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Generalized parton distributions and exclusive processes

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Summary. — Generalized parton distributions (GPDs) accessed in hard exclusive reactions encode information on such aspects of the QCD structure of the nucleon as its 3D parton image and the spin content. This contribution presents an overview of the status and recent progress in the phenomenology of GPDs and hard exclusive reactions.

PACS `16.60.-r` – Photon and charged-lepton interactions with hadrons.
PACS `24.85.+p` – Quarks, gluons, and QCD in nuclear reactions.

1. – QCD structure of the nucleon

One of central goals of modern nuclear physics is to understand the internal structure of nucleons and nuclei on the basis of Quantum Chromodynamics (QCD). A key outstanding problem is the spin and 3D quark-gluon structure of the nucleon which can be split into the following questions:

– What is the gluon polarization? What is partons' orbital motion? How do quarks and gluons combine to give the nucleon spin?

– How are partons distributed in the momentum and impact parameter spaces?

– What is the parton picture in the momentum space?

The master method to study the quark and gluon structure of the nucleon and nuclei has been hard processes with a large momentum transfer; the most notable example is deep inelastic scattering (DIS). In an analysis of hard processes in QCD, the principal theoretical tool is factorization theorems allowing one to introduce various universal parton distributions and to study their scale dependence.

In the case of hard exclusive processes, such as deeply virtual Compton scattering ($ep \to e'\gamma p'$) and exclusive electroproduction of mesons ($ep \to e'\rho\ (J/\psi, \pi, \dots)p'$), the QCD factorization theorems [1,2] allow us to describe these processes using the same set of universal generalized parton distributions (GPDs). Other examples of hard exclusive processes giving an access to GPDs include timelike Compton scattering ($\gamma p \to \gamma^* p' \to l^+ l^- p'$) [3] and certain hadron-initiated processes [4].

2. – GPDs: factorization, properties and physical significance

2˙1. *Factorization for DVCS*. – QCD factorization for exclusive processes can be illustrated using the case of DVCS as an example. At sufficiently large photon virtuality Q^2 and fixed momentum transfer $t = (p'-p)^2$, the DVCS amplitude can be factorized into a hard (perturbative) part and a soft (non-perturbative) part, which is parameterized in terms of GPDs. Thus, for the DVCS hadronic tensor at the leading order, one obtains [5-7]:

$$
\begin{aligned}
H^{\mu\nu} &= \frac{1}{2}(-g^{\mu\nu})_\perp \int_{-1}^{1} \mathrm{d}x\, C^+(x,\xi) \left[H\, \bar{u}(p')\hat{n}u(p) + E\, \bar{u}(p') i\sigma^{k\lambda} \frac{n_k \Delta_\lambda}{2m_N} u(p) \right] \\
&+ \frac{i}{2}(\epsilon^{\nu\mu})_\perp \int_{-1}^{1} \mathrm{d}x\, C^-(x,\xi) \left[\tilde{H}\, \bar{u}(p')\hat{n}\gamma_5 u(p) + \tilde{E}\, \bar{u}(p')\gamma_5 \frac{\Delta\cdot n}{2m_N} u(p) \right],
\end{aligned} \tag{1}
$$

where $C^{\pm} = 1/(x-\xi+i\epsilon) \pm 1/(x+\xi-i\epsilon)$; H, E, $\tilde{H}$, and $\tilde{E}$ are combinations of the corresponding helicity-conserving quark GPDs. Each GPD depends on two light-cone fractions x and $\xi \approx x_B/(2-x_B)$, where x_B is Bjorken x, the momentum transfer t and the renormalization scale Q^2.

