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GPDs, their relationships with TMDs and related topics

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Summary. — Generalized Parton Distributions in the chiral odd sector are evaluated by examining their contribution to the quark proton helicity amplitudes, and by using Parity transformations to establish relations with the chiral even sector. The relations we find are valid in a spectator model where the recoiling system has total spin S=0,1. These relations allow us to fix the parameters for the chiral odd generalized parton distributions using results from a global analysis of unpolarized and longitudinally polarized experimental data in the kinematical region of intermediate Bjorken x and for Q^2 in the multi-GeV region. Quantitative relations between chiral odd generalized parton distributions and transverse momentum dependent distributions are given, and their extension beyond spectator models is discussed. Contributions to the Generalized Transverse Momentum Distributions are obtained and considered in relation to observables. The related question about recoiling baryon polarizations is discussed.

PACS 13.60.Hb – Total and inclusive cross sections (including deep-inelastic processes).
PACS 13.40.Gp – Electromagnetic form factors.
PACS 24.85.+p – Quarks, gluons, and QCD in nuclear reactions.

1. – Introduction

Understanding the structure of the hadrons and their interactions is one of the oldest quests of particle and nuclear physics. It is expected that the composition of the hadrons is determined by QCD, yet the strong interaction at low momenta or long distances makes that composition —in terms of quarks and gluons— a highly non-linear problem. Different aspects of hadron composition, whether momentum, configuration space or spin, are encoded by different distribution functions. Figure 1 is a schematic map indicating the different kinds of distributions, their kinematic variables and the connections among

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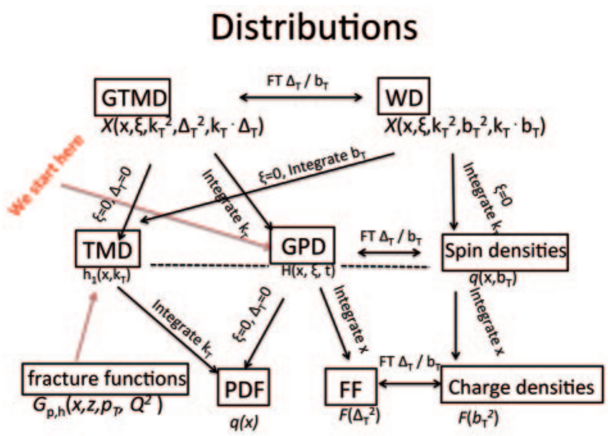

Fig. 1. – Relations among distributions.

them. The most general are the Wigner distributions that depend on the full phase space kinematics of the quark (or gluon) —the impact parameter and transverse momenta. The canonical variables are defined relative to the hadron's light cone frame. The GTMDs are the Fourier Transforms of the Wigner distributions in which the impact parameter $\vec{b}_T \to \vec{\Delta}_T$, so that the GTMDs are functions of the quark (or gluon) momenta and the nucleon (or hadron) momenta. Integrating over the quark transverse momentum gets down to the GPD structures. For every fixed $\vec{k}_T$ the GTMD resembles an amplitude for quark-nucleon scattering (at fixed $k^+ = xP^+$). This observation provides us with the starting point of a parametrization of all of the distributions, beginning with the GPDs. In the following we summarize our recent work on Hadron Spin Structure from GPD & TMD to GTMD & Fracture functions perspectives.

2. – Flexible hybrid parametrization

We have obtained a thorough parametrization of all quark-nucleon chiral even GPDs [1] using a diquark spectator model. The scalar diquarks (for u-quarks) and axial vector diquarks (for d-quarks and u-quarks) are considered. Regge behavior for small x_{Bj} is built in by replacing single mass diquarks with variable masses as determined by a spectral distribution that has high mass behavior $m^{2\alpha}_{diquark}$ as in ref. [2]. We use constraints from form factors, various sum rules, pdf's and DVCS data to determine the parameters. One of many examples of the predictions of the model is the beam spin asymmetry shown in fig. 2, left panel.

Having achieved a successful parametrization of the chiral-even GPDs, we applied the scheme to reevaluating the EM form factors for u- and d-quarks as measured most accurately at JLab. This involves the first moments of $H(x, \xi, t)$ and $E(x, \xi, t)$ and allows very precise parametrization, as shown in ref. [3].

