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Neutrino mass generation and $H \to \gamma\gamma/Z\gamma$ correlation with scalar multiplets

Chian-Shu Chen(1)(2), Chao-Qiang Geng(1)(2),
Da Huang(2) and Lu-Hsing Tsai(2)

(1) *Physics Division, National Center for Theoretical Sciences - Hsinchu, Taiwan*
(2) *Department of Physics, National Tsing Hua University - Hsinchu, Taiwan*

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Summary. — One natural way to understand the excess of the measured $H \to \gamma\gamma$ rate over the standard model (SM) expectation at the Large Hadron Collider (LHC) is to have charged scalar bosons, existing in most of the SM extensions. Motivated by this LHC result, we explore if it also sheds light on solving the small neutrino mass generation problem. We concentrate on a class of models with high-dimensional representations of scalars to realize Majorana neutrino masses at the two-loop level without imposing any new symmetry. In these models, multi scalars with electric charges higher than two are naturally expected, which not only enhance the $H \to \gamma\gamma$ rate, but provide more searching grounds at the LHC. In particular, the rate of $H \to Z\gamma$ is also correlated to that of the diphoton channel.

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1. – Introduction

Last year in July a boson with its mass around 125 GeV has been discovered by both ALTAS [1] and CMS [2] Collaborations, and its spin-parity property is further identified to be 0^+ according to the latest results based on the LHC full 2011+2012 dataset [3]. So far the scalar is most likely the Higgs particle as its properties are consistent with the SM predictions except the possible large production rate of $H \to \gamma\gamma$. In 2012, the excess in both experiments is around 2σ deviation from the SM prediction. Currently the measured signal strength of Higgs to diphoton by the ATLAS collaboration is $\mu = 1.6^{+0.3}_{-0.3}$ [4] while the CMS collaboration's result goes down to $\mu = 0.78^{+0.28}_{-0.26}$ [5]. In other words, the excess still survives in ATLAS but disappears in CMS. To eliminate the discrepancy between the two collaborations one has to rely on the future data accumulation and analysis in the LHC Run-II. At this moment, we consider the possibility that if the deviation is sustained, it is clearly a call for new physics. One of the natural mechanisms is to include new charged particles in the SM [6, 7], which would enhance the decay rate due to the new charged loop contributions.

In this paper we show that the inclusion of an extra scalar multiplet in the SM would naturally generate neutrino masses at the two-loop level and its charged components in turn would help in resolving the excess of the $H \to \gamma\gamma$ rate at the LHC. The correlation between $H \to \gamma\gamma$ and $H \to Z\gamma$ is also further studied. Consequently, multi scalar bosons with electric charges higher than 2 appear. These multi charged scalars clearly ensure some rich phenomenologies at the LHC. We will explore them along with the lepton-number–violating processes.

2. – Multi scalars and neutrino mass generation

If the extra scalars do not involve strong interaction with non-trivial $SU(2)_L \times U(1)_Y$ quantum numbers, there are only three possible renormalizable Yukawa interactions, given by

$$f_{ab}\bar{L}_a^{\mathrm{c}} L_b s, \qquad y_{ab}\bar{\ell}_{R_a}^{\mathrm{c}} \ell_{R_b} \Phi, \qquad \text{and} \qquad g_{ab}\bar{L}_a^{\mathrm{c}} L_b T, \tag{1}$$

where L (ℓ_R) stands for the left-handed (right-handed) lepton, a or b denotes e, μ and τ, c represents the charged conjugation, s and Φ are $SU(2)_L$ singlet scalar fields with $Y = 2$ and $Y = 4$, and T is an $SU(2)_L$ triplet with the hypercharge $Y = 2$, respectively. The third Yukawa interaction would generate neutrino masses at the tree level which is the so-called famous Type-II seesaw mechanism [8], and the explanation of tiny neutrino masses requires an extreme small value of the VEV or Yukawa couplings, which is obviously unnatural. Without introducing the triplet T, the interactions in eq. (1) are precisely given by the Zee-Babu model [9], which has been extensively studied in the literature, in particular its phenomenology of the doubly charged scalar at the LHC. In this work, we consider a new class of models by adding a scalar field ξ with $Y = 2$ and a non-trivial $SU(2)_L$ representation $\mathbf{n}$. To minimize our models, we disregard the singlet scalar s and keep the other singlet Φ so that only the second Yukawa interaction in eq. (1) can exist at the tree level.

