG cavities or waveguides with operational frequency within the forbidden windows.

This mechanism therefore allows to design a device with frequency-selective boundaries for the EM propagation, acting, *e.g.*, as a perfectly "reflecting wall" at a certain frequency, while exhibiting a transparent response in the remaining part of the spectrum. By adequately shaping the geometry around the defect, the trapped mode can be optimized so as to work as the operating accelerating mode. The cavity, suitably designed, can exhibit a single resonant mode (the fundamental), resulting by construction HOM-free.

Metallic PC structures have already been used to realize a new kind of large-gradient accelerator with an effective suppression of HOM wakefields [2]. Prototypes of fully metallic (super- and normal-conducting) mono-modal PC cavities have been constructed and tested at different working frequencies [3, 4]. In particular, when tested in the presence of short bunch beams, the metallic prototypes showed a higher breakdown rates in comparison with optimized pillbox cavity for comparable accelerating gradient, or peak surface electric field [5].

It has also been demonstrated that the use of *all-dielectric* structures can be used to realize accelerating cavities in order to minimize the conducting losses and cope with radio-frequency (RF) breakdown phenomena [6,7] that may easily occur in metallic structures where large accelerating field gradients need to be achieved.

Moreover, the existence of PBG and related phenomena is not restricted to *periodic* crystals. In fact, a large body of numerical and experimental studies have demonstrated the possibility of obtaining similar effects also in *aperiodically ordered* structures, typically referred to as "photonic quasicrystals" (PQCs) (see, *e.g.*, [8-10] for recent reviews of the subject). PQC structures are receiving a growing attention in a variety of fields and application scenarios (see, *e.g.*, [11]). Numerical and experimental investigations were devoted to systematically analyse the PBG isotropy of PC and PQC structures as a function of refractive index, light polarization and rotational symmetries [12-14], providing useful prescription to design PBG devices.

In particular, experimental studies on PQC-based optical microcavities have demonstrated the possibility of obtaining high quality factors and small modal volumes [15], thereby providing further degrees of freedom in tailoring the mode confinement in dielectric structures. In this framework, it is also worth mentioning possible alternative strategies based on irregular structures obtained by numerical local optimizations of the spatial lattice geometry [16].

For such reasons, in a previous study [17] we explored the PBG properties of point-defect resonators in aperiodic (Penrose and dodecagonal) structures composed of aperiodic arrangements of dielectric rods, with special emphasis on their use for the design of cavities for particle accelerators. More specifically, we carried out a parametric study of the confinement properties as a function of the structure size, filling fraction, and losses, so as to identify the best performing configurations, and we compared them with a reference periodic counterpart.

This paper, following up on our previous work, is aimed at highlighting the possible advantages of using *metallic* rods in the outermost regions of (periodic and aperiodic) dielectric structures, for the design and fabrication of PBG-based *hybrid* (normal metallo-dielectric and superconducting metallo-dielectric) high-quality-factor accelerating resonators. The basic underlying idea is to find out a suitable trade-off between dielectric and metal content, so as to improve the in-plane confinement without significantly increasing the conduction losses or breakdown phenomena. To this aim, we compare different configurations of these hybrid structures, with diverse sizes, lattice geometries, and dielectric/metal fractions.

Accordingly, the rest of the paper is laid out as follows. In sect. **2**, we present the results of our numerical full-wave studies, first on dielectric-rod structures (focusing on the effects of the metallic plates), and subsequently on *hybrid* structures consisting of dielectric and metallic rods. In sect. **3**, we validate the above results via experimental measurements at room and cryogenic temperatures. Finally, in sect. **4**, we provide some concluding remarks and hints for future research.

2. – Numerical studies

Our numerical full-wave studies of the EM response of the structures of interest are based on the combined use of the 3D commercial software CST Microwave Studio [18] (for modeling volumetric and surface losses) and an in-house 2D simulator based on the finite-difference-time-domain (FDTD) technique (for modeling the radiative losses not accounted for in the 3D simulator).

