Colloquia: QCDN12

Universality of TMD distribution functions of definite rank

P. J. Mulders[1], M. G. A. Buffing[1] and A. Mukherjee[2]

[1] *Nikhef and Department of Physics, Faculty of Sciences, VU University*
Amsterdam, The Netherlands

[2] *Department of Physics, Indian Institute of Technology*
Bombay, Powai, Mumbai 400076, India

Summary. — Transverse momentum dependent (TMD) distribution and fragmentation functions are described as Fourier transforms of matrix elements containing nonlocal combinations of quark and gluon fields. These matrix elements also contain a gauge link operator with a process dependent path, of which the process dependence that can be traced back to the color flow in the process. Expanding into irreducible tensors built from the transverse momenta p_T, we can define a universal set of TMD correlators of definite rank with a well-defined operator structure.

PACS `12.38.Aw` – General properties of QCD (dynamics, confinement, etc.).
PACS `12.38.-t` – Quantum chromodynamics.
PACS `13.88.+e` – Polarization in interactions and scattering.

1. – Introduction

Transverse momentum dependent (TMD) distribution functions (PDF) and fragmentation functions (PFF), also simply referred to as TMDs, basically are forward matrix elements of parton fields, for instance for quarks

$$(1) \qquad \Phi_{ij}(p|p) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4} \, e^{i\,p\cdot\xi} \, \langle P|\overline{\psi}_j(0)\,\psi_i(\xi)|P\rangle,$$

including a summation over color indices. For a single incoming fermion one would just have $\Phi \propto (\not{p} + m)$. In a diagrammatic approach in principle all sorts of correlators are needed, among them quark-quark-gluon correlators defined

$$(2) \qquad \Phi^{\mu}_{A\,ij}(p - p_1, p_1|p) = \int \frac{\mathrm{d}^4\xi\,\mathrm{d}^4\eta}{(2\pi)^8} \, e^{i\,(p-p_1)\cdot\xi} \, e^{i\,p_1\cdot\eta} \, \langle P|\overline{\psi}_j(0)\,A^{\mu}(\eta)\,\psi_i(\xi)|P\rangle.$$

The basic idea is to factorize these hadronic (soft) parts in a full diagrammatic approach and parametrize them in terms of PDFs. This will not work for the unintegrated correlators above. At high energies, however, the hard scale provides for each hadron light-like vectors P and n such that $P \cdot n = 1$. For instance $n = P'/P \cdot P'$, where P' is the momentum of another hadron obeying $P \cdot P' \propto s$. Using the light-like vectors, one writes down a Sudakov expansion of the parton momenta, $p = xP + p_T + (p \cdot P - xM^2)n$ with $x = p^+ = p \cdot n$. In any contraction with vectors not appearing in the correlator, the component xP contributes at order $\sqrt{s}$, the transverse component at order M and the remaining component contributes at order $M^2/\sqrt{s}$. This allows consecutive integration of the components. Starting from the fully unintegrated result in eq. (1) one obtains the TMD light-front (LF) correlator

$$(3) \qquad \Phi_{ij}(x, p_T; n) = \int \frac{\mathrm{d}\xi \cdot P \, \mathrm{d}^2\xi_T}{(2\pi)^3} \, e^{i\, p \cdot \xi} \, \langle P | \overline{\psi}_j(0) \, \psi_i(\xi) | P \rangle \Big|_{\xi \cdot n = 0},$$

and the collinear light-cone (LC) correlator

$$(4) \qquad \Phi_{ij}(x) = \int \frac{\mathrm{d}\xi \cdot P}{2\pi} \, e^{i\, p \cdot \xi} \, \langle P | \overline{\psi}_j(0) \, \psi_i(\xi) | P \rangle \Big|_{\xi \cdot n = \xi_T = 0 \text{ or } \xi^2 = 0},$$

or the local matrix element

$$(5) \qquad \Phi_{ij} = \langle P | \overline{\psi}_j(0) \, \psi_i(\xi) | P \rangle \Big|_{\xi = 0}.$$