The general scalar potential reads

$$\begin{aligned} V(H, \xi, \Phi^{\pm\pm}) = &-\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\xi^2|\xi|^2 + \lambda_\xi^\alpha|\xi|_\alpha^4 + \mu_\Phi^2|\Phi|^2 + \lambda_\Phi|\Phi|^4 \\ &+\lambda_{H\xi}^\beta(|H|^2|\xi|^2)_\beta + \lambda_{H\Phi}|H|^2|\Phi|^2 + \lambda_{\xi\Phi}|\xi|^2|\Phi|^2 + [\mu\xi\xi\Phi + \mathrm{h.c.}], \end{aligned} \tag{2}$$

where α, β are the short-handed notations denoting the possible invariant terms for higher representations in general. Also notice that all terms in the potential are self-Hermitian except the last μ-term, which is related to the dynamics of the lepton number breaking to be discussed later. For the even-dimensional representations, *i.e.* $\mathbf{n} = \mathbf{2}, \mathbf{4}, \mathbf{6}\ldots$, the products $\xi\xi$ vanish since

$$\xi\xi = \epsilon_{ii'}\epsilon_{jj'}\epsilon_{kk'}\ldots\xi_{ijk\ldots}\xi_{i'j'k'\ldots} = -\epsilon_{i'i}\epsilon_{j'j}\epsilon_{k'k}\ldots\xi_{ijk\ldots}\xi_{i'j'k'\ldots} = 0, \tag{3}$$

due to the antisymmetric matrix of ϵ_{ij} $(i, j = 1, 2)$. Subsequently, we only need to consider the odd dimensional representations of ξ, *i.e.* $\mathbf{n} = \mathbf{3}, \mathbf{5}, \mathbf{7}\ldots$. Since the triplet has been dropped out, the next minimal choice is $\mathbf{n} = \mathbf{5}$. From now on, we concentrate on this minimal one, *the quintuplet*, with $\xi = (\xi^{+++}, \xi^{++}, \xi^{+}, \xi^{0}, \xi^{-})^T$. One shall bear in mind that the results can be easily extended to those with higher representations of ξ. In these cases, there are three and two irreducible terms for $|\xi|^4$ and $|H|^2|\xi|^2$, respectively.

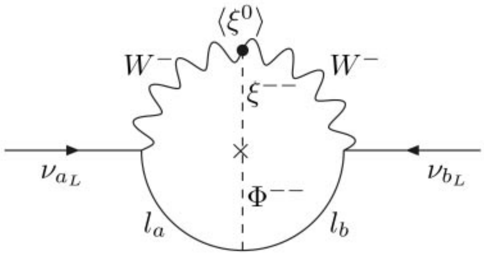

Fig. 1. – Two-loop contributions to neutrino masses.

With this setup, the neutrino masses can be generated in the two-loop diagrams as shown in fig. 1. In this mechanism, neutrino masses are calculable, and are given by

$$m_{\nu_{ab}} \simeq \frac{g^4}{\sqrt{2}(4\pi)^4} m_a m_b v_\xi y_{ab} \sin 2\theta \left[\frac{1}{M_{P_1}^2}\log^2\left(\frac{M_W^2}{M_{P_1}^2}\right) - \frac{1}{M_{P_2}^2}\log^2\left(\frac{M_W^2}{M_{P_2}^2}\right)\right], \tag{4}$$