The dielectric structures of interest are composed of sapphire cylindrical rods of radius r and relative dielectric permittivity 9.2, with typical loss-tangent values ranging between 10^{-6} and 10^{-8}, depending on the temperature of operation. As in [17], the rods are arranged according to two representative PQC geometries, based on the dodecagonal (12-fold–symmetric fig. 1(b)) and Penrose (10-fold–symmetric fig. 1(c)) tilings, respectively, and a reference periodic (triangular) PC structure (fig. 1(a)). The removal of the central rod creates the defect region and allows for the beam transit aperture. The mode of interest for the particle acceleration along the longitudinal direction is the TM_{010}-*like* (fundamental mode), with the electric field parallel to the rods.

All configurations are characterized by a lattice constant (corresponding to the period in the triangular case, or to the tile sidelength in the aperiodic cases, cf. fig. 1b) and c)) chosen as $a = 0.75\,\text{cm}$, so as to yield comparable values of the fundamental mode resonant frequency (around 16.5 GHz). The rods height is $h = 0.6\,\text{cm}$. Different transverse sizes are considered, by varying the radius R as an integer multiple of the lattice constant a (see fig. 1). As a figure of merit, we use the quality factor of the fundamental cavity mode Q.

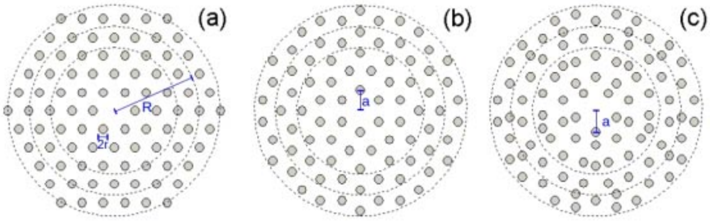

Fig. 1. – Point-defected PBG mono-modal cavities, with periodic triangular (a), and aperiodic dodecagonal (b) and Penrose (c) lattice geometries. Dotted line circles represent the three cavity extents taken into account in the present study.

In our simulations, the resonant frequency and the quality factors pertaining to volumetric and surface losses are computed via standard post-processing routines available in the CST Microwave Studio eigen-solver [18]. The radiative quality factor is instead computed from the 2-D FTDT analysis (with all dielectric and metallic elements assumed as lossless), by processing the time-signal evolution via a harmonic inversion tool [19] based on a low-storage "filter diagonalization method." The overall quality factor Q_T can be then obtained by combining the conducting (Q_C), dielectric (Q_D), and radiative (Q_R) quality factors via

$$\frac{1}{Q_T} = \frac{1}{Q_C} + \frac{1}{Q_R} + \frac{1}{Q_D}\,. \tag{1}$$

Note that, within the parametric ranges of interest, the dielectric quality factor Q_D is much higher than the other two factors, and its contribution in eq. (1) is accordingly negligible. As shown in [17], and compactly summarized in table I, for a moderate cavity size ($R \leq 5a$) and a filling factor $r/a = 0.2$, a judicious choice of a PQC geometry turns out to provide perceivable improvement in the field confinement as compared with the periodic reference configuration.

In dielectric-rod cavities, as expectable, the conducting quality factor Q_C is almost the same for all configurations and cavity sizes, since it depends mainly on the surface conductivity of the metallic plates.

Conversely, the radiative quality factor Q_R strongly depends on the geometry and size of each structure. The field confinement in these cavities is weaker than that achievable via fully metallic structures, and represents the main factor affecting the cavity performance when very compact ($R = 3a$) structures are needed.

For $R = 5a$, instead, Q_R improves from 4.08×10^4 (triangular) to 1.47×10^5 (dodecagonal). For this size, the total quality factor Q_T of dielectric (periodic or aperiodic) structures is much higher than those obtained in the case of fully-metallic periodic PBG cavities ($\sim 4 \times 10^3$ at room temperature) of comparable resonant frequency [5, 20]. This is mainly due to the reduction of conduction losses resulting from the use of dielectric (instead of metallic) rods.

