IL NUOVO CIMENTO Vol. 36 C, N. 5 Settembre-Ottobre 2013
DOI 10.1393/ncc/i2013-11594-2

Colloquia: QCDN12

Partonic angular momentum and the Sivers asymmetry

M. Radici
INFN - Sezione di Pavia - Pavia, Italy

ricevuto il 18 Aprile 2013

Summary. — We assume a connection between the forward limit of the GPD E and the first moment of the Sivers transverse-momentum distribution, inspired by Burkardt's idea of chromodynamic lensing. Then, we show that it is possible to fit at the same time the values of the nucleon anomalous magnetic moments and the data for semi-inclusive single-spin asymmetries originating from the Sivers effect. This opens a plausible way to quantifying the contribution of the partonic angular momentum to the spin of the nucleon, according to Ji's definition.

PACS `14.65.Bt` – Light quarks.
PACS `14.20.Dh` – Protons and neutrons.
PACS `13.60.-r` – Photon and charged-lepton interactions with hadrons.

1. – Introduction

The total angular momentum of a parton a (with $a = q, \bar{q}$) at some scale Q^2 can be computed as a specific moment of generalized parton distribution functions (GPD) [1]

$$2J^a(Q^2) = \int_0^1 \mathrm{d}x\, x \left(H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)\right). \tag{1}$$

The GPD $H^a(x,0,0;Q^2)$ corresponds to the familiar collinear parton distribution function (PDF) $f_1^a(x;Q^2)$. The forward limit of the GPD E^a does not correspond to any collinear PDF. It is not possible to probe the function E^a in experiments in the forward limit. Assumptions are eventually necessary to constrain $E^a(x,0,0;Q^2)$. The only model-independent constraint is the scale-independent sum rule

$$\sum_q e_{q_v} \int_0^1 \mathrm{d}x\, E^{q_v}(x,0,0) = \kappa, \tag{2}$$

where $E^{q_v} = E^q - E^{\bar{q}}$ and κ denotes the anomalous magnetic moment of the parent nucleon.

Denoting the Sivers function by $f_{1T}^{\perp a}$, we propose this simple relation at a scale Q_L,

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2), \tag{3}$$

where $f_{1T}^{\perp(0)a}$ is the integral of the Sivers function over transverse momentum. This assumption is inspired by theoretical considerations [2] and by results of the spectator model [3]. $L(x)$ is a flavor-independent function, representing the effect of the QCD interaction of the active quark with the rest of the spectators inside the nucleon.

The advantage of adopting the Ansatz of eq. (3) is twofold: first, the value of the anomalous magnetic moment fixes the integral of the GPD E and allows us to constrain the valence Sivers function also outside the region where SIDIS data are available; second, our Ansatz allows us to obtain flavor-decomposed information on the x-dependence of the GPD E [4] and, ultimately, on quark and antiquark total angular momentum.

2. – Extraction of Sivers function from data

To analyze SIDIS data, we use the same assumptions adopted in ref. [5]: we neglect the effect of TMD evolution, which has been studied only recently [6]; we assume a flavor- and scale-independent Gaussian transverse-momentum distribution for all involved TMDs, and we include the effect of the standard DGLAP evolution only in the collinear part of the parametrizing functions.

Neglecting c, b, t flavors, we parametrize the Sivers function in the following way:

$$f_{1T}^{\perp q_v}(x,k_\perp^2;Q_0^2) = C^{q_v}\frac{\sqrt{2e}MM_1}{\pi M_1^2\langle k_\perp^2\rangle}(1-x)f_1^{q_v}(x;Q_0^2)\,e^{-k_\perp^2/M_1^2}e^{-k_\perp^2/\langle k_\perp^2\rangle}\frac{1-x/\alpha^{q_v}}{|\alpha^{q_v}-1|}\,. \tag{4}$$

For $\bar{q}$, we use a similar function, excluding the last term. We used $\langle k_\perp^2\rangle = 0.14\,\mathrm{GeV}^2$. M_1 is a free parameter that determines the transverse-momentum width. We imposed constraints on the parameters C^a in order to respect the positivity bound for the Sivers function [7]. We multiply the unpolarized PDF by $(1-x)$ to respect the predicted high-x behavior of the Sivers function [8]. We introduce the free parameter α^{q_v} to allow for the presence of a node at $x=\alpha^{q_v}$, as suggested in refs. [3, 9-11].