where $m_{a,b}$ correspond to charged lepton masses; P_1 and P_2 are the mass eigenstates of doubly charged scalars with θ representing their mixing angle and $M_{P_i} > M_W$ is assumed. Note that the neutrino masses are suppressed by the two-loop factor, $SU(2)_L$ gauge coupling, charged lepton masses, mixing angle θ, and VEV of ξ, respectively, without fine-tuning Yukawa couplings y_{ab}. Due to the hierarchical structure in charged lepton masses, the model predicts the neutrino mass spectrum to be a normal hierarchy if one requires the perturbative bound on y_{ab}. Consequently, the neutrino mass matrix is given by

$$m_{\nu_{ab}} = U_{\text{PMNS}} m_{\nu_{diag}} U_{\text{PMNS}}^T \propto y_{ab}, \tag{5}$$

where $m_{\nu_{\text{diag}}} = \text{diag}\left(m_{\nu_1}, \sqrt{m_{\nu_1}^2 + \Delta m_{sol}^2}, \sqrt{m_{\nu_1}^2 + \Delta m_{atm}^2}\right)$ with m_{ν_1} the lightest ν mass. In other words, if one neglects the Majorana phases two unknown parameters are left in this expression, m_{ν_1} and Dirac CP phase, δ_{CP}. Interestingly, one is able to pin down the neutrino parameters via the leptonic processes governed by y_{ab} by utilizing the neutrino oscillation data. For example, the ratio $R_{\tau\mu} \equiv \frac{Br(\tau\to e\gamma)^*}{Br(\mu\to e\gamma)}$ (1) is related to the lightest neutrino mass, m_{ν_1}, as illustrated in fig. 2, with the use of the latest neutrino oscillation data [10]. In principle, many variants of such kind of quantity can be also defined from the leptonic rare decays or same-sign dilepton decays of $\Phi^{\pm\pm}$. Hence, our model provides a complementary way to determine neutrino parameters.

We now move to the lepton number (L) violation. In general, $U(1)_L$ can be either global or gauge symmetry. If the lepton number indeed comes from a global symmetry as that in the SM, the spontaneous symmetry breaking will generate a Nambu-Goldstone (NG) boson, usually called Majoron. In this model, the VEV of $v_\xi/\sqrt{2} = \langle\xi^0\rangle$ breaks $U(1)_L$ spontaneously. Since ξ is an $SU(2)$ multiplet, its corresponding Majoron has a direct coupling to the Z boson, which is strongly constrained by the LEP measurement

(1) Here, we have used $Br(\tau \to e\gamma)^* \equiv \Gamma(\tau \to e\gamma)/\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)$.

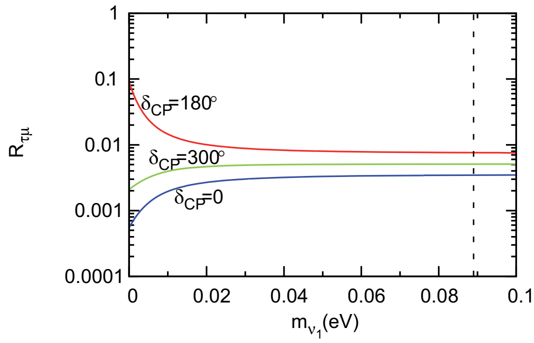

Fig. 2. – $R_{\tau\mu}$ *versus* m_{ν_1}, where the blue, red and green curves correspond to $\delta_{CP} = 0$, 180° and 300°, respectively, with the global χ^2 analysis in ref. [10], while the dashed black line represents the lower bound set by cosmology.