In such a case, therefore, the primary source of dissipation is given by the surface losses of the metallic plates. Although there is still room for further cavity design optimization, acting specially on the cavity height, the replacement of copper with a superconducting material appears the way to go, even if the required low-temperature operation implies an increased complexity. Nevertheless, in order to achieve the performance required for accelerating cavities, one still needs to reduce the radiation leaks, which would otherwise limit the total quality factor.

Table I. – *Simulation results for the selected lattice geometries, assuming cavity sizes $R = 3a$, $4a$, and $5a$ and filling factor $r/a = 0.2$. Q_C, Q_R, and Q_T are the conducting, radiative, and total quality factors, respectively. The last column indicates the weight of the conduction losses in the total quality factor.*

$R = 3a$	Q_C	Q_R	Q_T	$(1 - \frac{Q_T}{Q_R})$
Triangular	1.05×10^4	7.74×10^2	7.20×10^2	0.05
Dodecagonal	1.07×10^4	1.87×10^3	1.65×10^3	0.09
Penrose	1.06×10^4	4.37×10^2	4.23×10^2	0.02
$R = 4a$				
Triangular	1.19×10^4	5.70×10^3	3.80×10^3	0.32
Dodecagonal	1.18×10^4	1.72×10^4	7.00×10^3	0.59
Penrose	1.14×10^4	5.00×10^3	3.48×10^3	0.30
$R = 5a$				
Triangular	1.19×10^4	4.08×10^4	9.20×10^3	0.77
Dodecagonal	1.19×10^4	9.30×10^4	1.05×10^4	0.89
Penrose	1.22×10^4	1.47×10^5	1.13×10^4	0.92

In order to reduce the radiative losses, without sacrificing the performance improvement attainable via superconducting technologies, one may think of replacing *some* dielectric rods (intuitively, those located in the outermost regions, where the field is weaker), with metallic ones, thereby obtaining a "hybrid" metallo-dielectric PBG cavity. We present here the results pertaining to hybrid structures of size $R = 5a$, based on the above selected lattice geometries (triangular, dodecagonal, Penrose).

Table II compares the FDTD-simulated radiative quality factors pertaining to the dielectric-rod reference case with those pertaining to hybrid PBG cavities featuring one or two peripheral "rings" made of metallic (copper) rods. Here, and henceforth, the hybrid structures are labelled as $D + M$, with D and M denoting the number of rings made of dielectric and metallic rods, respectively. Note that, in the aperiodic cases, the "rings" are not regularly shaped, and their definition may be ambiguous. In our simulations, they were defined via radial inequalities (*e.g.*, for a total cavity size of

Table II. – *Simulated radiative quality factors Q_R for hybrid PBG cavities of total size $R = 5a$ (with $r/a = 0.2$), featuring zero (i.e., fully dielectric), one, and two peripheral rings of metallic (copper) rods.*

Dielectric+Metallic	Triangular	Dodecagonal	Penrose
$5 + 0$	4.08×10^4	9.30×10^4	1.47×10^5
$4 + 1$	1.78×10^6	7.44×10^6	2.51×10^6
$3 + 2$	2.50×10^8	5.85×10^7	3.93×10^8