The importance of integrating at least the light-cone (minus) component $p^- = p \cdot P$ is that the nonlocality is at equal (light-cone) time, *i.e.* time-ordering is not relevant anymore for TMD or collinear PDFs [1]. For local matrix elements one can calculate the anomalous dimensions, which show up as the Mellin moments of the splitting functions that govern the scaling behavior of the collinear correlator $\Phi(x)$. We note that the collinear correlator is not simply an integrated TMD. The dependence on upper limit $\Phi(x; Q^2) = \int^Q \mathrm{d}^2 p_T \, \Phi(x, p_T)$ is governed by the anomalous dimensions (splitting functions). One has a α_s/p_T^2 behavior of TMDs that is calculable using collinear TMDs and which matches to the intrinsic non perturbative p_T-behavior [2]. We note that in operator product expansion language, the collinear correlators involve operators of definite twist, while TMD correlators involve operators of various twist.

In order to determine the importance of a particular correlator in a hard process, one can do a dimensional analysis to find out when they contribute in an expansion in the inverse hard scale. Dominant are the ones with lowest canonical dimension obtained by maximizing contractions with n, for instance for quark or gluon fields the minimal canonical dimensions $\dim[\overline{\psi}(0) \not{n} \, \psi(\xi)] = \dim[F^{n\alpha}(0) \, F^{n\beta}(\xi)] = 2$, while an example for a multi-parton combination gives $\dim[\overline{\psi}(0) \not{n} \, A_T^\alpha(\eta) \, \psi(\xi)] = 3$. Equivalently, one can maximize the number of P's in the parametrization of Φ_{ij}. Of course one immediately sees that any number of collinear $n \cdot A(\eta) = A^n(\eta)$ fields doesn't matter. Furthermore one must take care of color gauge-invariance, for instance when dealing with the gluon fields and one must include derivatives in color gauge-invariant combinations. With dimension zero there is $iD^n = i\partial^n + gA^n$ and with dimension one there is $iD_T^\alpha = i\partial_T^\alpha + gA_T^\alpha$. The color gauge-invariant expressions for quark and gluon distribution functions actually

include gauge link operators,

$$(6) \qquad U_{[0,\xi]} = \mathcal{P} \exp\left(-i \int_0^\xi \mathrm{d}\zeta_\mu A^\mu(\zeta) \right)$$

connecting the nonlocal fields,

$$(7) \quad \Phi_{q\,ij}^{[U]}(x,p_T;n) = \int \frac{\mathrm{d}\xi \cdot P\, \mathrm{d}^2\xi_T}{(2\pi)^3} \, e^{i\,p\cdot\xi} \, \langle P|\overline{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)|P\rangle \Big|_{LF},$$

$$(8) \quad \Phi_g^{[U,U']\,\mu\nu}(x,p_T) = \int \frac{\mathrm{d}(\xi\cdot P)\, \mathrm{d}^2\xi_T}{(2\pi)^3} \, e^{ip\cdot\xi}\, \mathrm{Tr}\langle P,S|\, F^{n\mu}(0)\, U_{[0,\xi]}\, F^{n\nu}(\xi)\, U'_{[\xi,0]}\, |P,S\rangle \Big|_{LF}.$$

For transverse separations, the gauge links involve gauge links running along the minus direction to $\pm\infty$ (dimensionally preferred), which are closed with one or more transverse pieces at light-cone infinity [3, 4]. The two simplest possibilities are $U^{[\pm]} = U_{[0,\pm\infty]}^n\, U_{[0_T,\xi_T]}^T\, U_{[\pm\infty,\xi]}^n$, leading to gauge link dependent quark TMDs $\Phi_q^{[\pm]}(x,p_T)$. Which correlator is relevant in which process is a matter of doing the (diagrammatic) calculation and resummation [5,6]. For gluons, the correlator involves color gauge-invariant traces of field operators $F^{n\alpha}$, which are written in the color-triplet representation, requiring the inclusion of *two* gauge links $U_{[0,\xi]}$ and $U'_{[\xi,0]}$. Thus even with the simplest links, one has already four gluon TMDs $\Phi_g^{[\pm,\pm]}(x,p_T)$.