of the invisible Z decay width. There are many ways to resurrect it, we illustrate here by adding another $SU(2)_L$ triplet (**3**) scalar field $\Delta = (\Delta^+, \Delta^0, \Delta^-)^T$ with $Y = 0$ to the model. The potential involving Δ is given by

$$(6) \qquad \begin{aligned} V = &+\frac{1}{2}\mu_\Delta^2 \Delta^2 + \lambda_\Delta \Delta^4 + f_H \Delta^2 |H|^2 + f_\Phi \Delta^2 |\Phi|^2 + f_\xi^{(\kappa)} (\Delta^2 \xi^* \xi)_\kappa \\ &+\tilde{\mu}_2 \Delta H H^* + \tilde{\mu}_3 \Delta \xi \xi^* + [\tilde{\lambda}_1 \Delta \xi H^* H^* + \text{h.c.}], \end{aligned}$$

where $\kappa = 1, 2$, corresponding two gauge invariant terms. Note that the cubic term Δ^3 automatically vanishes in eq. (6). It is easy to see that the coexistences of $\bar{l}_R^c l_R \Phi$, $\mu\xi\xi\Phi$ and $\Delta\xi H^* H^*$ in eqs. (1), (2) and (6), respectively, break the lepton number explicitly.

The Higgs mass spectrum can be solved by analyzed the potential. Assuming $\mu_\xi^2, \mu_\Delta^2 > 0$, then $\mu_H^2 > 0$ leads to $\langle H^0 \rangle = \frac{v}{\sqrt{2}}$, $\langle \xi^0 \rangle = \frac{v_\xi}{\sqrt{2}}$, $\langle \Delta^0 \rangle = v_\Delta$. We take $\mu_\Delta^2 \gg v^2 \gg v_\xi^2 \approx v_\Delta^2$, say $\mu_\Delta \approx 1\,\text{TeV}$, $v_\xi \approx v_\Delta \approx 1\,\text{GeV}$ as constrained by ρ parameter, and denote $\mu_\xi^2 = \alpha v^2$. The mass eigenvalues of scalar fields are approximately given by $m_h^2 = 2\lambda_H v^2$, $m_{\text{Re}\xi^0}^2 = \frac{\tilde{\lambda}_1}{2} v^2$, and $m_{\Delta^0}^2 = \mu_\Delta^2 + f_H v^2$ where we identify h as the SM-like Higgs boson. The pseudo-scalar fields are $m_{G^0}^2 = 0$ which is the NG boson absorbed by the Z boson and $m_{\text{Im}\xi^0}^2 = \frac{\tilde{\lambda}_1}{2} v^2$. The singly charged scalars we obtain $m_{G^+}^2 = 0$ is the NG boson providing mass to the W boson, and three physical fields with masses $m_{\xi^+}^2 = v^2\Big(\alpha + \frac{\lambda_{H\xi}^{(1)}}{2} + \frac{\lambda_{H\xi}^{(2)}}{4}\Big)$, $m_{\xi^-}^2 = v^2\Big[-\Big(\alpha + \frac{\lambda_{H\xi}^{(1)}}{2} + \frac{\lambda_{H\xi}^{(2)}}{4}\Big) + \tilde{\lambda}_1\Big]$, and $m_{\Delta^+}^2 = \mu_\Delta^2 + f_H v^2$. From the positive-definite condition for $m_{\xi^-}^2$ we have the relation $\alpha + \frac{\lambda_{H\xi}^{(1)}}{2} + \frac{\lambda_{H\xi}^{(2)}}{4} \lesssim \tilde{\lambda}_1$. For doubly charged scalars in the basis (ξ^{++}, Φ^{++}), the mass matrix is

$$(7) \qquad \begin{pmatrix} \dfrac{v^2}{2}(4\alpha + 2\lambda_{H\xi}^{(1)} + \lambda_{H\xi}^{(2)} - \tilde{\lambda}_1) & -\sqrt{2}\tilde{\mu}_1 v_\xi \\ -\sqrt{2}\tilde{\mu}_1 v_\xi & \mu_\Phi^2 + \dfrac{v^2}{2}\lambda_{H\Phi} \end{pmatrix}$$

with mixing angle θ given by $\tan 2\theta = \frac{\sqrt{2}\tilde{\mu}_1 v_\xi}{v^2(4\alpha + 2\lambda_{H\xi}^{(1)} + \lambda_{H\xi}^{(2)} - \tilde{\lambda}_1)/2 - \mu_\Phi^2 - v^2 \lambda_{H\Phi}/2}$. The off-

