IL NUOVO CIMENTO Vol. 36 C, N. 5 Settembre-Ottobre 2013
DOI 10.1393/ncc/i2013-11585-3

Colloquia: QCDN12

Canonical and kinetic decompositions of the proton spin

C. Lorcé

IPNO, Université Paris-Sud, CNRS/IN2P3 - 91406 Orsay, France and
LPT, Université Paris-Sud, CNRS - 91405 Orsay, France

ricevuto il 18 Aprile 2013

Summary. — We propose a short summary of the present situation concerning the proton spin decomposition. We briefly discuss some of the main controversies about the issues of gauge invariance, uniqueness and measurability. As a conclusion, we argue that part of the controversies is actually undecidable.

PACS 11.15.-q – Gauge field theories.
PACS 12.20.-m – Quantum electrodynamics.
PACS 12.38.Aw – General properties of QCD (dynamics, confinement, etc.).
PACS 13.88.+e – Polarization in interactions and scattering.

1. – Introduction

The last few years have seen numerous developments concerning the proper definition of quark and gluon contributions to the proton spin. In particular, the status and the physical relevance of the canonical angular momentum operators has been clarified thanks to the notion of gauge-invariant extensions. This allows one to render gauge invariant the interpretation of Δg as the gluon spin contribution. Moreover, it has been shown that one can access the canonical orbital angular momentum provided that one is able to extract experimentally either the Wigner distributions or particular twist-3 distributions. For a recent review of the discussions, see ref. [1].

In this short letter, we summarize the recent developments about the proton spin decomposition and briefly discuss the issues of gauge invariance, uniqueness and measurability. In sect. **2**, we present the suggestion made by Chen *et al.* to separate the gauge field into pure-gauge and physical terms. Although gauge invariant, this approach is not unique owing to the Stueckelberg symmetry which reflects the freedom in defining what is exactly meant by pure-gauge and physical contributions. In sect. **3**, we recall the kinetic and canonical gauge-invariant definitions of quark orbital angular momentum (OAM) and argue that there exist actually infinitely many inequivalent canonical OAM operators, raising the question of deciding which is the physical one. In sect. **4**, we show that the Wigner operator gives access to both the kinetic and canonical angular momentum operators, provided that one uses the appropriate Wilson lines. Finally, we conclude this letter with sect. **5**.

2. – Chen *et al.* decomposition and Stueckelberg symmetry

In order to unambiguously define what is meant by gluon spin and orbital angular momentum, Chen *et al.* proposed to separate explicitly the gauge degrees of freedom from the physical ones [2-5]

$$A_\mu(x) = A_\mu^{\text{pure}}(x) + A_\mu^{\text{phys}}(x), \tag{1}$$

where the pure-gauge and physical parts satisfy specific gauge transformation laws

$$A_\mu^{\text{pure}}(x) \mapsto \tilde{A}_\mu^{\text{pure}}(x) = U(x)\left[A_\mu^{\text{pure}}(x) + \frac{i}{g}\,\partial_\mu\right]U^{-1}(x), \tag{2}$$

$$A_\mu^{\text{phys}}(x) \mapsto \tilde{A}_\mu^{\text{phys}}(x) = U(x)A_\mu^{\text{phys}}(x)U^{-1}(x). \tag{3}$$

Since $A_\mu^{\text{pure}}(x)$ is a pure gauge, it can be written as

$$A_\mu^{\text{pure}}(x) = \frac{i}{g}\,U_{\text{pure}}(x)\partial_\mu U_{\text{pure}}^{-1}(x), \tag{4}$$

where $U_{\text{pure}}(x)$ is some unitary gauge matrix with the gauge transformation law

$$U_{\text{pure}}(x) \mapsto \tilde{U}_{\text{pure}}(x) = U(x)U_{\text{pure}}(x). \tag{5}$$

Clearly, in the gauge $U(x) = U_{\text{pure}}^{-1}(x)$ the pure-gauge term vanishes.

