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On singularities of the TMDs, their origin and treatment

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Summary. — The origin of the specific singularities arising in transverse-momentum dependent parton distribution functions is discussed and a new method of their treatment is proposed.

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Quantum Chromodynamics provides consistent description of the strong interactions at small distance (large characteristic momentum), where the standard methods of perturbation theory supplied with the renormalization-group based resummations of large logarithms are directly applicable. The non-Abelian nature of QCD plays a crucial role at large distance (small momentum scale), where the coupling constant α_s grows, thus breaking down the applicability of the standard perturbative methods. In the large distance domain set by the typical hadronic length $R_H \geq \Lambda_{\text{QCD}}$ the intrinsic structure of hadrons has to be taken into account. Internal hadronic degrees of freedom can be "seen" in different experiments with different kinematical setup. The resulting images of the hadrons are, therefore, also different. In particular, the longitudinal structure of hadrons is accessible in fully inclusive deeply inelastic electron-nucleon scattering experiments. It is successfully described in terms of the collinear (integrated) parton distribution functions (PDFs). The novel view of the quark and gluon content of nucleons arises as the result of the research programs dealing with high-energy semi-inclusive reactions with polarized and unpolarized hadrons in initial and final states, where the transverse motion and the spin-orbit correlations of the partons can be directly probed [1]. Understanding the partonic structure of nucleons beyond the collinear approximation requires appropriate development of the theory. Multi-dimensional (beyond-the-collinear) imaging of hadrons is an ideal test field of QCD as the true theory of strong interactions. The novel

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information of the spin and transverse partonic degrees of freedom will result from the current and planned experiments at DESY, CERN, RHIC, JLab 12 GeV and Electron-Ion Collider (see, *e.g.*, [1] and references therein).

The present consideration addresses several important issues related to the study of the three-dimensional image of nucleons within the transverse momentum-dependent (TMD) factorization approach to the semi-inclusive hadronic processes [2]. Given that the TMD matrix elements involve several local field operators defined at different space-time points separated by non-light-like intervals, one has to introduce a system of the Wilson lines that saves gauge invariance, although giving rise to a functional dependence on path [3-5]. The crucial observation is that the requirement of the complete gauge invariance calls for the very specific structure of the gauge links (WIlson lines) in the operator definition of TMDs. Derivation of the proper evolution equations for the TMD correlators is intimately related to the peculiar singularity structure of the light-like cusped Wilson lines and loops of various shapes [6, 7]. Strong interest in the mathematical structure of the corresponding loop space arises also in the context of the duality between the $n-$gluon scattering amplitudes in the $\mathcal{N} = 4$ super-Yang-Mills theory and vacuum expectation values of the Wilson loops of special form (see, *e.g.*, the last entree in [8]). Therefore, we observe possible relationship between the generic properties of the polygonal Wilson loops and gauge-invariant hadronic correlators. Recently we proposed a new approach to the analysis of the correlation functions which contain cusped light-like Wilson lines [9]. This approach is based on a generalization of the universal quantum dynamical principle by J. Schwinger and allows us to derive a differential equation which connects the area variations and renormalization group behavior of those objects [10].

In a few words, the logical structure of this approach is as follows: We first made use of the result that for the light-like planar dimensionally regularized (not renormalized) Wilson rectangles the area variations can be introduced in terms of the ordinary derivatives. The area differential equations in the configuration space determine then the evolution of the light-like Wilson polygons and represent, therefore, the equations of motion in the loop space. As the result, the derived differential equations provide a closed set of dynamical equations for the loop functionals, and can be explicitly solved in several interesting cases.

We start with the quantum dynamical principle proposed by Schwinger [10], which we adopt in the following form: the area variations of a generic path-dependent functional $\Phi(\Gamma)$ are governed by the equation

$$\frac{\delta}{\delta\sigma} \langle\, \alpha' \,|\Phi(\Gamma)|\, \alpha'' \,\rangle = \frac{i}{\hbar} \langle\, \alpha' \,| \frac{\delta \hat{S}}{\delta\sigma} \Phi(\Gamma) |\, \alpha'' \,\rangle, \tag{1}$$

where $\hat{S}$ is an analogue of the Schwinger quantum action operator applicable to "highly-singular" (for example, light-like) field correlators. The equations of motion in the loop space must be the laws which prescribe how the loops change their shape. We have to find the proper operator $\hat{S}$, which governs the shape variations of the Wilson planar polygons.

For an arbitrary *smooth* Wilson loop $\mathcal{W}(\Gamma)$, the functional reads $\Phi(\Gamma) = 1/N_c\, \mathrm{Tr}_c$-$\mathcal{P} \exp[ig \oint_\Gamma dz\, \mathcal{A}(z)]$ and the leading $O(g^2)$ non-trivial term of its perturbative series reads

$$\mathcal{W}^{(1)} = 1 - \frac{g^2 C_F}{2} \oint_\Gamma \oint_\Gamma \mathrm{d}z_\mu \mathrm{d}z'_\nu\, D^{\mu\nu}(z - z'), \tag{2}$$

where $D^{\mu\nu}$ is the dimensionally regularized ($\omega = 4 - 2\epsilon$) gluon propagator (we adopt the Feynman gauge)

$$D^{\mu\nu} = -g^{\mu\nu}\,\Delta(z - z'), \qquad \Delta(z - z') = \frac{\Gamma(1-\epsilon)}{4\pi^2}\,\frac{(\pi\mu^2)^\epsilon}{[-(z-z')^2 + i0]^{1-\epsilon}}\,. \tag{3}$$

