

More on the relation between the two physically inequivalent decompositions of the nucleon spin

M. WAKAMATSU

Department of Physics, Faculty of Science, Osaka University - Osaka, Japan

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Summary. — The recent controversy on the nucleon spin decomposition problem is critically overviewed. We argue that there exist two and only two physically inequivalent gauge-invariant decompositions of the longitudinal nucleon spin, contrary to the rapidly spreading view in the QCD spin physics community that there are infinitely many decompositions of the nucleon spin.

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1. – Introduction

Is a gauge-invariant complete decomposition of the nucleon spin possible? It is a fundamentally important question of QCD as a color gauge theory. The reason is that the gauge-invariance is a *necessary condition of observability*. Unfortunately, this is quite a delicate problem, which is still under intense debate. We feel that the recent INT workshop on “Orbital Angular Momentum in QCD” increased controversy rather than settled it. We therefore believe it an urgent task to correct widespread misunderstanding on the meaning of *true* gauge-invariance in the nucleon spin decomposition problem.

2. – Decomposition of gauge field into physical and pure-gauge components

In a series of papers [1-4], we have established that there are two physically inequivalent gauge-equivalent decompositions of the nucleon spin, which we call the decomposition (I) and (II). The decomposition (I) and (II) are respectively characterized by two different orbital angular momenta (OAMs) for both of quarks and gluons, *i.e.* the “dynamical” OAMs and the generalized “canonical” OAMs. The basic assumption for deriving these two gauge-invariant decompositions of the nucleon spin is that the total gluon field can be decomposed into the two parts as

$$(1) \quad A^\mu(x) = A_{phys}^\mu(x) + A_{pure}^\mu(x),$$

satisfying the following conditions, *i.e.* the pure-gauge condition for the pure-gauge component of A^μ ,

$$(2) \quad F_{pure}^{\mu\nu} \equiv \partial^\mu A_{pure}^\nu - \partial^\nu A_{pure}^\mu - ig[A_{pure}^\mu, A_{pure}^\nu] = 0,$$

and the transformation properties for the physical and pure-gauge components of the gluon field A^μ given by

$$(3) \quad A_{phys}^\mu(x) \rightarrow U(x) A_{phys}^\mu(x) U^{-1}(x),$$

$$(4) \quad A_{pure}^\mu(x) \rightarrow U(x) \left(A_{pure}^\mu(x) + \frac{i}{g} \partial^\mu \right) U^{-1}(x),$$

under general gauge transformation of QCD. A question is whether the conditions (2), (3) and (4) are enough to uniquely fix the decomposition (1). Naturally, the answer is No! Note however that the decomposition (1) is proposed as a covariant generalization of Chen *et al.*'s decomposition given in a noncovariant form [5, 6]:

$$(5) \quad \mathbf{A}(x) = \mathbf{A}_{phys}(x) + \mathbf{A}_{pure}(x),$$

One must know the fact that, at least in the QED case, this decomposition is nothing new. It just corresponds to the standardly known transverse-longitudinal decomposition of the 3-vector potential of the photon field,

$$(6) \quad \mathbf{A}(x) = \mathbf{A}_\perp(x) + \mathbf{A}_\parallel(x),$$

satisfying the conditions

$$(7) \quad \nabla \cdot \mathbf{A}_\perp = 0, \quad \nabla \times \mathbf{A}_\parallel = 0.$$

It is a well-established fact that this decomposition is *unique* once the Lorentz frame is fixed. A crucially important ingredient here is the transversality condition $\nabla \cdot \mathbf{A}_\perp = 0$ for the transverse component \mathbf{A}_\perp . Naturally, an analogous condition is necessary to uniquely fix the physical component of A_{phys}^μ in the decomposition (1) given in the (seemingly) covariant form. This fundamental fact of gauge theory is not properly understood in the community, and conflicting views have rapidly spread around. On the one hand, Lorcé claims that the decomposition (1) is not unique because of the presence of the hidden Stückelberg-like symmetry, which alters both of A_{phys}^μ and A_{pure}^μ while keeping their sum intact [7]. This misapprehension comes from the oversight of the importance of the transversality condition that should be imposed on the physical component. We shall demonstrate this fact in the next section through a simple example from electrodynamics.