By construction, the decomposition (1) is gauge invariant $\tilde{A}_\mu = \tilde{A}_\mu^{\text{pure}} + \tilde{A}_\mu^{\text{phys}}$. However, it is not unique since we still have some freedom in defining exactly what we mean by "pure-gauge" and "physical". The reason is that the pure-gauge and physical terms remain respectively pure-gauge and physical under the following transformation leaving $A_\mu(x)$ invariant

$$A_\mu^{\text{pure}}(x) \mapsto A_\mu^{\text{pure},g}(x) = A_\mu^{\text{pure}}(x) + \frac{i}{g}\,U_{\text{pure}}(x)U_0^{-1}(x)\left[\partial_\mu U_0(x)\right]U_{\text{pure}}^{-1}(x), \tag{6}$$

$$A_\mu^{\text{phys}}(x) \mapsto A_\mu^{\text{phys},g}(x) = A_\mu^{\text{phys}}(x) - \frac{i}{g}\,U_{\text{pure}}(x)U_0^{-1}(x)\left[\partial_\mu U_0(x)\right]U_{\text{pure}}^{-1}(x), \tag{7}$$

where $U_0(x)$ is a gauge-invariant unitary matrix. At the level of $U_{\text{pure}}(x)$, this transformation reads

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}^g(x) = U_{\text{pure}}(x)U_0^{-1}(x). \tag{8}$$

While the ordinary gauge transformation acts on the left of $U_{\text{pure}}(x)$ as in eq. (5), this new transformation acts on the right. It is therefore important to distinguish them. Noting that the pure-gauge term A_μ^{pure} plays a role similar to the derivative of the Stueckelberg field, we refer to this transformation as the Stueckelberg (gauge) transformation [1]. Explicit realizations of the Chen *et al.* decomposition are usually non-local. Gauge invariance is then assured by the use of Wilson lines whose path dependence is at the origin of the Stueckelberg symmetry [6].

3. – Kinetic and canonical orbital angular momentum

There exist essentially two kinds of gauge-invariant quark orbital angular momentum. One is the *kinetic* OAM [7]

$$\mathcal{M}^{\mu\nu\rho}_{q,\text{OAM}}(x) = \frac{i}{2}\,\overline{\psi}(x)\gamma^\mu x^{[\nu}\overleftrightarrow{D}^{\rho]}(x)\psi(x) \tag{9}$$

and the other one is the *canonical* OAM [2,3]

$$\mathsf{M}^{\mu\nu\rho}_{q,\text{OAM}}(x) = \frac{i}{2}\,\overline{\psi}(x)\gamma^\mu x^{[\nu}\overleftrightarrow{D}^{\rho]}_{\text{pure}}(x)\psi(x), \tag{10}$$

where the covariant derivatives at the point x are defined as $D^\mu(x) = \partial^\mu - igA^\mu(x)$ and $D^\mu_{\text{pure}}(x) = \partial^\mu - igA^\mu_{\text{pure}}(x)$. We used for convenience the notations $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$ and $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$. These two OAMs differ by a so-called potential term [4,5]

$$\mathsf{M}^{\mu\nu\rho}_{\text{pot}}(x) = -g\,\overline{\psi}(x)\gamma^\mu x^{[\nu}A^{\rho]}_{\text{phys}}(x)\psi(x), \tag{11}$$

which is usually non-vanishing. In the gauge $U(x) = U^{-1}_{\text{pure}}(x)$, the canonical OAM simply reduces to the same expression as in the definition of the Jaffe-Manohar OAM [8] and can then be thought of as a gauge-invariant extension (GIE) of the latter [9-11].

Contrary to the kinetic quark OAM, the canonical quark OAM is not Stueckelberg invariant, *i.e.* it depends on how one explicitly separates the gauge field into pure-gauge and physical terms. There is consequently an infinite number of possible different definitions of canonical OAM, all sharing the same formal structure (10). The reduction to the Jaffe-Manohar OAM occurs in different gauges, which implies that the different canonical OAMs are not equivalent. This raises the question of deciding which canonical decomposition is the physical one.