2. – Parametrization of TMDs

In principle quark and gluon TMDs including a gauge link need to be parametrized with a set of PDFs (or in the case of fragmentation PFFs), which just as the correlator depend on the gauge link U. For quarks [7,8] one finds the following set of functions depending on x and p_T^2:

$$(9) \quad \Phi^{[U]}(x,p_T;n) = \left\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 \right.$$
$$\left. + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\slashed{S}_T + h_{1s}^{\perp[U]}(x,p_T)\frac{\gamma_5\,\slashed{p}_T}{M} + ih_1^{\perp[U]}(x,p_T^2)\frac{\slashed{p}_T}{M} \right\} \frac{\slashed{P}}{2},$$

with the spin vector parametrized as $S^\mu = S_L P^\mu + S_T^\mu + M^2 S_L n^\mu$ and shorthand notations for $g_{1s}^{[U]}$ and $h_{1s}^{\perp[U]}$,

$$(10) \qquad g_{1s}^{[U]}(x,p_T) = S_L g_{1L}^{[U]}(x,p_T^2) - \frac{p_T \cdot S_T}{M} g_{1T}^{[U]}(x,p_T^2).$$

For quarks, these include not only the functions that survive upon p_T-integration, $f_1^q(x) = q(x)$, $g_1^q(x) = \Delta q(x)$ and $h_1^q(x) = \delta q(x)$, which are the well-known collinear spin-spin densities (involving quark and nucleon spin) but also momentum-spin densities such as the Sivers function $f_{1T}^{\perp q}(x,p_T^2)$ (unpolarized quarks in a transversely polarized nucleon) and spin-spin-momentum densities such as $g_{1T}(x,p_T^2)$ (longitudinally polarized quarks in

a transversely polarized nucleon). The parametrization for gluons, following the naming convention in ref. [9], reads

$$(11) \quad 2x\,\Gamma^{\mu\nu[U]}(x,p_{\scriptscriptstyle T}) = -g_T^{\mu\nu}\,f_1^{g[U]}(x,p_{\scriptscriptstyle T}^2) + g_T^{\mu\nu}\,\frac{\epsilon_T^{p_{\scriptscriptstyle T} S_{\scriptscriptstyle T}}}{M}\,f_{1T}^{\perp g[U]}(x,p_{\scriptscriptstyle T}^2)$$

$$+i\epsilon_T^{\mu\nu}\,g_{1s}^{g[U]}(x,p_{\scriptscriptstyle T}) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_{\scriptscriptstyle T}^2}{2M^2}\right)h_1^{\perp g[U]}(x,p_{\scriptscriptstyle T}^2)$$

$$-\frac{\epsilon_T^{p_{\scriptscriptstyle T}\{\mu}p_T^{\nu\}}}{2M^2}\,h_{1s}^{\perp g[U]}(x,p_{\scriptscriptstyle T}) - \frac{\epsilon_T^{p_{\scriptscriptstyle T}\{\mu}S_T^{\nu\}} + \epsilon_T^{S_{\scriptscriptstyle T}\{\mu}p_T^{\nu\}}}{4M}\,h_{1T}^{g[U]}(x,p_{\scriptscriptstyle T}^2).$$

For TMD correlators time-reversal does not provide constraints as future- and past-pointing is interchanged. Depending on the behavior of the Dirac structure, one can distinguish T-even and T-odd functions satisfying $f^{[U]} = \pm f^{[U^t]}$, where U^t is the time-reversed link of U. In the quark-quark correlator $f_{1T}^{\perp q}$ and $h_1^{\perp q}$ are T-odd, in the gluon-gluon correlator $f_{1T}^{\perp g}$, h_{1T}^g, $h_{1L}^{\perp g}$ and $h_{1T}^{\perp g}$ are T-odd. Since time reversal is a good symmetry of QCD, the appearance of T-even or T-odd functions in the parametrization of the correlators is linked to specific observables with this same character. In particular single spin asymmetries are T-odd observables.