On the other, another argument against the uniqueness of the decomposition (1) is advocated by Ji *et al.* [8]. According to them, the Chen decomposition is a gauge-invariant extension (GIE) of the Jaffe-Manohar decomposition based on the Coulomb gauge, while the Bashinsky-Jaffe decomposition is a GIE of the Jaffe-Manohar decomposition based on the light-cone gauge. They claim that, since the way of GIE is not unique, there is no need that these two decompositions give the same physical predictions. This made Ji reopen his longstanding claim that the gluon spin ΔG in the nucleon is not a gauge-invariant quantity in a *true* or *traditional* sense, although it is a measurable quantity in

polarized deep-inelastic scatterings. One should recognize a self-contradiction inherent in this claim. In fact, first remember the fundamental proposition of physics: “Observables must be gauge-invariant”. The contraposition of this proposition (it is always correct if the original proposition is correct) is “Gauge-variant quantities cannot be observables”. This dictates that, if ΔG is claimed to be observable, it must be gauge-invariant also in a traditional sense.

3. – The Chen decomposition is not a GIE à la Stückelberg

In this section, we clarify the following two facts in easier QED case given by the following Hamiltonian:

$$(8) \quad H = \sum_i \frac{1}{2} m_i \dot{\mathbf{r}}_i^2 + \frac{1}{2} \int d^3r (\mathbf{E}^2 + \mathbf{B}^2),$$

which describes an interacting system of charged particles and photons. First, Chen *et al.*'s decomposition is not a GIE à la Stückelberg. Second, there are two and only two physically inequivalent decompositions of total angular momentum of charged particle and photon system. Let us start with the expression for the total angular momentum of this system.

$$(9) \quad \mathbf{J} = \sum_i \mathbf{r}_i \times m_i \dot{\mathbf{r}}_i + \int d^3r \mathbf{r} \times (\mathbf{E} \times \mathbf{B}).$$

Here, the 1st term represents the OAM carried by the charged particles, while the 2nd term does the total angular momentum of the photon. There is no doubts that the two terms on the r.h.s are both gauge-invariant. As already noticed, the vector potential \mathbf{A} of the photon field can be decomposed into longitudinal and transverse components as (6). We emphasize again that this longitudinal-transverse decomposition is *unique*, once the Lorentz frame of reference is fixed. Under a general gauge-transformation given by the following equations:

$$(10) \quad A^0 \rightarrow A'^0 = A^0 - (\partial/\partial t)\Lambda(x), \quad \mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla\Lambda(x),$$

the longitudinal and transverse components transform as follows:

$$(11) \quad \mathbf{A}_{\parallel} \rightarrow \mathbf{A}'_{\parallel} = \mathbf{A}_{\parallel} + \nabla\Lambda(x), \quad \mathbf{A}_{\perp} \rightarrow \mathbf{A}'_{\perp} = \mathbf{A}_{\perp}.$$

This means that \mathbf{A}_{\parallel} carries unphysical gauge degrees of freedom, while \mathbf{A}_{\perp} is intact under gauge transformations. To avoid misunderstanding, we emphasize that the above longitudinal-transverse decomposition should clearly be distinguished from the Coulomb gauge fixing. The Coulomb gauge fixing is to require $\nabla \cdot \mathbf{A} = 0$. Because $\nabla \cdot \mathbf{A}_{\perp} = 0$ by definition, this is equivalent to requiring that $\nabla \cdot \mathbf{A}_{\parallel} = 0$. This is the Coulomb gauge fixing condition. After this gauge choice, \mathbf{A}_{\parallel} is divergence-free as well as irrotational by definition, so that one can set $\mathbf{A}_{\parallel} = 0$ without loss of generality.

Another important remark is as follows. Naturally, the longitudinal-transverse decomposition of the 3-vector potential is *Lorentz-frame dependent*. (Anyhow, the whole treatment above is *non-covariant*.) It is true that the Coulomb gauge condition $\nabla \cdot \mathbf{A} = 0$

is not preserved, once we move to different Lorentz frame. Here, we need another gauge-transformation to get vector potential satisfying the Coulomb gauge condition. Nonetheless, the Lorentz-frame dependence of the longitudinal-transverse decomposition does not make any trouble, because one can start this decomposition in an arbitrarily chosen Lorentz frame. After all, the gauge- and frame-independence of observables is the core of Maxwell's electrodynamics as a Lorentz-invariant gauge theory!

Now we come back to our original task. As written very clearly in the text book of electrodynamics [9], the total angular momentum of the photon can actually be split into three gauge-invariant pieces as

$$(12) \quad \mathbf{J}^\gamma = \int d^3r \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = \mathbf{J}_{long} + \mathbf{J}_{trans},$$

with

$$(13) \quad \mathbf{J}_{long} \equiv \int d^3r \mathbf{r} \times (\mathbf{E}_\parallel \times \mathbf{B}) = \sum_i q_i \mathbf{r}_i \times \mathbf{A}_\perp(\mathbf{r}_i),$$

$$(14) \quad \mathbf{J}_{trans} \equiv \int d^3r \mathbf{r} \times (\mathbf{E}_\perp \times \mathbf{B}) = \int d^3r E_\perp^l (\mathbf{r} \times \nabla) A_\perp^l + \int d^3r \mathbf{E}_\perp \times \mathbf{A}_\perp.$$

Here, \mathbf{J}_{long} is nothing but the *potential angular momentum* in our terminology [1]. Each term of the above decomposition is separately gauge-invariant, because \mathbf{A}_\perp is gauge invariant.

