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Overview of TMD phenomenology

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Summary. — We will give a brief review of the current phenomenological extractions of the Transverse Momentum Dependent distribution and fragmentation functions (TMDs).

PACS 13.66.Bc – Hadron production in e^-e^+ interactions.
PACS 13.85.Qk – Inclusive production with identified leptons, photons, or other non-hadronic particles.
PACS 13.85.Ni – Inclusive production with identified hadrons.
PACS 13.88.+e – Polarization in interactions and scattering.

1. – Introduction

Transverse Momentum Dependent (TMD) distribution and fragmentation functions are currently the subject of an intense theoretical and experimental investigation. TMD parton distribution functions, in particular, allow us to explore the three-dimensional structure of the nucleons in momentum space and therefore to shed some light onto peculiar properties of the nucleon dynamics otherwise inaccessible. The first part of this review will be devoted to the phenomenology of the Sivers function and its extraction from Semi-Inclusive Deep Inelastic Scattering (SIDIS) data. In the second part we will present the so called TMD way to extraction of the transversity function and a comparison with an alternative, collinear, approach. Finally in the last section we will discuss the Cahn effect and the extraction of the Boer-Mulders function from unpolarized SIDIS data.

2. – The Sivers function from SIDIS data

The Sivers function describes the distortion, in the transverse momentum space, of the distribution of an unpolarized quark in an a transversely polarized nucleon. If we denote with $f_{q/p^{\uparrow(\downarrow)}}(x, \boldsymbol{k}_\perp)$ the distribution of an unpolarized quark with lightcone momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$ in transversely polarized proton with

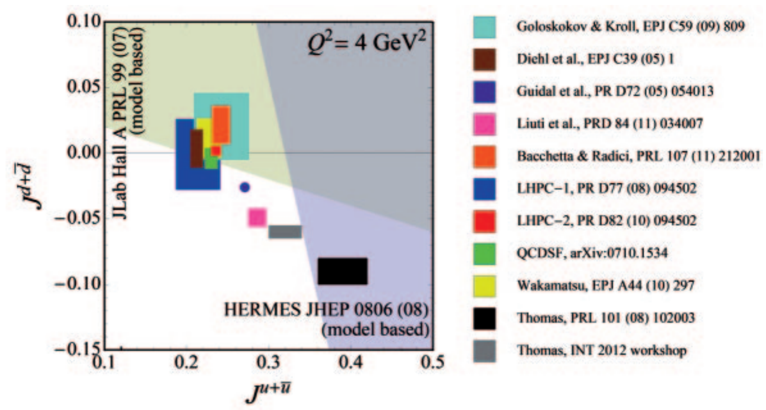

Fig. 1. – Quark angular momentum as extracted in ref. [7], compared with several calculations. Figure from ref. [8].

momentum $\boldsymbol{P}$ and transverse polarization vector $\boldsymbol{S}_T$ up, $\uparrow$ (down, $\downarrow$) then

$$f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{q/p^\downarrow}(x, \boldsymbol{k}_\perp) = \Delta^N f_{q/p^\uparrow}(x, k_\perp)\, \boldsymbol{S}_T \cdot (\hat{\boldsymbol{P}} \times \hat{\boldsymbol{k}}_\perp), \tag{1}$$

where the symbol $\hat{}$ denotes unitary vectors and $\Delta^N f_{q/p^\uparrow}$ is the Sivers function in the Turin notation. This function vanishes in collinear approximation and correlates the transverse momentum of the quarks with the polarization of the parent proton.

In 2009, Anselmino and collaborators [1] performed a fit of SIDIS data for pion and kaon production collected by the HERMES [2] and COMPASS [3,4] Collaborations. They found that the Sivers functions for u and d quarks are sizable, opposite in sign and very similar in magnitude. Sea quarks, instead, were not well constrained with the exception of the $\bar{s}$ quark contribution which was found to be quite large. This result was mainly driven by the large K^+ asymmetry found by the HERMES Collaboration.