3. – Moments of TMDs

In order to study the gauge link dependence, it is convenient to construct moments of TMDs. The procedure of moments is well-known for the moments of collinear functions. For $\Phi(x)$ in eq. (4) one constructs moments

$$(12) \quad x^N\Phi(x) = \int \frac{d\xi\cdot P}{2\pi}\,e^{i\,p\cdot\xi}\,\langle P|\overline{\psi}(0)\,(i\partial^n)^N\,U_{[0,\xi]}^n\,\psi(\xi)|P\rangle\Big|_{LC}$$

$$= \int \frac{d\xi\cdot P}{2\pi}\,e^{i\,p\cdot\xi}\,\langle P|\overline{\psi}(0)\,U_{[0,\xi]}^n\,(iD^n)^N\,\psi(\xi)|P\rangle\Big|_{LC}.$$

Integrating over x one finds the connection of the Mellin moments of PDFs with local matrix elements having specific anomalous dimensions, which via an inverse Mellin transform define the splitting functions. In the same way one can consider transverse moments [4] starting with the light-front TMD in eq. (3),

$$(13) \quad p_{\scriptscriptstyle T}^\alpha\,\Phi^{[\pm]}(x,p_{\scriptscriptstyle T};n) =$$
$$\int \frac{d\xi\cdot P\,d^2\xi_{\scriptscriptstyle T}}{(2\pi)^3}\,e^{i\,p\cdot\xi}\,\langle P|\overline{\psi}(0)\,U_{[0,\pm\infty]}^n\,U_{[0_T,\xi_T]}^T\,iD_{\scriptscriptstyle T}^\alpha(\pm\infty)\,U_{[\pm\infty,\xi]}^n\psi(\xi)|P\rangle\Big|_{LF}.$$

Integrating over $p_{\scriptscriptstyle T}$ gives the lowest transverse moment. This moment involves twist-3 (or higher) collinear multi-parton correlators, in particular light-cone quark-quark-gluon correlator $\Phi_F^{n\alpha}(x-x_1,x_1|x)$ starting from eq. (2), and the similarly defined correlator $\Phi_D^\alpha(x-x_1,x_1|x)$. The particular combinations that are needed in the moments are

$$(14) \quad \widetilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$$

$$= \int dx_1\,\Phi_D^\alpha(x-x_1,x_1|x) - \int dx_1\,PV\frac{1}{x_1}\,\Phi_F^{n\alpha}(x-x_1,x_1|x),$$

$$(15) \quad \Phi_G^\alpha(x) = \pi\,\Phi_F^{n\alpha}(x,0|x).$$

The latter is referred to as a gluonic pole or ETQS-matrix element [10-12,4]. The collinear correlators have trivial gauge links bridging the light-like separation and thus they have no gauge link dependence. The functions are T-even or T-odd under time reversal. While the collinear function $\Phi(x)$ is T-even, the functions contributing to the first moment involve the T-even correlator $\widetilde{\Phi}_\partial^\alpha(x)$ and the T-odd correlator $\Phi_G^\alpha(x)$. For the higher moments one finds that the relevant correlators involve similar multi-partonic correlators, *e.g.* for the double weighting one needs the T-even correlator $\widetilde{\Phi}_{\partial\partial}^{\alpha\beta}(x)$, the symmetrized T-odd correlator $\widetilde{\Phi}_{\{\partial G\}}^{\alpha\beta}(x)$ and the T-even double gluonic pole correlator $\Phi_{GG}^{\alpha\beta}(x)$. In terms of these functions we get for the moments $\Phi_{\partial...\partial}^{\alpha_1...\alpha_n[U]} \equiv \int \mathrm{d}^2 p_T \, p_T^{\alpha_1} \ldots p_T^{\alpha_n} \, \Phi^{[U]}(x, p_T)$

$$(16) \qquad \Phi^{[U]}(x) = \Phi(x), \quad \Phi_\partial^{[U]\alpha}(x) = \widetilde{\Phi}_\partial^\alpha(x) + C_G^{[U]} \, \Phi_G^\alpha(x),$$