Quark and gluon helicity-conserving and helicity-flip (transversity) GPDs can be defined through appropriate matrix elements of quark and gluon operators at a light-like separation sandwiched between the proton states with non-equal momenta. For example, for parton helicity-conserving quark GPDs, one has:

$$
\begin{aligned}
&\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ix\bar{P}^+ z^-} \left\langle p'|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|p \right\rangle_{|z^+=z_\perp=0} = \\
&\frac{1}{2\bar{P}^+}\left[H^q\, \bar{u}(p')\gamma^+ u(p) + E^q\, \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} u(p) \right] \\
&\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ix\bar{P}^+ z^-} \left\langle p'|\bar{q}(-\frac{z^-}{2})\gamma^+\gamma_5 q(\frac{z^-}{2})|p \right\rangle_{|z^+=z_\perp=0} = \\
&\frac{1}{2\bar{P}^+}\left[\tilde{H}^q\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q\, \bar{u}(p')\frac{\gamma_5\Delta^+}{2m_N} u(p) \right],
\end{aligned} \tag{2}
$$

where q is the quark flavor; $\bar{P}^+ = (p^+ + p'^+)/2$. Selecting particular final states filters out different GPDs [8]. For instance, in the case of DVCS, in addition to the helicity-conserving quark GPDs appearing at the leading order (1), the next-to-leading order (NLO) brings both helicity-conserving and transversity gluon GPDs [9]. Helicity-conserving gluon GPDs also enter the leading-order amplitude of vector meson (ρ, J/ψ) electroproduction. To access quark transversity GPDs, one needs to study electroproduction of pseudoscalar mesons, where the former appear in the twist-three contribution [10].

2˙2. *Properties of GPDs*. – Using the operator definition of GPDs (2), one can readily establish such formal properties of GPDs as their connection to the usual parton distributions in the $\xi = t = 0$ forward limit and the connection to the nucleon elastic form factors. Further, as a consequence of Lorentz invariance, one finds that Mellin moments of GPDs are polynomials (of a given finite order) in even powers of ξ. Finally, unitarity imposes positivity constraints on GPDs that provide upper bounds on the GPDs. It is a challenge to construct a model/parameterization of GPDs satisfying all these criteria.

The initial interest in GPDs—which is still topical—was ignited by the observation that GPDs quantify the total angular momentum carried by quarks (J^q) and gluons (J^g) in the nucleon [11]:

$$(3) \qquad J^q = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x x \left[H^q(x,\xi,t=0) + E^q(x,\xi,t=0)\right],$$
$$J^g = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x \left[H^g(x,\xi,t=0) + E^g(x,\xi,t=0)\right],$$

such that $\sum_q J^q + J^g = 1/2$. There are two types of further decomposition of eq. (3). The first one [11] is the explicitly gauge-invariant decomposition of the quark contribution, $J^q = S^q + L^q$ (S^q is the quark helicity and L^q is the orbital angular momentum), which does not allow for explicit parton interpretation. The second one [12] is the gauge-dependent decomposition into the quark and gluon spin and orbital angular momentum contributions, $J^q = S^q + l^q$ and $J^g = \Delta g + l^g$, both of which have a parton model interpretation. The ambiguity of the proton spin decomposition has recently become a topic of active research, for the discussion and references, see, *e.g.* [13, 14].

2˙3. *Physical significance of GPDs*. – In general, the first Mellin moments of GPDs are related to form factors of the QCD energy-momentum tensor $\hat{T}^{q,g}_{\mu\nu}$ [11]:

$$(4) \qquad \langle p'|\hat{T}^{q,g}_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[M_2^{q,g}(t)\frac{\bar{P}_\mu\bar{P}_\nu}{m_N} + J^{q,g}(t)\frac{i(\bar{P}_\mu\sigma_{\nu\rho} + \bar{P}_\nu\sigma_{\mu\rho})\Delta^\rho}{2m_N}$$
$$+d_1^{q,g}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5m_N} \pm \bar{c}(t)g_{\mu\nu}\Bigg]u(p)\,.$$

The form factors $J^{q,g}(t)$ give the parton contribution to the proton spin (see the discussion above). The form factors $d_1^{q,g}(t)$ are given by the traceless part of the three-dimensional stress tensor and, thus, describe sheer forces acting on partons in the nucleon [15]. Indeed, in the Breit reference frame, one obtains for the quark d_1 [15]:

$$(5) \qquad d_1^q(0) = -\frac{m_N}{2}\int \mathrm{d}^3\vec{r}\, T^q_{ij}(\vec{r})\left(r^i r^j - \frac{1}{3}\delta^{ij}r^2\right).$$

An explicit calculation of the form factors in eq. (4) was carried out using the chiral quark soliton model [16].