Intuitively a distortion in the transverse momentum space corresponds in the impact parameter space to an orbiting quark along the direction of polarization of the parent proton. Therefore the Sivers function is somehow related to quark angular momentum. An exact relation between the Sivers function and the quark angular momentum is not known yet. However, lensing models [5] provide a link between the Sivers function and the Generalized Parton Distribution function (GPD) E_q which ultimately is related to the quark angular momentum [6]:

$$f_{1T}^{\perp(0)}(x) = -L(x) E_q(x,0,0), \qquad J_q = \frac{1}{2}\int_0^1 \mathrm{d}x\, x\,[H_q(x,0,0) + E_q(x,0,0)]. \tag{2}$$

Here $f_{1T}^{\perp(0)}(x)$ denotes the integral on $k_\perp$ of the Sivers function in the Amsterdam notation ($\Delta^N f_{q/p^\uparrow}(x, k_\perp) = -2(k_\perp/m_p) f_{1T}^\perp(x, k_\perp)$, where m_p is the proton mass); $L(x)$ is the lensing function; J_q denote the quark angular momentum, and the GPD $H_q(x,0,0)$ correspond to the collinear parton distribution function (PDF) $f_1(x)$. Using the model in eq. (2) and further constraints on the GPDs E_q coming from the anomalous magnetic moments, Bacchetta and Radici [7] extracted the Sivers function and the quark angular momenta from the Sivers asymmetries in SIDIS. Their results are reported in fig. 1 where they are compared to several calculations.

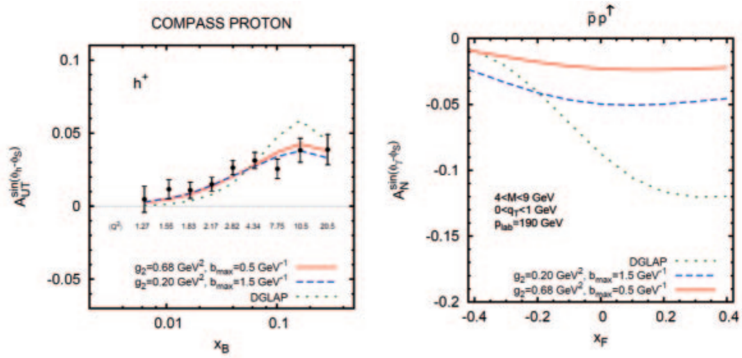

Fig. 2. – The left panel shows the result of three different fits of SIDIS data using standard DGLAP evolution (green dotted line) and TMD evolution with two different values for b_{max} and g_2: $b_{max} = 0.5\,\mathrm{GeV}^{-1}$, $g_2 = 0.68\,\mathrm{GeV}^2$ (red solid line) and $b_{max} = 1.5\,\mathrm{GeV}^{-1}$, $g_2 = 0.20\,\mathrm{GeV}^2$ (blue dashed line). In the right panel the corresponding predictions for an hypothetical $\bar{p}p^\uparrow$ Drell-Yan experiment.

In 2011 J. Collins and collaborators presented their evolution equations [9-11] for the unpolarized TMD PDF and the Sivers function. A first application [12] of the new TMD evolution equations to some limited samples of the HERMES [13] and COMPASS [14] data has shown signs of the Q^2 TMD evolution. Similarly in ref. [15] Anselmino and collaborators performed a fit of all sets of HERMES and COMPASS data using, for the first time, the TMD evolution scheme and finding a good agreement with experimental data. These analyses are very preliminary and more data, in a broader range of Q^2, are certainly needed to confirm or disprove the Collins' approach. In particular Drell-Yan data will be very fruitful in this sense. In fact, the Sivers effect in Drell-Yan processes results to be strongly sensitive to the parameter g_2 in the TMD evolution equation

$$\tilde{F}(x, b_T; Q) = \tilde{F}(x, Q_0)\ \tilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}. \tag{3}$$

Here $\tilde{F}(x, b_T; Q)$ denotes either the unpolarized TMD or the first derivative of the Sivers function in the impact parameter space; $\tilde{R}(Q, Q_0, b_T)$ is the perturbative evolution kernel; α and g_2 are the parameters governing the non-perturbative part of the evolution. The parameter g_2 is responsible of the broadening of the b_T distribution when Q increases, *i.e.* of the broadening of the transverse momentum distribution. In refs. [12, 15] g_2 is a fixed parameter coming from previous analysis of the unpolarized Drell-Yan data. Usually the parameter g_2 is strongly correlated to the parameter b_{max}, *i.e.* the parameter governing the scale at which the non-perturbative effects start. As one can see from the left panel of fig. 2, fitting SIDIS data with different g_2-b_{max} pairs leads to similar results. Notice that the pairs g_2-b_{max} used in the fit give similar, acceptable, description of the unpolarized Drell-Yan data. However if the extracted Sivers functions are then used to predict the corresponding Drell-Yan asymmetries, one obtains quite different results (left panel of fig. 2). Thus future Drell-Yan data will certainly help to further constraint the non-perturbative parameters of the TMD evolution.