4. – Wigner operator and its relation with orbital angular momentum

The gauge-invariant quark Wigner operator is defined as [12,13]

$$W^{[\gamma^\mu]q}(x,k) \equiv \int \frac{\mathrm{d}^4z}{(2\pi)^4}\,e^{ik\cdot z}\,\overline{\psi}(x-\tfrac{z}{2})\gamma^\mu\,\mathcal{W}_\mathcal{C}(x-\tfrac{z}{2},x+\tfrac{z}{2})\,\psi(x+\tfrac{z}{2}). \tag{12}$$

It can be interpreted as a phase-space density operator. It then is natural to define the quark OAM density as [14,15]

$$M^{\mu\nu\rho}_{q,\text{OAM}}(x) = \int \mathrm{d}^4k\,x^{[\nu}k^{\rho]}\,W^{[\gamma^\mu]q}(x,k). \tag{13}$$

In order to be gauge invariant, the definition of the Wigner operator involves a gauge link. The consequence of this gauge link is that the Wigner distribution inherits a path dependence. Using a straight gauge link in eq. (12) leads to the *kinetic* OAM L_z [6,9]. With the view of connecting the Wigner distributions to the Transverse-Momentum dependent parton Distributions (TMDs) [16,17] appearing in the description of high-energy semi-inclusive processes like Semi-Inclusive DIS and Drell-Yan, it is more natural to consider instead a staple-like gauge link consisting of two longitudinal straight lines connected at $x^- = \pm\infty$ by a transverse straight line. In this case, eq. (12) gives the *canonical* OAM ℓ_z appearing in the light-front GIE [6, 9, 15, 18].

As emphasized in the previous section, there exist formally an infinity of gauge-invariant canonical quark OAM. Note however that the proton structure is usually probed in high-energy scattering experiments. Even though physics is invariant under rotations, actual high-energy experiments provide us with a specific direction and make therefore the light-front GIE more natural, just like a Stern-Gerlach experiment provides us with a natural basis for describing the spin states.

5. – Conclusion

Separating explicitly the gauge degrees of freedom from the physical ones led to the notion of gauge-invariant extension, and allowed the definition of gauge-invariant canonical angular momentum operators. This approach has been shown to be tightly connected with the use of non-local operators and Wilson lines. However, the gauge-invariant canonical operators are not unique owing to the Stueckelberg symmetry that can be thought of as the path dependence in the non-local approach. This is nicely reflected in the definition of quark orbital angular momentum based on the Wigner operators. Depending on the choice of the path for the Wilson line, one obtains different gauge-invariant results. We stress that it is the experimental conditions that fix the Wilson lines and therefore the gauge-invariant extension to use.

* * *

I would like to thank E. Leader, A. Metz, B. Pasquini, L. Szymanowski, M. Wakamatsu and F. Wang for many helpful comments and discussions. This work was supported by the P2I ("Physique des deux Infinis") network.

REFERENCES

[1] Lorcé C., *Phys. Rev. D*, **87** (2013) 034031.
[2] Chen X.-S., Lu X.-F., Sun W.-M., Wang F. and Goldman T., *Phys. Rev. Lett.*, **100** (2008) 232002.
[3] Chen X.-S., Sun W.-M., Wang F. and Goldman T., *Phys. Rev. D*, **83** (2011) 071901.
[4] Wakamatsu M., *Phys. Rev. D*, **81** (2010) 114010.
[5] Wakamatsu M., *Phys. Rev. D*, **83** (2011) 014012.
[6] Lorcé C., *Phys. Lett. B*, **719** (2013) 185.
[7] Ji X.-D., *Phys. Rev. Lett.*, **78** (1997) 610.
[8] Jaffe R. L. and Manohar A., *Nucl. Phys. B*, **337** (1990) 509.
[9] Ji X., Xiong X. and Yuan F., *Phys. Rev. Lett.*, **109** (2012) 152005.
[10] Ji X., Xu Y. and Zhao Y., *JHEP*, **08** (2012) 082.
[11] Ji X., Xiong X. and Yuan F., arXiv:1207.5221 [hep-ph].
[12] Ji X., *Phys. Rev. Lett.*, **91** (2003) 062001.
[13] Belitsky A. V., Ji X. and Yuan F., *Phys. Rev. D*, **69** (2004) 074014.
[14] Lorcé C. and Pasquini B., *Phys. Rev. D*, **84** (2011) 014015.
[15] Lorcé C., Pasquini B., Xiong X. and Yuan F., *Phys. Rev. D*, **85** (2012) 114006.
[16] Meissner S., Metz A. and Schlegel M., *JHEP*, **08** (2009) 056.
[17] Lorcé C., Pasquini B. and Vanderhaeghen M., *JHEP*, **05** (2011) 041.
[18] Hatta Y., *Phys. Lett. B*, **708** (2012) 186.