3. – π^0 electroproduction

The fits to data on EM Form Factors, DVCS cross sections and asymmetries and sum rules gave us confidence in extending the parametrization to the meson production

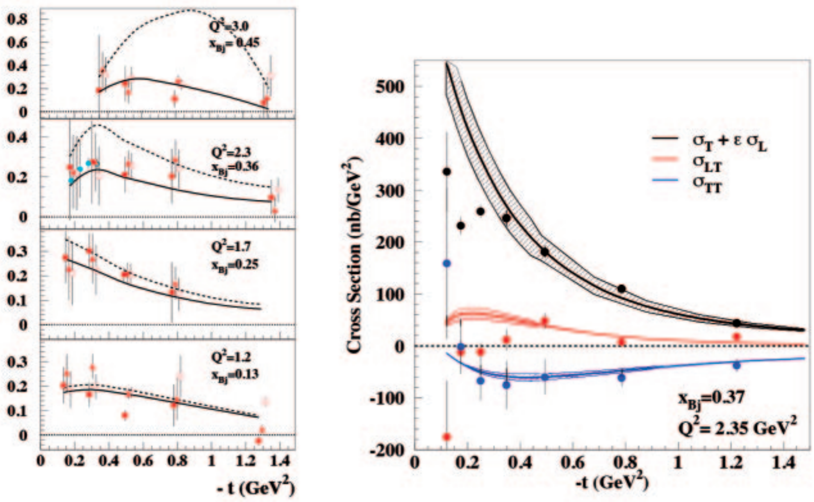

Fig. 2. – Left: Beam spin asymmetry, $A_{LU}(\phi = 90°)$ in 12 of the x_{Bj} and Q^2 bins measured in Hall B [6]. The second panel from the top includes also data from Hall A [7]. See ref. [1] for details. Right: $\mathrm{d}\sigma_T/\mathrm{d}t + \epsilon_L \mathrm{d}\sigma_L/\mathrm{d}t$, $\mathrm{d}\sigma_{TT}/\mathrm{d}t$, and $\mathrm{d}\sigma_{LT}$ calculated from parametrization of ref. [8], along with data from Hall B [5]. The hatched areas represent the theoretical errors for the parametrization, see refs. [1, 3].

processes. Of particular interest has been π^0 electroproduction where the small t behavior is not dominated by the pion pole exchanged in the t-channel of the process. The quantum numbers exchanged in the t-channel lead to the convolution of the hard part of the process, $\gamma + \text{quark} \to \pi^0 + \text{quark}$, with the two chiral even GPDs $\widetilde{H}$ and $\widetilde{E}$ and the four chiral odd GPDs, H_T, E_T, $\widetilde{H}_T$, $\widetilde{E}_T$. Can all of these contribute? It was shown a while ago [4] that to leading order in Q^2 the amplitude for the exclusive electroproduction *through virtual longitudinal photons* of pseudoscalar and vector mesons factorizes into the hard part and the GPDs. This involves having "leading twist" distribution amplitudes for the quark + antiquark to the meson. For transverse virtual photons there is no proof of factorization, this being down by $1/Q^2$ in the cross sections. We developed a different approach, for which we were guided by several observations: The measured cross sections [5] have sizable contributions from transverse virtual photons without the expected Q^2 behavior in the accessible range of Q^2. The distribution amplitude for the π^0 has a general form $\mathcal{P} = K f_\pi \{\gamma_5 \;\not{q}' \phi_\pi(\tau) + \gamma_5 \mu_\pi \phi_\pi^{(3)}(\tau)\}$, and the "twist 3" part contributes to the transverse photon amplitude and the chiral odd GPDs. The chiral even GPDs are primarily coupled to the longitudinal photon. In terms of the chiral odd GPDs, the cross sections have the form that is exemplified by two of them

$$\frac{\mathrm{d}\sigma_T}{\mathrm{d}t} \approx \mathcal{N}^2 [g_\pi^{odd}(Q)]^2 \, \frac{1}{(1+\xi)^4} \left[\mid \mathcal{H}_T \mid^2 + \tau \left(\mid \overline{\mathcal{E}}_T \mid^2 + \mid \widetilde{\mathcal{E}}_T \mid^2 \right) \right],$$

$$\frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t} \approx \mathcal{N}^2 [g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4} \, \tau \left[\mid \overline{\mathcal{E}}_T \mid^2 - \mid \widetilde{\mathcal{E}}_T \mid^2 + \Re e \mathcal{H}_T \, \frac{\Re e(\overline{\mathcal{E}}_T - \mathcal{E}_T)}{2} \right.$$
$$\left. + \, \Im m \mathcal{H}_T \, \frac{\Im m(\overline{\mathcal{E}}_T - \mathcal{E}_T)}{2} \right],$$

where $\tau = (t_o - t)/2M^2$, and $\overline{\mathcal{E}}_T = 2\mathcal{H}_T + \mathcal{E}_T$.