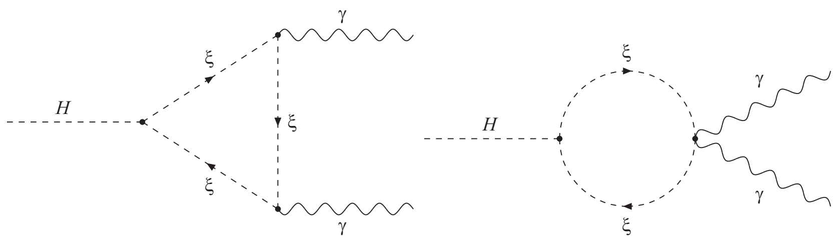

Fig. 3. – Contributions to $H \to \gamma\gamma$ from charged scalar exchanges in the loops.

diagonal elements determine the scale of neutrino mass. The diagonal terms, which are equal to mass eigenvalues in leading order, are given by $M^2_{P_1} = v^2\Big[2\Big(\alpha+\frac{\lambda^{(1)}_{H\xi}}{2}+\frac{\lambda^{(2)}_{H\xi}}{4}\Big)-\frac{\tilde{\lambda}_1}{2}\Big]$ and $M^2_{P_2} = \mu^2_\Phi + \frac{v^2}{2}\lambda_{H\Phi}$. The mass of triply charged scalar ξ^{+++} is given by $m^2_{\xi^{+++}} = v^2\Big[3\Big(\alpha + \frac{\lambda^{(1)}_{H\xi}}{2} + \frac{\lambda^{(2)}_{H\xi}}{4}\Big) - \tilde{\lambda}_1\Big]$. It leads to $\alpha + \frac{\lambda^{(1)}_{H\xi}}{2} + \frac{\lambda^{(2)}_{H\xi}}{4} \gtrsim \frac{\tilde{\lambda}_1}{3}$. Combining with the upper limit from singly charged scalar mass, one has $\frac{\tilde{\lambda}_1}{3} \lesssim \alpha + \frac{\lambda^{(1)}_{H\xi}}{2} + \frac{\lambda^{(2)}_{H\xi}}{4} \lesssim \tilde{\lambda}_1$. Note that the mass formulae of ξ's implies that $m^2_{\xi^0} = \frac{1}{4}m^2_{\xi^{+++}} + \frac{3}{4}m^2_{\xi^-}$, $m^2_{\xi^+} = \frac{1}{2}m^2_{\xi^{+++}} + \frac{1}{2}m^2_{\xi^-}$, $m^2_{\xi^{++}} = \frac{3}{4}m^2_{\xi^{+++}} + \frac{1}{4}m^2_{\xi^-}$. Therefore both hierarchy $m^2_{\xi^{+++}} > m^2_{\xi^{++}} > m^2_{\xi^+} > m^2_{\xi^0} > m^2_{\xi^-}$ and $m^2_{\xi^{+++}} < m^2_{\xi^{++}} < m^2_{\xi^+} < m^2_{\xi^0} < m^2_{\xi^-}$ are possible.