Then one gets

$$\frac{\delta \mathcal{W}(\Gamma)}{\delta\sigma_{\mu\nu}} = \frac{g^2 C_F}{2}\,\frac{\delta}{\delta\sigma_{\mu\nu}}\oint_\Gamma\oint_\Gamma \mathrm{d}z_\lambda \mathrm{d}z'^{\lambda}\,\Delta(z - z') + O(g^4). \tag{4}$$

Applying the Stokes theorem, one obtains $\oint_\Gamma dz_\lambda\,\mathcal{O}^\lambda = \frac{1}{2}\int_\Sigma \mathrm{d}\sigma_{\lambda\rho}(\partial^\lambda\mathcal{O}^\rho - \partial^\rho\mathcal{O}^\lambda)$, $\mathcal{O}^\lambda = \oint_\Gamma \mathrm{d}z^\lambda\,\Delta(z)$, where Γ is the boundary of the surface Σ. We get therefore the leading perturbative term of the Makeenko-Migdal equation [3]:

$$\partial_\mu\frac{\delta\mathcal{W}(\Gamma_{\text{smooth}})}{\delta\sigma_{\mu\nu}(x)} = \frac{g^2 N_c}{2}\oint_{\Gamma_{\text{smooth}}}\mathrm{d}y_\nu\,\delta^{(\omega)}(x-y) + O(g^4). \tag{5}$$

Computing the NLO terms, one obtains the Makeenko-Migdal equation. However we are mostly interested in the Wilson functionals which do not satisfy the conditions of the applicability of the Stokes theorem, thus the derivation must be revisited.

In the present analysis we restrict ourselves to the shape variations of the planar light-like Wilson rectangles preserving (classically) conformal invariance [8]. The leading non-trivial term reads

$$\begin{aligned}\frac{\delta W^{(1)}(\Gamma_{\text{rect.}})}{\delta\sigma_{\mu\nu}} = \frac{g^2 C_F}{2}\frac{\Gamma(1-\epsilon)(\pi\mu^2)^\epsilon}{4\pi^2}\,\frac{\delta}{\delta\sigma_{\mu\nu}}\sum_{i,j}(v_j^\lambda v_j^\lambda)\,\cdot \\ \times\int_0^1\int_0^1\frac{\mathrm{d}\tau\mathrm{d}\tau'}{[-(x_i - x_j - \tau_i v_i + \tau_j v_j)^2 + i0]^{1-\epsilon}}\,,\end{aligned} \tag{6}$$

where the sides of the rectangle are parameterized as $z_i^\mu = x_i^\mu - v_i^\mu\tau$ with the light-like vectors v_i. For this special class of the Wilson functionals, the area gets factorized from the integrals and can be evaluated explicitly

$$W^{(1)}(\Gamma_{\text{rect.}}) = -\frac{\alpha_s N_c}{2\pi}\Gamma(1-\epsilon)(\pi\mu^2)^\epsilon(-2N^+N^-)^\epsilon\,\frac{1}{2}\int_0^1\int_0^1\frac{\mathrm{d}\tau\mathrm{d}\tau'}{[(1-\tau)\tau']^{1-\epsilon}}\,, \tag{7}$$

where $2(v_1v_2) = 2N^+N^-$. Setting the area differentiation operator to be $\delta/\delta\ln\sigma = \delta/\delta\ln(2N^+N^-)$ we obtain the combined differential equation

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left[\frac{\delta}{\delta\ln\sigma}\ln W(\Gamma_{\text{rect.}})\right] = -\sum\Gamma_{\text{cusp}}(\alpha_s), \tag{8}$$

where $\Gamma_{\text{cusp}}(\alpha_s)$ is the universal cusp anomalous dimension which plays a crucial role in a number of important applications [11]. Let us emphasize that it is independent of the smooth parts of the contours under consideration. This result resembles the situation in $2D$-QCD, where the Makeenko-Migdal equations form a closed system [3, 4].

We apply then the method described above to the TMDs with the light-like longitudinal Wilson lines $\mathcal{F}(x, \vec{k}_\perp)$ [7] (justification of this procedure will be given in a dedicated work):

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \left[\frac{\mathrm{d}}{\mathrm{d}\ln\theta} \mathcal{F}(x, \vec{k}_\perp; \mu, \theta) \right] = 2\Gamma_{\mathrm{cusp}}(\alpha_s) \cdot \mathcal{F}(x, \vec{k}_\perp \mu, \theta), \tag{9}$$

with the "area" being hidden in the rapidity cutoff $\theta \sim (pN^-)^{-1}$ [7]. It is worth noting that only the overlapping (UV and rapidity simultaneously) divergences contribute to the combined evolution. This equation can be formally integrated to give an explicit expression for the evolution of the quark TMD in two scales: ultraviolet cutoff and rapidity parameter. Nonperturbative information gets thus encoded in the integration constants, while evolution itself is treated perturbatively. This solution will be reported separately.

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