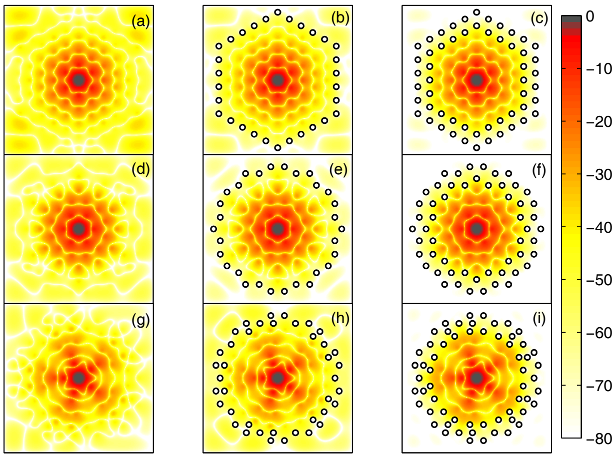

Fig. 2. – Electric field intensity maps (in dB) for the hybrid PBG cavities of size $R = 5a$ (with $r/a = 0.2$), featuring zero, one or two peripheral "rings" of copper rods (displayed as black empty circles), and different lattice geometries: triangular ((a), (b), (c)), dodecagonal ((d), (e), (f)), Penrose ((g), (h), (i)).

$R = 5a$, the outermost ring is defined as exterior to the radial domain $R' = 4a$, and so on); this ensures the inclusion of a comparable number of metallic rods for the different lattice geometries.

One readily observes that the inclusion of metallic rods dramatically improves the confinement properties of the PBG cavities. In particular, in the periodic case, there is a two-order-of-magnitude step increase in the radiative quality factor, which brings its value to over 10^8 (very close to what predicted for the Penrose geometry) when two peripheral copper rings are included. The dodecagonal geometry, which exhibits the best performance in the 4+1 configuration, is outperformed by the other geometries in the 3+2 configuration. The field confinement improvements are also evident in the corresponding (transverse) electric-field maps shown in fig. 2.

Specifically, figs. 2(a), 2(d), and 2(g) show the results pertaining to the dielectric-rod cavities (5+0 configuration). The field maximum intensity (centered at the defect position) is almost the same in the three different cases, but the spatial distribution evidences the better confinement properties of the Penrose geometry, as confirmed by the data reported in table I. The improvement in the radiative quality factor of the hybrid cavities is already sensible when the first (outermost) peripheral metallic ring is included (see figs. 2(b), 2(e), and 2(h)), and becomes striking in the 3+2 configuration (see figs. 2(c), 2(f), and 2(i)). This is in agreement with the trend shown in table II. More difficult, however, is to discern from the plots the different performance in terms of Q_R values exhibited by the three geometries for each hybrid (4+1 or 3+2) configuration.

Figure 3 shows the simulated total quality factors pertaining the three lattice geometries, as a function of the temperature T and of the number of metallic rings. Direct-current conductivity is assumed to vary (with the temperature) according to the data reported in [21] for high-purity electro-polished copper. As a reference, the behaviour of a fully-metallic periodic structure is also displayed.

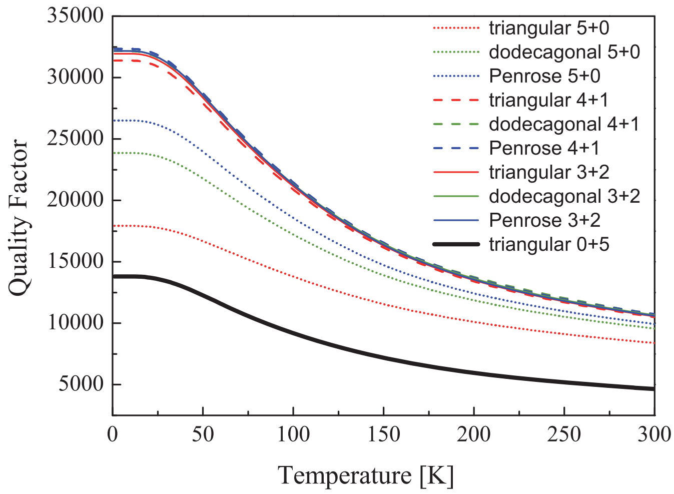

Fig. 3. – Simulated temperature dependence of the total quality factor for triangular, dodecagonal and Penrose PBG hybrid cavities of size $R = 5a$ (with $r/a = 0.2$), and featuring zero, one or two copper rings, compared with the expected for a fully metallic triangular cavity (solid curve).