3. – $h \to \gamma\gamma/Z\gamma$ correlation

Since ξ does not directly interact with the SM fermions, the Higgs production is not expected to be modified. However, as promised, the decay rate of $H \to \gamma\gamma$ receives extra contributions from the new charged scalars in the loops as shown in fig. 3, the decay rate which is relative to the SM prediction can be expressed as

$$(8)\quad R_{\gamma\gamma} \equiv \frac{\Gamma(h\to\gamma\gamma)}{\Gamma_{SM}(h\to\gamma\gamma)} = \left|1 + \tilde{N}\frac{\mu_s v}{2m^2_S}\left(\sum_{I_3} Q^2_S\right)\frac{A^{\gamma\gamma}_0(\tau_S)}{A^{\gamma\gamma}_1(\tau_W) + N_c Q^2_t A^{\gamma\gamma}_{1/2}(\tau_t)}\right|^2,$$

where Q_S is the electric charges of the component of the scalar multiplet ξ, (I_3+1), I_3 runs from $-(\mathbf{n}-1)/2$ to $(\mathbf{n}-1)/2$, $\tilde{N}$ represents the degeneracy of the multiplet, and μ_s is the trilinear coupling to the SM Higgs. The amplitudes $A_{0,\frac{1}{2},1}$ and the mass ratios $\tau_{f,W,S}$ are defined in ref. [11]. Here, for simplicity, we have taken the same trilinear coupling μ_s and charged scalar mass m_s. The new contributions from the multi charged scalars interfere constructively with that of the SM if $\mu_s < 0$. $h \to Z\gamma$ receives similar contributions from the new scalar, given by [12]

$$(9)\qquad R_{Z\gamma} \equiv \frac{\Gamma(h\to Z\gamma)}{\Gamma_{SM}(h\to Z\gamma)} = \left|1 - \tilde{N}\frac{\mu_s v}{m^2_S}\left(2\sum_{I_3} Q_S\cdot g_{ZSS}\right)\frac{A^{Z\gamma}_0(\tau_S,\lambda_S)}{v\mathcal{A}^{Z\gamma}_{SM}}\right|^2.$$

It is straightforward to show that our formula in eq. (9) for the $Z\gamma$ decay can be retrieved to the one in eq. (8) for the $\gamma\gamma$ mode when taking $m_Z \to 0$ and making the replacement

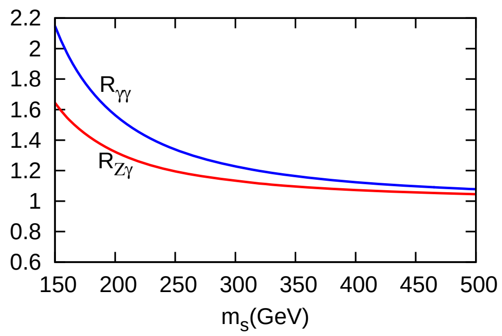

Fig. 4. – $R_{\gamma\gamma} \equiv \Gamma(H \to \gamma\gamma)/\Gamma(H \to \gamma\gamma)_{\mathrm{SM}}$ and $R_{Z\gamma} \equiv \Gamma(H \to Z\gamma)/\Gamma(H \to Z\gamma)_{\mathrm{SM}}$ as functions of the degenerate mass factor m_s of the multi charged scalar states with $\mathbf{n} = \mathbf{5}$ and the universal trilinear coupling to Higgs, $\mu_s = -100\,\mathrm{GeV}$.

$g_{ZSS} \to Q_S$. We plot the ratio of $R_{\gamma\gamma} \equiv \Gamma(H \to \gamma\gamma)/\Gamma(H \to \gamma\gamma)_{SM}$ and $R_{Z\gamma} \equiv \Gamma(H \to Z\gamma)/\Gamma(H \to Z\gamma)_{SM}$ in fig. 4 with a typical value of $\mu_s = -100\,\mathrm{GeV}$ and $\mathbf{n} = \mathbf{5}$. It is clear that the excess rate in the diphoton decay channel reported at the ATLAS and CMS collaborations can be easily explained by these multi charged scalars. It is interesting to see that the factor $Q_S \cdot g_{ZSS}$ can be examined in a general scalar multiplet. By using the identity $Q = I_3 + Y/2$, we have

$$Q_S \cdot g_{ZSS} = \frac{1}{s_W c_W} \left(I_3 + \frac{Y}{2} \right) \left(I_3 c_W^2 - \frac{Y s_W^2}{2} \right). \tag{10}$$