For the lensing function we use the Ansatz $L(x) = K/(1-x)^\eta$. The choice of this form is guided by model calculations [3, 9], by the large-x limit of the GPD E [8], and by the phenomenological analysis of the GPD E proposed in ref. [12]. We checked *a posteriori* that there is no violation of the positivity bound on the GPD E^{q_v}.

We performed a combined χ^2 fit to the nucleon anomalous magnetic moments (for our present purposes, we take $\kappa^p = 1.793\pm 0.001$, $\kappa^n = -1.913\pm 0.001$) and the Sivers asymmetry with identified hadrons from refs. [13-15].

We set the gluon Sivers function to zero (its influence through evolution is anyway limited) and we chose $Q_0 = Q_L = 1\,\mathrm{GeV}$. We fixed $\alpha^{d_v,s_v} = 0$ (no nodes in the valence down and strange Sivers functions). We explored several scenarios characterized by different choices of the parameters related to the strange quark. In all cases, we obtained very good values of χ^2 per degree of freedom (χ^2/dof), around 1.34.

All fits lead to a negative Sivers function for u_v and large and positive for d_v, in agreement with previous studies (see ref. [5] and references therein). The data are compatible with vanishing sea-quark contributions (with large uncertainties). However, in the x range where data exist, large Sivers functions for $\bar{u}$ and $\bar{d}$ are excluded, as well as large and negative for $\bar{s}$. The Sivers function for s_v is essentially unconstrained. It turns

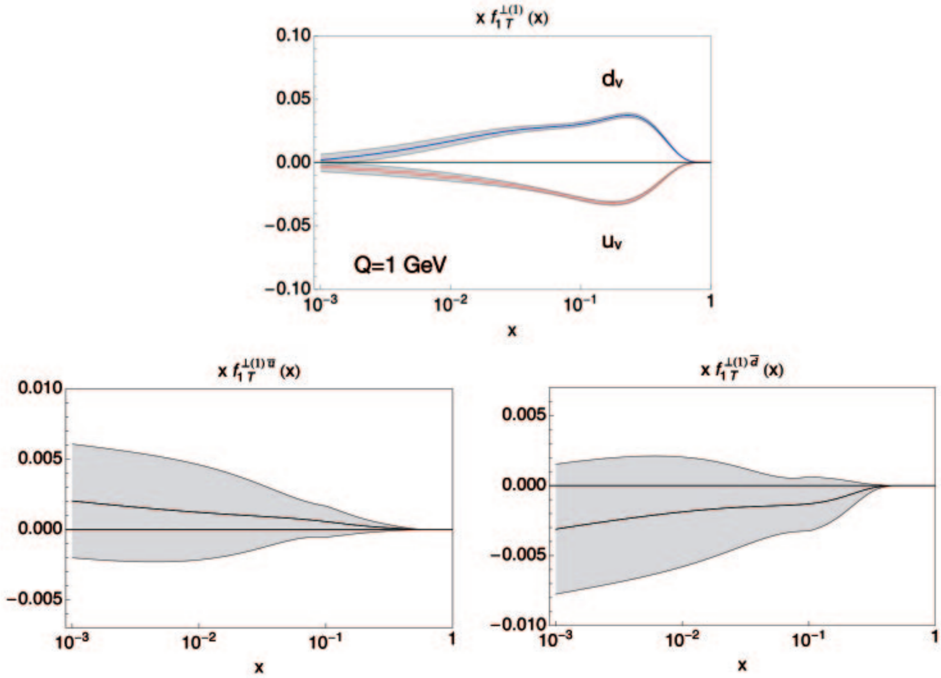

Fig. 1. – The moment $xf_{1T}^{\perp(0)\,a}(x;Q_L^2)$ of the Sivers function at the scale $Q_L^2 = 1\,\mathrm{GeV}^2$ for $a = u_v, d_v$ (top panel) and $\bar{u}$, $\bar{d}$ (lower panel). The uncertainty bands are produced by statistical errors on the fit parameters.

out $\alpha^{u_v} \approx 0.78$, so there is little room for a node in the up Sivers function, also because of the constraint imposed by the anomalous magnetic moments.