Very little is known about the size and overall behavior of the chiral odd GPDs, besides that H_T becomes the transversity structure function, h_1, in the forward limit, $\overline{E}_T$'s first moment is the proton's transverse anomalous magnetic moment, and $\widetilde{E}_T$'s first moment is null. To evaluate the chiral odd GPDs in ref. [8] we used Parity transformations in a spectator picture, which allows us to write them as linear combinations of the better determined chiral even GPDs [1]. Along with the π^0 distribution amplitude and the separation of the natural and unnatural parity transition form factors, the amplitudes for the process were completely determined and all experimentally accessible observables calculated [8]. Figure 2, right panel, shows the results for one pair of x_{Bj} and Q^2 values.

4. – Generalized distributions

As illustrated in fig. 1, the GPDs can be linked to GTMDs and Wigner Distributions [9]. The flexible parametrization that we have used for GPDs will yield functional forms for each of the distributions. We have been studying some of the resulting Wigner Distributions [10]. Among the leading twist, spin dependent GTMDs, there are twice as many as either GPDs or TMDs. Do these extra GTMD distributions provide information that cannot be extracted from the measurable GPDs and TMDs? We have been studying this question from the symmetry perspective. GTMDs correspond to quark (or gluon) + nucleon scattering amplitudes. It is known that many GTMDs would violate parity conservation in either the $\Delta_T \to 0$ limit, TMDs, or in the integration over parton momenta, GPDs. Those GTMDs, particularly F_{14} and G_{11} in the notation of ref. [9], correspond to forbidden observables in either limit. For example, F_{14} would correspond to a distribution in which the nucleon is longitudinally polarized, but the quark is unpolarized. Such an asymmetry measurement would violate parity conservation. This leaves open many questions about the meaning of such distributions.

5. – Sources of hyperon polarization

There is a long-standing puzzle in hadronic spin physics, the inclusive production of highly polarized strange hyperons. The expectation from Perturbative QCD based on the Kane, Pumplin, Repko (KPR) calculation [11] was that such polarization should be small as governed by the quantity $\alpha_s(\hat{s})m_q/\sqrt{\hat{s}}$, where m_q is the effective mass of the strange quark (or the hyperon) that normalizes the helicity flip vertices, and $\sqrt{\hat{s}}$ a characteristic energy of the hard production process for the quark. To explain why measurements did not agree with the expectation, soft processes or semi-classical mechanisms of various kinds have been proposed over the years. One approach by Dharmaratna and Goldstein (DG) [12] calculated all the 4th order hard QCD amplitudes that enter in hadron+hadron collisions via parton+parton producing s-quark pairs or heavier flavors. A simple ansatz for that soft process used the analogy with the "Thomas precession" mechanism [13] to boost the x_F(quark) to x_F(hadron) while enhancing the scale of the polarization by $\approx 2\pi$. Taken at face value this predicted [14] the behavior of the Λ_c data from Fermilab experiment E791 [15]. The attractive features of the DG "hybrid model" suggest a more rigorous QCD based formulation.

Several directions are being pursued to explore the hyperon and heavy flavor baryon polarization phenomena as applied to LHC energies, where the $g + g$ mechanism is dominant, especially at small Bjorken x for both protons. To begin with, the leptoproduction framework is used in preparation for considering purely hadronic production. One approach is through the "fracture function" picture of Trentadue and Veneziano [16], as

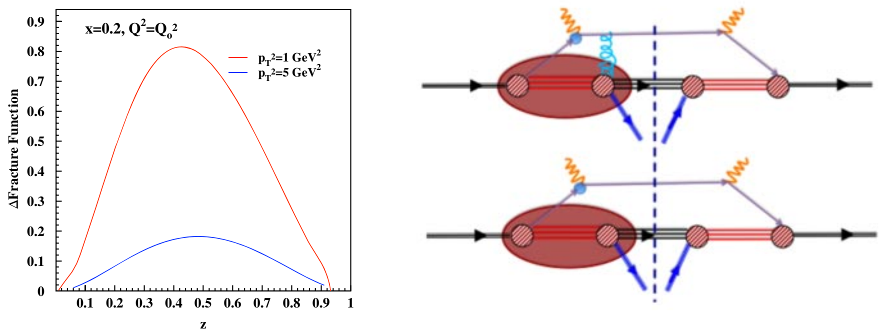

Fig. 3. – Left: Tree level Fracture Function for Λ production at $x = 0.2$ for two values of p_T. Right: Diagram for polarized Λ fracture function (upper graph); tree level contribution to unpolarized Λ fracture function (lower graph).