Looking at these behaviours, the advantage of using a dielectric-rod structure (dotted lines) instead of a fully-metallic one (bold line) is fairly clear. Similarly, hybrid structures with one (dashed lines) or two (solid lines) metallic rings outperform (the more the lower the temperature) the fully-dielectric ones. It is also evident that the inclusion of metallic rings progressively brings the radiation losses to a negligible level; in this regime, the dominant source of dissipation comes from the metallic plates, and the responses (as a function of the temperature) of the different geometries tend to become identical.

A possible solution might be the replacement of copper with a high-temperature superconductor (HTS), to cover the inner surface of the confinement plates. Setting the operational temperature at about 30 K, this would determine a three-order-of-magnitude reduction of the surface losses, and therefore a corresponding increase of the related conduction quality factor. It is important to stress that use of HTS peripheral rod rings would only add further complexity to the structure (because of the necessity of efficiently cooling down them too), without any significant improvement in the overall quality factor.

It therefore appears that a judicious combination of i) superconducting plates, ii) low-loss dielectric rods (in the interior region), and iii) metallic rods (in the outermost region) may open up new perspectives in the development of novel monomodal, PBG based, high-quality factor open cavities for the acceleration of energetic particle beams at very high operational frequencies.

The use of peripheral metallic rings certainly improves the confinement properties of the PBG resonators, while maintaining the advantages foreseen for dielectric cavities (reduction of breakdown phenomena, moderate fabrication complexity, etc.). Figure 4 displays the total quality factor Q_{LP} of a hybrid PBG cavity of size $R = 5a$ as a function of the number of metallic rings for the geometries of interest, at a temperature of operation of 30 K, assuming lossless plates and a dielectric loss-tangent of 10^{-8} (which is a reasonable value for single-crystal sapphire at low temperatures), so as to better highlight the role of the copper rods.

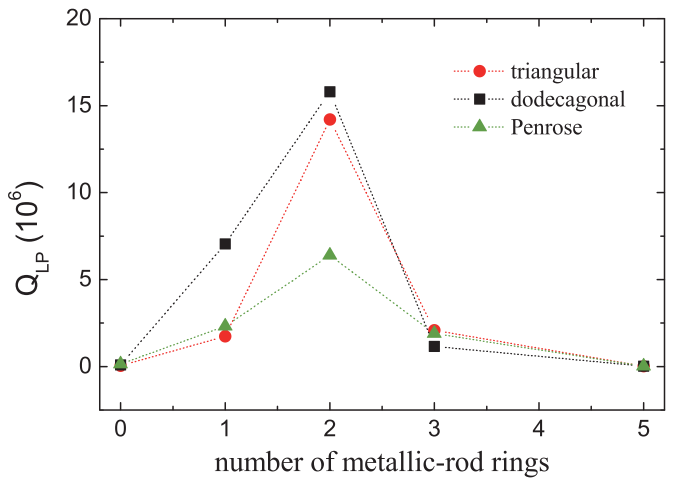

Fig. 4. – Estimated total quality factor Q_{LP} as a function of the number of metallic (copper) rod rings for triangular, dodecagonal, and Penrose PBG hybrid structures of size $R = 5a$ (with $r/a = 0.2$) at a temperature of 30 K, assuming lossless plates and dielectric loss-tangent of 10^{-8}. Dotted lines are guide-to-eye only.

As can be observed, Q_{LP} first increases (reaching its maximum for the 3+2 configuration), and then it starts decreasing (reaching its minimum for the fully metallic 0+5 configuration). As expected, increasing the number of metallic rings *levels* the performance of the hybrid cavities, irrespective of the different spatial arrangements of the rods. Another interesting feature observable in fig. 4 is that the Penrose geometry, in spite of its higher radiative quality factor, is largely outperformed by the other two geometries in the 3+2 configuration. This is attributable to the slight ($\sim 10\%$) larger number of copper rods in outermost metallic rings (as compared to the triangular and dodecagonal cases), with a detrimental effect on the level of conduction losses for this geometry.