$$(17) \qquad \Phi_{\partial\partial}^{[U]\alpha\beta}(x) = \widetilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_G^{[U]} \, \Phi_{\{\partial G\}}^{\alpha\beta}(x) + \sum_{c=1}^2 C_{GG,c}^{[U]} \, \Phi_{GG,c}^{\alpha\beta}(x),$$

etc. The gauge link dependence is contained in calculable gluonic pole factors $C_G^{[U]}$, etc. For instance the factors $C_G^{[\pm]} = \pm 1$. An additional summation is needed if there are multiple color arrangements possible for the fields, *e.g.* the summation over c in the double gluonic pole contribution of the quark-quark correlator is needed because one can have color structures $\mathrm{Tr}(GG\psi\overline{\psi})$ ($c = 1$) or $\mathrm{Tr}(\psi\overline{\psi}) \, \mathrm{Tr}(GG)/N_c$ ($c = 2$). For the simplest gauge links in quark correlators one has $C_{GG,1}^{[\pm]} = 1$ and $C_{GG,2}^{[\pm]} = 0$, but if Wilson loops appear in U the latter coefficient is nonzero.

4. – Universal TMDs of definite rank

The parametrization of quark-quark correlators in eq. (9) shows that all weightings beyond double p_T-weighting are zero. From it we deduce for a nucleon [13]

$$(18) \qquad \Phi^{[U]}(x, p_T) = \Phi(x, p_T^2) + \frac{p_{Ti}}{M} \widetilde{\Phi}_\partial^i(x, p_T^2) + \frac{p_{Tij}}{M^2} \widetilde{\Phi}_{\partial\partial}^{ij}(x, p_T^2)$$
$$+ C_G^{[U]} \left(\frac{p_{Ti}}{M} \Phi_G^i(x, p_T^2) + \frac{p_{Tij}}{M^2} \widetilde{\Phi}_{\{\partial G\}}^{ij}(x, p_T^2) \right)$$
$$+ \sum_{c=1}^2 C_{GG,c}^{[U]} \frac{p_{Tij}}{M^2} \Phi_{GG,c}^{ij}(x, p_T^2),$$

where p_T^{ij} is the second rank traceless tensor $p_T^{ij} = p_T^i p_T^j - \frac{1}{2} p_T^2 g_T^{ij}$. We refer to the correlators in eq. (18) as TMD correlators of definite rank. Each of these definite rank correlators is parametrized in terms of universal TMD PDFs. Using the T-behavior of the functions and their tensorial structure the identification is straightforward, with for instance $f_{1T}^\perp$ and $h_1^\perp$ appearing in Φ_G. Furthermore, one obtains

$$(19) \qquad f_1^{[U]}(x, p_T^2) = f_1(x, p_T^2), \quad g_{1T}^{[U]}(x, p_T^2) = g_{1T}(x, p_T^2), \quad \ldots,$$

$$(20) \qquad f_{1T}^{\perp[U]}(x, p_T^2) = C_G^{[U]} f_{1T}^\perp(x, p_T^2), \quad h_1^{\perp[U]}(x, p_T^2) = C_G^{[U]} h_1^\perp(x, p_T^2),$$

$$(21) \qquad h_{1T}^{\perp[U]}(x, p_T^2) = h_{1T}^{\perp(A)}(x, p_T^2) + C_{GG,1}^{[U]} h_{1T}^{\perp(B1)}(x, p_T^2) + C_{GG,2}^{[U]} h_{1T}^{\perp(B2)}(x, p_T^2).$$

Functions like f_1 and g_{1T} are universal. The T-odd rank-1 TMD PDFs like $h_1^\perp$ also are universal but appear in cross sections with calculable process dependent gluonic pole

factors. At rank-2, however, one has three universal T-even pretzelocity functions $h_{1T}^{\perp\cdots}$ appearing in different, but calculable,combinations in hard cross sections. For a spin-1/2 target the rank-2 quark-quark correlator $\Phi_{\{\partial G\}} = 0$, but such a rank-2 correlator is relevant for a spin one hadron [13] or for gluon-gluon TMD correlators [14]. The analogous treatment for fragmentation functions is simpler. In that case there is no process dependence [15]. Phrased in terms of operators, the gluonic pole matrix elements vanish in that case [16-18]. Nevertheless, there exist T-odd fragmentation functions, but their QCD operator structure is T-even, similar to the structure of $\widetilde{\Phi}_{\partial}^{\alpha}$.