What happens if we combine the potential angular momentum term with the ‘‘mechanical’’ angular momentum of charged particles? We get

$$(15) \quad \sum_i \mathbf{r}_i \times m_i \dot{\mathbf{r}}_i + \sum_i \mathbf{r}_i \times q_i \mathbf{A}_\perp(\mathbf{r}_i) = \sum_i \mathbf{r}_i \times (\mathbf{p}_i - q_i \mathbf{A}_\parallel(\mathbf{r}_i)).$$

Here, we have used the usual definition of the canonical momentum:

$$(16) \quad \mathbf{p}_i \equiv \partial L / \partial \dot{\mathbf{r}}_i = m_i \dot{\mathbf{r}}_i - q_i \mathbf{A}(\mathbf{r}_i) = m_i \dot{\mathbf{r}}_i - q_i (\mathbf{A}_\parallel(\mathbf{r}_i) + \mathbf{A}_\perp(\mathbf{r}_i)).$$

Note that, on the l.h.s. of (15), the \mathbf{A}_\perp terms cancel out and \mathbf{A}_\parallel remains.

This leads to a gauge-invariant decomposition corresponding to Chen *et al.*'s.

$$(17) \quad \mathbf{J} = \mathbf{L}'_p + \mathbf{S}'_\gamma + \mathbf{L}'_\gamma,$$

where

$$(18) \quad \mathbf{L}'_p = \sum_i \mathbf{r}_i \times (\mathbf{p}_i - q_i \mathbf{A}_\parallel(\mathbf{r}_i)) \Rightarrow \sum_i \mathbf{r}_i \times (1/i) \mathbf{D}_{i,pure},$$

$$(19) \quad \mathbf{S}'_\gamma = \int d^3r \mathbf{E}_\perp \times \mathbf{A}_\perp,$$

$$(20) \quad \mathbf{L}'_\gamma = \int d^3r E_\perp^k (\mathbf{r} \times \nabla) A_\perp^k.$$

The gauge-invariance of the first term can easily be convinced from the gauge transformation property of the longitudinal component

$$(21) \quad \mathbf{A}_{\parallel}(\mathbf{r}_i) \rightarrow \mathbf{A}_{\parallel}(\mathbf{r}_i) + \nabla \Lambda(\mathbf{r}_i),$$

combined with the gauge transformation property of quantum mechanical wave function of the charged particle system:

$$(22) \quad \Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) \rightarrow \left(\prod_i^N e^{i q_i \Lambda(\mathbf{r}_i)} \right) \Psi(\mathbf{r}_1, \dots, \mathbf{r}_N).$$

We emphasize that the pure-gauge covariant derivative in the Chen formalism appears automatically. The gauge degrees of freedom, carried by the longitudinal component is *not* introduced *by hand*. It exists from the beginning in the original gauge theory! This means that the Chen decomposition is not a GIE by the Stückelberg trick. Note however that the Chen decomposition is not only one GI decomposition. Because the potential angular momentum \mathbf{J}_{long} is solely gauge-invariant, we can leave it in the photon OAM part, which leads to another GI decomposition, *i.e.* the decomposition (I), according to our classification [2].

Another very important remark is as follows. It is a wide-spread belief that, among the following two quantities, *i.e.* the canonical OAM and the mechanical OAM,

$$(23) \quad \mathbf{L}_{can} = \mathbf{r} \times \mathbf{p} \Leftrightarrow \mathbf{L}_{mech} = \mathbf{r} \times (\mathbf{p} - e \mathbf{A}_{\perp}),$$

what is closer to physical image of orbital motion is the former, because the latter appears to contain an *extra interaction term* with the gauge field. This is a totally mistaken idea. In fact, the truth is just opposite. We have shown above that the “canonical” OAM is a sum of the mechanical OAM and the potential angular momentum as

$$(24) \quad \mathbf{L}^{“can”} = \mathbf{L}_{mech} + \sum_i \mathbf{r}_i \times q_i \mathbf{A}_{\perp}(\mathbf{r}_i) = \sum_i m_i \mathbf{r}_i \times \dot{\mathbf{r}}_i + \int d^3r \mathbf{r} \times (\mathbf{E}_{\parallel} \times \mathbf{B}_{\perp}).$$

As is clear from the expression of mechanical OAM given as an outer product of \mathbf{r} and $\dot{\mathbf{r}} = \mathbf{v}$, it is the “mechanical” OAM not the “canonical” OAM that has a natural physical interpretation as orbital motion of particles. It may sound paradoxical, but what contains an extra interaction term is rather the “canonical” angular momentum than the “mechanical” angular momentum.