3. – Experimental results

In order to validate the above findings and explore their technological viability, we fabricated some prototypes of the simulated structures, by suitably placing the dielectric (single-crystal sapphire) and metallic (oxygen-free high-conductivity copper) rods (of radius $r = 0.15$ cm and height $h = 0.6$ cm) between two copper plates, inserted in a cryogenic box. The whole system is then cooled down to about 100 K by introducing liquid nitrogen. The operation temperature is monitored using a Si-diode sensor placed on one of the copper plates. Both the feed and pick-up antennas are placed on the top plate, far enough from the central region where the EM field reaches its maximum to ensure that all measurements are performed in the weak-coupling limit. The resonant cavity is then connected to a HP8720C network analyzer and the quality factors at room and cryogenic temperatures are evaluated by the standard -3 dB method, looking at the frequency transmission curve at the resonance. For the experimental characterization, we considered hybrid cavities with the three different geometries and $R = 5a$, having a single peripheral metallic ring (4+1 configuration).

TABLE III. – *Measured total quality factor at room* (300 K) *and cryogenic* (100 K) *temperatures for triangular, dodecagonal and Penrose PBG hybrid cavities with* $R = 5a$ *and a single peripheral metallic ring (i.e.,* $4 + 1$ *configuration).*

Geometry	Q_{exp}^{100K}	Q_{sim}^{100K}	Q_{exp}^{300K}	Q_{sim}^{300K}
Triangular	1.84×10^4	1.90×10^4	1.14×10^4	1.05×10^4
Dodecagonal	1.95×10^4	1.94×10^4	1.12×10^4	1.07×10^4
Penrose	2.00×10^4	1.96×10^4	1.05×10^4	1.07×10^4

In table III, the experimental quality factors are reported at 300 K and 100 K, and compared with the results obtained from the numerical simulations. Note that, unlike in [21], the surface of the copper used in our experiments was not chemically treated or polished (to remove impurities, oxide layers, etc.). This was accordingly taken into account in the numerical simulations by slightly (10%) increasing the value of the copper surface impedance with respect to the data reported in [21].

There is a very nice agreement between measurements and simulations, confirming the validity of our initial assumptions. The data show that losses, at both room and cryogenic temperatures, are essentially dominated by the conductive contribution due to the metallic plates, and consequently determine an upper limit for the total quality factor of the order of 10^4, irrespective of the geometrical configurations, as already evidenced in fig. 3. One can accordingly conjecture that the insertion of superconducting plates, with the corresponding reduction of conduction losses of three or more orders of magnitude, would have already for this configuration a tremendous impact on the quality factor (and hence on the overall performance) of an accelerating cavity operating at such high frequencies. Another interesting conclusion that can be drawn is that the use of conventional low critical temperature materials like niobium, very common in the development of superconducting accelerating cavities, in this case is unnecessary, since the overall quality factor of hybrid cavities of such compact size would be inherently limited (by radiation losses) to values on the order of 10^7 (see fig. 4). HTS materials may be used instead, with an obvious simplification of the related cryogenic technology, and corresponding cost reduction.

4. – Conclusions

In this paper, we have explored hybrid configurations of point-defected PBG cavities, showing that a clever blend of superconducting materials, low-loss dielectrics, and highly conducting metals may pave the way to the development of novel monomodal, compact, high-performance, accelerating cavities. Via a systematic study of geometrical configurations, size, and dielectric/metal fractions, we showed that suitably dimensioned hybrid open structures may attain high in-plane EM radiation confinement, without significant increase in the conduction losses. The exploitation of superconducting materials (in the terminating plates) would render the fabrication of this new type of resonators extremely rewarding, even if at the expense of a higher operational complexity introduced by the cryogenic technology. Our preliminary experimental results at 100 K show that this route is technologically viable, especially for the development of very compact, hybrid, PBG cavities based on HTS materials.

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