The best fit, with a χ^2/dof 1.32, is reached for vanishing strange contribution $C^{s_v} = C^{\bar{s}} = 0$. Our results for the Sivers function are shown in fig. 1 and is comparable with other extractions(for example, see ref. [6]). The results for the forward limit of the GPD E also turn out qualitatively similar to available extractions [12, 16].

3. – Quark and gluon contributions to the nucleon spin

Using eq. (1), we can compute the total longitudinal angular momentum carried by each flavor q and $\bar{q}$ at our initial scale $Q_L^2 = 1\,\mathrm{GeV}^2$. Using the standard evolution equations for the angular momentum (at leading order, with 3 flavors only, and $\Lambda_{\mathrm{QCD}} = 257\,\mathrm{MeV}$), we obtain the following results at $Q^2 = 4\,\mathrm{GeV}^2$:

$$
\begin{aligned}
J^u &= 0.229 \pm 0.002^{+0.008}_{-0.012}, & J^{\bar{u}} &= 0.015 \pm 0.003^{+0.001}_{-0.000},\\
J^d &= -0.007 \pm 0.003^{+0.020}_{-0.005}, & J^{\bar{d}} &= 0.022 \pm 0.005^{+0.001}_{-0.000},\\
J^s &= 0.006^{+0.002}_{-0.006}, & J^{\bar{s}} &= 0.006^{+0.000}_{-0.005}.
\end{aligned}
$$

The first symmetric error is statistical and related to the errors on the fit parameters, while the second asymmetric error is theoretical and reflects the uncertainty introduced by other possible scenarios. In the present approach, we cannot include the (probably large) systematic error due to the rigidity of the functional form in eq. (4). The bias induced by the choice of the functional form may affect in particular the determination of the sea quark angular momenta, since they are not directly constrained by the values of the nucleon anomalous magnetic moments. Our present estimates (at $Q^2 = 4\,\mathrm{GeV}^2$) agree well with other estimates, particularly with those ones based on the extraction of the GPD E [12, 16, 17] and on lattice simulations [18, 19]. Our study indicates a total contribution to the nucleon spin from quarks and antiquarks of $0.271 \pm 0.007^{+0.032}_{-0.028}$, of which 85% is carried by the up quark.

Our approach can be used also to estimate the size of the total angular momentum carried by the gluons. In this case, we expect the lensing function to be different from that of the quarks. However, our extraction leaves little room for a nonzero gluon Sivers function, since the quark Sivers function already saturates the so-called Burkardt sum rule [20]. If the Sivers function of the gluons is zero, our reasoning allows us to conclude that E^g is also zero, independent of the details of the lensing function. This would lead to a value of $J^g = 0.215$ at $4\,\mathrm{GeV}^2$, which agrees with the result in refs. [16, 17], and seems compatible within errors with our finding of $J^{q+\bar{q}} = 0.271 \pm 0.007^{+0.032}_{-0.028}$. However, these considerations are strongly affected by the uncertainties on the sea-quark Sivers functions outside the x range where data exists. Direct measurements of the sea-quark and gluon Sivers functions are therefore highly necessary.

At this point, we add a remark on the effect of TMD evolution on the Sivers function. The discussion in ref. [6] suggests that the inclusion of TMD-evolution effects might lead to larger values of the Sivers function at the starting scale Q_0^2. If this were the case, we would need to compensate the effect by a smaller size of the lensing function in order to have an agreement with the anomalous magnetic moments. However, this will have a negligible net effect on the results for J^a, also because TMD evolution mildly modifies the Sivers function for sea quarks [6].

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