Another important property of GPDs that fuels interest in them is the observation that GPDs encode information on simultaneous longitudinal (momentum space) and transverse (coordinate space) correlations/distributions of partons in the nucleon [17, 18]. In particular,

$$(6) \quad \int \frac{\mathrm{d}^2 D_\perp}{(2\pi)^2} e^{-iD_\perp\cdot b_\perp}\left[H^q - \frac{\xi^2}{1-\xi^2}E^q\right] \propto \left\langle p'^+, -\frac{\xi b_\perp}{1-\xi}, +\frac{1}{2}|\mathcal{O}(b_\perp)|p^+, \frac{\xi b_\perp}{1+\xi}, +\frac{1}{2}\right\rangle,$$

where $D_\perp = p'_\perp/(1-\xi) - p_\perp/(1+\xi)$; the impact parameter $b_\perp$ is the distance between the active parton and the nucleon transverse center of momentum; $\mathcal{O}(b)$ is the Fourier transform of the $\bar{q}\gamma^+ q$ operator. The nucleon states in the right-hand side of eq. (6) are hybrid states characterized by the plus-momentum and the position in the transverse plane as well as by polarization.

In the $\xi = 0$ limit, the nucleon states in eq. (6) are the same and, thus, the GPDs can be given a probabilistic interpretation. In particular,

$$\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} H^q(x, 0, -\Delta_\perp^2) \equiv q(x, b_\perp)\,, \tag{7}$$

where $q(x, b_\perp)$ can be interpreted as the distribution of unpolarized quarks in the unpolarized nucleon with the given light-cone fraction x at the distance $b_\perp$ from the nucleon transverse center of momentum.

Taking the nucleon states in eq. (6) to be transversely polarized (corresponding to the x-direction in the non-relativistic limit), one finds that [18]

$$q_X(x, b_\perp) = q(x, b_\perp) - \frac{1}{2m_N} \frac{\partial}{\partial b_y} \mathcal{E}^q(x, b_\perp)\,, \tag{8}$$

where

$$\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} E^q(x, 0, -\Delta_\perp^2) \equiv \mathcal{E}^q(x, b_\perp)\,. \tag{9}$$

Therefore, the GPD E^q describes the transverse distortion of the unpolarized quark distribution in the transverse plane when the nucleon is polarized in the transverse direction.

3. – Constraining GPDs from exclusive processes

Full extraction of GPDs from experiments is very difficult because i) there is a large number of GPDs (there are 16 quark and gluon leading twist GPDs) depending on four variables, ii) observables always involve convolution (integrals) of GPDs, iii) exclusive processes are rare which means that errors are large and the kinematic coverage is limited.

Therefore, one needs to set less ambitious and more realistic goals, *e.g.*, one can try to extract the average transverse parton size $\langle b^2 \rangle$ ($\langle b^2 \rangle$ is defined through the Fourier transform of the corresponding physical observables). Exactly this strategy has been adopted (for now) for the discussed physics program of a future Electron-Ion Collider (EIC) [19,20]. An example of simulations of the DVCS cross section in the EIC kinematics and the resulting accuracy of the extraction of $\langle b^2 \rangle$ is presented in fig. 1.

Further advances in constraining GPDs are only possible by combining the full machinery of the GPD theory and phenomenology—factorization theorems for exclusive processes, modern flexible parameterizations of GPDs, dispersion relations, input from lattice QCD—with new high precision data from Hermes, Jefferson Lab at 6 and 12 GeV, Compass and the EIC.