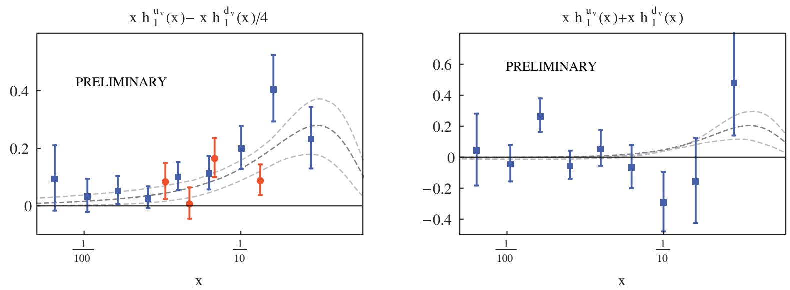

Fig. 3. – The circle red dots are the extracted transversities at HERMES; the squared blue dots are the extracted transversities at COMPASS. The dashed lines correspond to Torino's transversity [17].

3. – Transversity and Collins functions from SIDIS and e^+e^- data

The transversity function is one of the fundamental PDFs appearing in collinear approximation. Due to its chiral-odd nature, it cannot be observed in deep inelastic processes. Transversity can be studied in SIDIS processes where it appears convoluted with the chiral-odd Collins fragmentation function [16]. The Collins fragmentation function can be probed in $e^+e^- \to h_1h_2X$ processes. Therefore a combined fit of SIDIS asymmetries $A_{UT}^{\sin(\phi_h+\phi_S)}$ together with $e^+e^- \to h_1h_2X$ data allows the simultaneous extraction of the transversity distribution and the Collins fragmentation functions. In ref. [17] Anselmino and collaborators analyzed the data released by the HERMES [2] and COMPASS [3] Collaborations for SIDIS, and the A_{12}^{UL} data by the BELLE Collaboration [18] for the $e^+e^- \to \pi_1\pi_2X$ process. They found a sizable transversity, opposite in sign for u and d quarks. Similarly the Collins function exhibits an opposite sign between favored and unfavored contributions. The TMD approach used in refs. [17, 19] does not take into account the TMD evolution of the Collins function which is still unknown. The Collins function is assumed to be proportional to the unpolarized collinear fragmentation function. Only this function is evolved in Q^2 using the DGLAP evolution equation. Therefore the ratio Collins function/unpolarized fragmentation function does not evolve in Q^2. Data do not seem to contradict this model; however they are not enough to make a conclusive statement on the Collins evolution. There are instead theoretical indications that the Collins function could be strongly suppressed by TMD evolution [20, 21]. In this case, the Collins function at SIDIS scale ($Q^2 \simeq 2\,\mathrm{GeV}^2$) would be much larger than that at the BELLE scale ($Q^2 \simeq 100\,\mathrm{GeV}^2$) and therefore we would expect a smaller transversity function. An alternative way to extract the transversity function is to consider in the final state, instead of a Collins functions, a chiral odd dihadron function. Dihadron functions are collinear objects and their evolution is known. Again, a combined analysis of SIDIS and e^+e^- data allows to extract information on transversity. Analyzing the BELLE, the HERMES and the COMPASS data, the Pavia group extracted transversity using for the first time the dihadron production [22]. In fig. 3 we present the refined results published in ref. [23]. As one can see, the extraction is compatible with the results of ref. [17], based on the TMD approach. Notice, however, that the dihadron analysis suffers from the fact that the corresponding unpolarized dihadron fragmentation function is itself unknown and the extraction in refs. [22, 23] relies on the BELLE Monte Carlo description of the unpolarized data.