extended to include angular momentum dependences by Sivers [17]. The fracture functions are joint probabilities that relate the beam direction hard scattered quark distribution from the target nucleon (a function of x_{Bj}) to the baryon fragmentation of the target (a function of z). Another approach is from the GPD perspective.

The *Fracture Functions* (FF) provide a QCD-based perspective on target fragmentation. For a quark-nucleon FF,

$$\mathcal{F}^{\lambda_q}_{\Lambda_N;\Lambda'_\Lambda,\Lambda_\Lambda}(x, k_T, z, p_T, Q^2) = \sum_{\Lambda_X} \int \frac{\mathrm{d}^3 P_X}{(2\pi)^3 2E_X} \int \frac{\mathrm{d}^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \times \langle P, \Lambda_N \mid \bar{\psi}^{\lambda_q}(\xi) \mid P_h, \Lambda'_\Lambda; X\rangle \times \langle P_h, \Lambda_\Lambda; X \mid \psi^{\lambda_q}(0) \mid P, \Lambda_N\rangle. \tag{1}$$

where transverse momenta k_T, p_T are for the quark and the outgoing hadron, respectively.We evaluated the behavior of FFs in describing leptoproduction of polarized hyperons within the spectator model - the diquark spectator model [1]. To obtain polarization in this picture a phase difference has to be generated between interfering helicity flip and non-flip amplitudes. As Sivers has noted [17], there has to be a means to generate a "Boer-Mulders" FF or a "Polarizing" FF. The mechanism we considered is illustrated by having an additional gluon exchange as in the upper right diagram of fig. 3 connecting the outgoing parton to the spectator or the fragmenting inclusive state X.

The resulting Fracture Function $\mathcal{F}$, shown for two values of p_T produces the polarization pattern in fig. 3, left. The corresponding diagram is the lower right diagram of fig. 3. This picture is completed for the leptoproduction and is being extended to the hadronic process, $p + p \to \Lambda + X$. Results will be presented elsewhere.

In the *GPD description* the process amplitudes are linear combinations of the complex Compton Form Factors (CFFs). Again, we are interested in the $p + p$ process, so the virtual photon will have to be replaced by an antiquark or a gluon. Then the variables are related as $x_1 \equiv X$, the quark, gluon or antiquark fraction of the initial proton momentum, $x_F = 1 - \zeta > 0$, the hyperon's momentum fraction, $p_T^2 = \Delta_\perp^2$ which is related to t. The scale of the process is Q^2.

When the protons interact through having virtual photons replaced by antiquarks or gluons, then the general form of the cross section will be

$$\begin{aligned} (2) \qquad & \int \mathrm{d}x_2 \;[\mathcal{H}^*_{N\to Y}\mathcal{H}_{N\to Y}]\,(\zeta(x_F), t(p_T^2), Q^2) \\ & \times f(x_2, Q^2)\hat{\sigma}_{12\to sX}(x_2, x_F^s, p_T) \end{aligned}$$

where $N \to Y$ represents the nucleon to hyperon transition. The hard subprocess involves helicity conserving interactions for zero mass quarks. The only contributions to the overall convoluted helicity amplitudes will be those for which the Extended GPD ($p \to u : s \to \Lambda$) has the form $A_{\Lambda_\Lambda,\lambda_s=+;\Lambda_p,\lambda_u=+}$ or $A_{\Lambda_\Lambda,\lambda_s=-;\Lambda_p,\lambda_u=-}$. We evaluate these off-diagonal GPDs using the parameterization of ref. [1] with $\mathrm{SU}(3)_{flavor}$ for the Lambda-diquark-quark vertex. Once the off-diagonal CFFs have been determined, the resulting complex, helicity dependent amplitude structures are folded into the $p + p \to \Lambda + X$ helicity amplitudes to obtain cross sections and polarizations. This will be presented elsewhere.

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