An example of the future experiment aiming to help constrain GPDs is the measurement of timelike Compton scattering (TCS) at Jefferson Lab at 12 GeV [21], see the left panel of fig. 2. The two principal goals of the experiment are i) to test universality of GPDs and to enlarge the data set for future global QCD fits of GPDs and ii) to gain an access to the poorly constrained real part of Compton form factors (CFFs) via the interference between the TCS and Bethe-Heitler (BH) amplitudes.

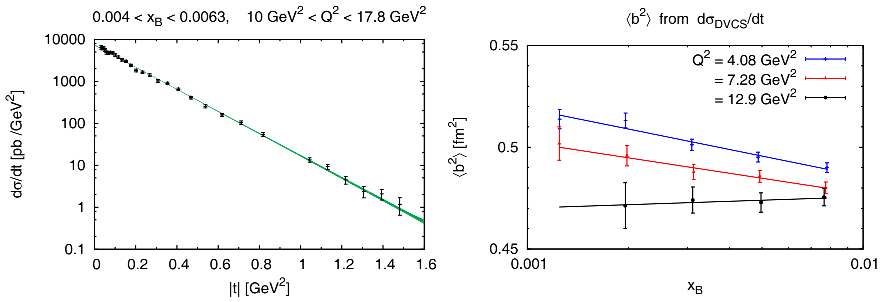

Fig. 1. – Left: Simulated DVCS cross section $d\sigma_{\mathrm{DVCS}}/dt$ at EIC for a bin in x_B and Q^2. Right: $\langle b^2 \rangle$ obtained from $d\sigma_{\mathrm{DVCS}}/dt$ for different bins in x_B and Q^2. The figure is from [20].

For the latter, it is convenient to introduce the new observable R which is constructed by special weighting of the $\gamma p \to \gamma^* p' \to e^+e^- p'$ cross section and its integration over the angles of the final e^+e^- pair [3]. Thus constructed, R singles out the TCS-BH interference term and is directly proportional to the real part of the TCS CFFs. The right panel of fig. 2 presents predictions for R as a function of $-t$ in the typical kinematics of the measurement. As one can see, future measurements of R should have the discriminating power to distinguish between different models of GPDs, thus providing additional constraints for them. (The curves labeled "DD+D-term" correspond to the double distribution model with the D-term added with different weights and the curve labeled "Dual" corresponds to the calculation with the dual parameterization of GPDs, see the details in [21].)

4. – Summary

In last fifteen years, GPDs have emerged as a powerful tool to reveal such important aspects of the QCD structure of the nucleon as 3D parton correlations and distributions in the nucleon and the spin content of the nucleon. Further advances in the field of GPDs and hard exclusive processes rely on

- developments in theory, *e.g.*, calculations of next-to-leading order and higher twist corrections,
- new methods in phenomenology, *e.g.*, the use of novel flexible parameterizations of GPDs, neural networks, and global QCD fits, and
- new high-precision data covering previously unexplored kinematic regions: JLab at 6 and 12 GeV, Hermes with recoil detector, Compass II, EIC.

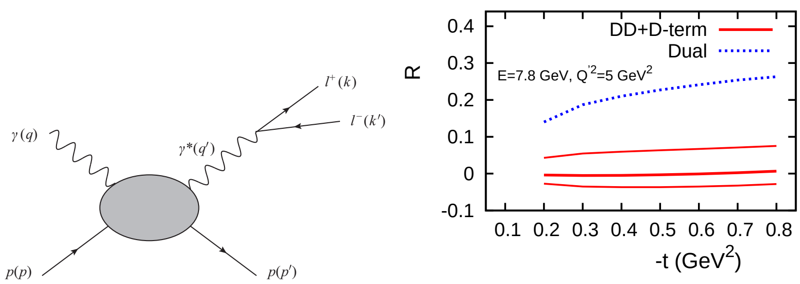

Fig. 2. – Left: Timelike Compton scattering (TCS). Right: The observable R as a function of $-t$ in the typical kinematics of the TCS measurement at JLab at 12 GeV.

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