The universal p_T^2-dependent TMD functions and correlators show up in azimuthal asymmetries, with the azimuthal dependence contained in the tensors in eq. (18). For a given rank m they just are $|p_T|^m e^{\pm im\varphi}$. For the study of the p_T^2-dependence, it is convenient to use Bessel transforms [19] of the $(-p_T^2/2M^2)$-moments,

$$(22)\quad f_{\cdots}^{(m/2)}(x, p_T^2) = \left(\frac{-p_T^2}{2M^2}\right)^{m/2} f_{\cdots}(x, p_T^2) = \int_0^\infty \mathrm{d}b \ \sqrt{|p_T|b} \, J_m(|p_T|b) \, f_{\cdots}^{(m/2)}(x, b).$$

For quarks one gets multiple color possibilities at rank two, leading to three universal pretzelocities. For gluons [14] there are more situations in which multiple color configurations show up, among them in the study of the gluon Boer-Mulders function $h_1^{\perp g}(x, p_T)$ corresponding to linear gluon polarization in an unpolarized nucleon.

$$* \ * \ *$$

REFERENCES

[1] DIEHL M. and GOUSSET T., *Phys. Lett. B*, **428** (1998) 359.
[2] COLLINS J. C., SOPER D. E. and STERMAN G. F., *Nucl. Phys. B*, **250** (1985) 199.
[3] BELITSKY A. V., JI X. and YUAN F., *Nucl. Phys. B*, **656** (2003) 165.
[4] BOER D., MULDERS P. J. and PIJLMAN F., *Nucl. Phys. B*, **667** (2003) 201.
[5] BACCHETTA A., BOMHOF C. J., MULDERS P. J. and PIJLMAN F., *Phys. Rev. D*, **72** (2005) 034030.
[6] BOMHOF C. J. and MULDERS P. J., *JHEP*, **02** (2007) 029.
[7] MULDERS P. J. and TANGERMAN R. D., *Nucl. Phys. B*, **461** (1996) 197 (Erratum **484**, (1997) 538).
[8] BACCHETTA A. *et al.*, *JHEP*, **02** (2007) 093.
[9] MEISSNER S., METZ A. and GOEKE K., *Phys. Rev. D*, **76** (2007) 034002.
[10] EFREMOV A. V. and TERYAEV O. V., *Phys. Lett. B*, **150** (1985) 383.
[11] QIU J.-W. and STERMAN G. F., *Phys. Rev. Lett.*, **67** (1991) 2264.
[12] BOER D., MULDERS P. J. and TERYAEV O. V., *Phys. Rev. D*, **57** (1998) 3057.
[13] BUFFING M. G. A., MUKHERJEE A. and MULDERS P. J., *Phys. Rev. D*, **86** (2012) 074030.
[14] BUFFING M. G. A., MUKHERJEE A. and MULDERS P. J., arXiv:1306.5897 [hep-ph].
[15] COLLINS J. C. and METZ A., *Phys. Rev. Lett.*, **93** (2004) 252001.
[16] MEISSNER S. and METZ A., *Phys. Rev. Lett.*, **102** (2009) 172003.
[17] GAMBERG L. P., MUKHERJEE A. and MULDERS P. J., *Phys. Rev. D*, **77** (2008) 114026.
[18] GAMBERG L. P., MUKHERJEE A. and MULDERS P. J., *Phys. Rev. D*, **83** (2011) 071503.
[19] BOER D., GAMBERG L., MUSCH B. and PROKUDIN A., *JHEP*, **10** (2011) 021.