As already pointed out, Lorcé claims that the decomposition of the gauge field into the physical and pure-gauge components is not unique because of the presence of the hidden Stückelberg-like symmetry, which changes both of A_{phys}^{μ} and A_{pure}^{μ} while keeping their sum intact [7]. This contradicts the above-explained common knowledge of electrodynamics that the transverse-longitudinal decomposition is unique once the Lorentz-frame of reference is specified. In the noncovariant treatment, the Stückelberg transformation introduced by Lorcé corresponds to a simultaneous transformation of \mathbf{A}_{\parallel} and \mathbf{A}_{\perp} :

$$(25) \quad \mathbf{A}_{\parallel} \rightarrow \mathbf{A}_{\parallel}^g = \mathbf{A}_{\parallel} - \nabla C(x), \quad \mathbf{A}_{\perp} \rightarrow \mathbf{A}_{\perp}^g = \mathbf{A}_{\perp} + \nabla C(x),$$

with $C(x)$ being an arbitrary function of space-time. (The similarity and the vital difference between the above Stückelberg transformation (25) and the standard gauge transformation (11) should be clearly recognized.) It was argued that, since this transformation alters both of \mathbf{A}_{\parallel} and \mathbf{A}_{\perp} while keeping the sum of them is intact, there are infinitely many decompositions of \mathbf{A} into \mathbf{A}_{\parallel} and \mathbf{A}_{\perp} . Here is an important oversight. The above transformation certainly keeps the irrotational-free condition for \mathbf{A}_{\parallel} , since

$$(26) \quad \nabla \times \mathbf{A}_{\parallel}^g = \nabla \times (\mathbf{A}_{\parallel} - \nabla C(x)) = 0.$$

However, it does not maintain the divergence-free (transversality) condition for \mathbf{A}_{\perp} , since

$$(27) \quad \nabla \cdot \mathbf{A}_{\parallel}^g = \nabla \cdot (\mathbf{A}_{\parallel} + \nabla C(x)) = \Delta C(x) \neq 0,$$

unless $\Delta C(x) = 0$. In the usual circumstances of electrodynamics, the harmonic function $C(x)$ satisfying $\Delta C(x) = 0$ can be set equal to zero without loss of generality owing to the Helmholtz theorem. Thus, the Stückelberg symmetry does not exist, and the transverse-longitudinal decomposition is unique.

4. – What is needed to settle the controversies

We have shown that each term of our nucleon spin decomposition (I) and (II) is separately gauge invariant, as long as the two parts of the decomposition of A^μ satisfy the conditions (2)–(4) under general color $SU(3)$ gauge transformation. The fact that we did not give explicit formula for A_{phys}^μ and A_{pure}^μ caused misunderstandings, however. To resolve this misapprehension, we emphasize again the fact that the underlying physics idea implicit in this decomposition is the transverse-longitudinal decomposition. From the physical viewpoint, the massless gauge field has only 2 transverse degrees of freedom, and the other components are not independent dynamical degrees of freedom. As was pointed out before, however, the transverse-longitudinal decomposition can be made, only after specifying a particular Lorentz frame. Fortunately, there exists a convenient method, with which we can make this decomposition in a seemingly covariant form which is convenient for perturbative calculations of Feynman diagrams. The key is a introduction of a constant 4-vector n^μ . A typical example is Coulomb gauge-type projector in QED case, which projects out the physical components of the photon field as extensively discussed by Lavelle and McMullan [10]:

$$(28) \quad A_{phys}^\mu(x) = P_{phys}^{\mu\nu} A_\nu(x),$$

where the projection operator is given by

$$(29) \quad P_{phys}^{\mu\nu} = g^{\mu\nu} + \frac{\partial^\mu \partial^\nu - \partial \cdot n (\partial^\mu n^\nu + \partial^\nu n^\mu) + n^\mu n^\nu \square}{(\partial \cdot n)^2 - \square}.$$

with $n^\nu = (1, 0, 0, 0)$ being a temporal vector. One can easily check that this projection operator satisfies the transversality condition $k_\mu P_{phys}^{\mu\nu} = P_{phys}^{\mu\nu} k_\nu = 0$. More convenient for our purpose is a general axial-gauge type projector given as follows:

$$(30) \quad P_{phys}^{\mu\nu} = g^{\mu\nu} - \frac{\partial^\mu n^\nu + \partial^\nu n^\mu}{\partial \cdot n} + \frac{n^\mu n^\nu \square}{(\partial \cdot n)^2}.$$