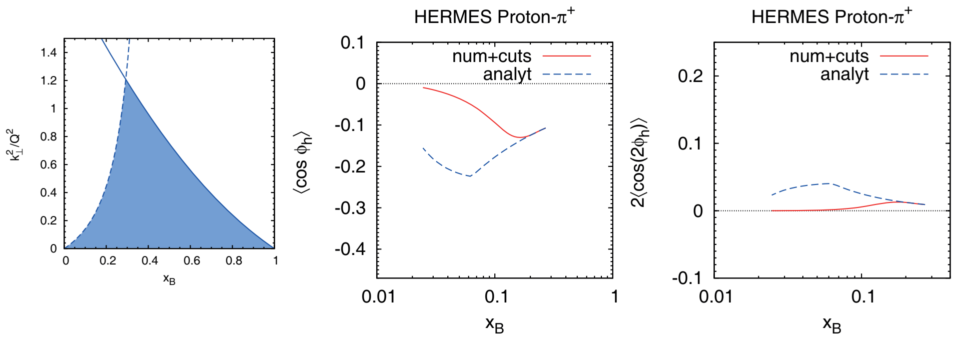

Fig. 4. – In the left panel the $k_\perp^2/Q^2$ phase space as determined by the bounds of eqs. (6) and (7). The allowed region, which fulfills both bounds, is represented by the shaded area. The Cahn contribution to the $\langle\cos\phi_h\rangle$ and $\langle\cos 2\phi_h\rangle$ asymmetries is shown in the central and right panels. The solid (red) line corresponds to the Cahn contribution calculated with a numerical $k_\perp$ integration over the range $[0, k_\perp^{max}]$ given by eqs. (6) and (7). The dashed (blue) line is obtained by integrating over $k_\perp$ analytically.

4. – Boer-Mulders function and Cahn effect from unpolarized SIDIS data

In ref. [24] Barone *et al.* performed an analysis of the $\cos 2\phi$ preliminary asymmetry measured by the COMPASS [25, 26] and HERMES [27] Collaborations in unpolarized SIDIS. At leading-twist the only $k_\perp$-dependent term contributing to the $\cos 2\phi$ asymmetry contains the Boer-Mulders distribution $h_1^\perp$ coupled to the Collins fragmentation function. Another contribution to the $\cos 2\phi$ asymmetry comes from the twist-4 Cahn contribution. Notice that the Cahn term is only a part of the complete, still unknown, twist-4 contribution.

The available data on $\langle\cos 2\phi\rangle$ do not allow for a full extraction of the Boer-Mulders function. Thus $h_1^\perp$, as suggested by lensing models [28], is assumed to be proportional to the Sivers function $f_{1T}^\perp$,

$$h_1^{\perp q}(x, k_\perp^2) = \lambda_q\, f_{1T}^{\perp q}(x, k_\perp^2), \tag{4}$$

with a coefficient λ_q to be fitted to the data. The Sivers functions and the Collins functions are taken, respectively, from ref. [29] and ref. [17]. Barone *et al.* found:

$$\lambda_u = 2.0, \quad \lambda_d = -1.1, \tag{5}$$

thus, in very nice agreement with lattice QCD calculations [30]. However, the χ^2 per degree of freedom of the fit was $\chi^2_{dof} = 3.73$. This large value reflects our poor knowledge of the underlying Cahn mechanism. In ref. [31] Boglione *et al.* observed that the very large Cahn contributions found in ref. [24] come from a kinematical region were the ratio $k_\perp^2/Q^2$ is large, thus violating the usual assumption in the TMD factorization $k_\perp/Q \ll 1$. In phenomenological analysis, the transverse momentum distribution of the TMDs is usually assumed to be a Gaussian. This is a convenient approximation as it allows to solve the $k_\perp$ integration analytically, integrating over the full $k_\perp$ range, $[0, \infty]$, and it leads to a successful description of many sets of data. However, in some kinematical region the Gaussian smearing is not sufficient to prevent large $k_\perp^2/Q^2$ contributions to the cross section. The large twist-4 Cahn effect found in ref. [24] is a remarkable example of such contributions. A physical picture that allows us to put some further constraints on the partonic intrinsic motion is provided by the parton model, where kinematical limits on the transverse momentum size can be obtained by requiring the energy of the parton to be less than the energy of the parent hadron and by preventing the parton to move backward with respect to the parent hadron direction ($k_z < 0$). They give, respectively,

$$k_\perp^2 \leq (2 - x_B)(1 - x_B)Q^2, \quad 0 < x_B < 1. \tag{6}$$

$$k_\perp^2 \leq \frac{x_B(1 - x_B)}{(1 - 2x_B)^2}Q^2, \quad x_B < 0.5. \tag{7}$$

The ratio $k_\perp^2/Q^2$, as constrained by eqs. (6) and (7), is shown in the left panel of fig. 4 as a function of x_B: from this plot it is immediately evident that, in the region spanned by present data from HERMES and COMPASS experiments ($x_b < 0.3$), eq. (7) gives a stringent limit on $k_\perp^2/Q^2$. This leads to a suppression of the Cahn effect and to a better description of some observables like the $\langle \cos\phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$ asymmetries (central and right panel of fig. 4). Moreover, it introduces some interesting effects in the $\langle P_T^2 \rangle$ behaviors, see ref. [31] for more details.

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