Here, n^μ is an arbitrary constant 4-vector, which can be either of time-like, light-like, or space-like one. Note that the above projection operator also satisfies the transversality condition $k_\mu P_{phys}^{\mu\nu} = P_{phys}^{\mu\nu} k_\nu = 0$. This ensures that A_{phys}^μ is gauge-invariant in the case of abelian gauge theory. In the case of nonabelian gauge theory, however, the physical (or transverse) component of the gluon field given by (28) and (30) satisfies the desired covariant gauge transformation property only at the lowest order in the gauge coupling constant. Accordingly, the gluon spin operator

$$(31) \quad M_{G-spin}^{\mu\nu\lambda} = 2 \text{Tr} \left[F^{\mu\lambda} A_{phys}^\nu + F^{\nu\lambda} A_{phys}^\mu \right],$$

in which $A_{phys}^\nu(x)$ and $A_{phys}^\mu(x)$ are replaced by this approximate form is regarded as a lowest order expression of more rigorously defined gluon spin operator, so that it is expected to be used in the calculation of the corresponding anomalous dimension at the 1-loop level. (See [11], for more detail.) On the basis of this expression of the gluon spin operator containing arbitrary 4-vector n^μ , which is thought to specify the Lorentz frame in which the transversality condition is given and also the quantization of the gauge field is carried out, we have calculated the 1-loop anomalous dimension matrix for the quark and gluon spin operators in the nucleon, to find that it reproduces the standardly known answer irrespectively of the choice of n^μ . This is thought to give a further evidence to the gauge-independence of gluon spin *in a traditional sense*.

5. – What is a problem of GIE approach?

Lorcé and Pasquini gave a useful relation between OAM and Wigner distribution [12]. However, gauge-invariant definition of Wigner distribution generally depends on the path of gauge link. Hatta showed that the LC-like path choice gives “canonical” OAM [13]. On the other hand, Ji, Xiong, and Yuan argued that the straight path connecting the relevant two space-time points gives “dynamical” OAM [14]. What plays a crucial role in these formulations is the so-called gauge-links. The Wigner distributions defined through such gauge-links are gauge-invariant by construction, but they are generally path-dependent. The idea of gauge-link is of general nature and has a long history. Once, DeWitt tried to formulate the quantum electrodynamics in a gauge-invariant way, *i.e.* without introducing gauge-dependent potential [15]. However, it was recognized soon that, although the framework is manifestly gauge-invariant it *does* depend on the choice of path defining the gauge-invariant potential [16-18]. It was also demonstrated that path-dependence is eventually a reflection of the gauge-dependence [19]. This dictates that, if a quantity in question is seemingly gauge-invariant but path-dependent, it is not a gauge-invariant quantity in a *true* or *traditional* sense, so that it may not correspond to observables. Undoubtedly, the GIE approach is equivalent to the standard treatment of gauge theory, only when its extension by means of gauge link is path-independent. By the standard treatment of the gauge theory, we mean the following. Start with a gauge-invariant quantity or expression. Fix gauge in response to necessity of practical calculation. Answer should be independent of gauge choice.

6. – Summary and conclusion

We have argued that there exist two and only two physically inequivalent gauge-invariant decompositions of the nucleon spin, in sharp contrast to the conflicting viewpoint that there are infinitely many decompositions of the nucleon spin. These two

decompositions, which we call (I) and (II), are characterized by two different OAMs for quarks and gluons, *i.e.* the “dynamical” OAM and the generalized “canonical” OAM. We have established the fact that the dynamical OAMs of quarks and gluons appearing in the decomposition (I) can in principle be extracted model-independently from combined analysis of GPD and polarized PDF measurements [2].

On the other hand, the observability of the OAM appearing in the decomposition (II), *i.e.* the generalized “canonical” OAM is not clear yet. This is because, although the relation between the “canonical” OAM and a Wigner distribution is suggested, its path-dependence or path-independence should be clarified more thoroughly. Moreover, once quantum loop effects are included, the very existence of TMDs as well as Wigner distributions satisfying gauge-invariance and factorization (or universality) at the same time is under investigation. Is process-independent extraction of canonical OAM possible? One must say that this is still a challenging open question.

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