When the isoweak charge I of the scalar multiplet is larger than its hypercharge Y, we find $Q_S \cdot g_{ZSS} > 0$, which indicates the enhanced behavior in the $Z\gamma$ channel compared with the SM prediction. In contrast, for the multiplet with $Y \gg I$, we get $Q_S \cdot g_{ZSS} < 0$,

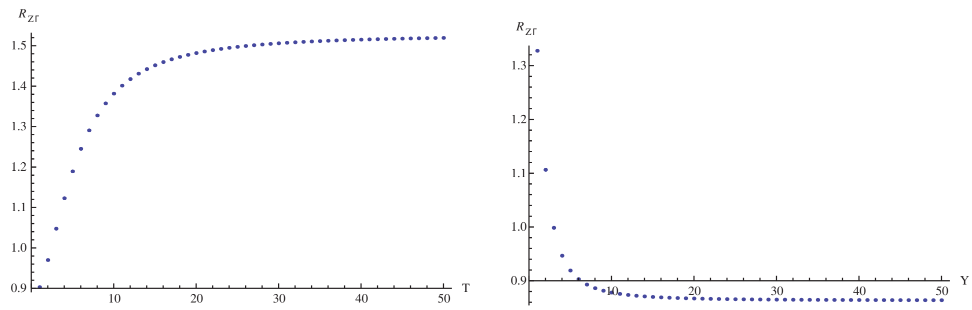

Fig. 5. – $R_{Z\gamma}$ as a function of the isospin I and Y with $R_{\gamma\gamma} = 1.5$ and $m_S = 300\,\mathrm{GeV}$, respectively.

leading to a suppressed rate of $h \to Z\gamma$. Moreover, if we take the ratio

$$\frac{\sqrt{R_{Z\gamma}}-1}{\sqrt{R_{\gamma\gamma}}-1} = -\frac{4[A_1^{\gamma\gamma}(\tau_W)+N_cQ_t^2A_{1/2}^{\gamma\gamma}(\tau_t)]}{v\mathcal{A}_{SM}^{Z\gamma}}\frac{A_0^{Z\gamma}(\tau_S,\lambda_S)}{A_0^{\gamma\gamma}(\tau_S)}\frac{\sum_{I_3}Q_S\cdot g_{ZSS}}{\sum_{I_3}Q_S^2}$$
$$= 2.71\cdot\frac{A_0^{Z\gamma}(\tau_S,\lambda_S)}{A_0^{\gamma\gamma}(\tau_S)}\frac{4I(I+1)c_W^2-3Y^2s_W^2}{4I(I+1)+3Y^2}, \tag{11}$$

we find the two limitations for the $Z\gamma$ and $\gamma\gamma$ correlations as a function of isospin I and hypercharge Y, respectively. The figures are plotted in fig. 5. It shows that $\Gamma(h \to Z\gamma)$ only depends on the relative size of the isospin T and the absolute value of the hypercharge Y. In particular, we have shown that the enhancement factor $R_{Z\gamma}$ is a monotonically increasing function of I and a monotonically decreasing one of $|Y|$ with a fixed value of $R_{\gamma\gamma}$. This observation enables us to predict that $0.76 < R_{Z\gamma} < 2.05$ by imposing the observed range $1.5 < R_{\gamma\gamma} < 2$, if the scalars are heavier than 200 GeV. Note that this range is irrelevant to the number of the scalar multiplets, their representations, and the couplings to the Higgs particle. The neutrino models and our results on the $h \to Z\gamma$ decay clearly can be tested at the LHC. Also by studying the correlations between $R_{\gamma\gamma}$ and $R_{Z\gamma}$, the simultaneous observations of $\gamma\gamma$ and $Z\gamma$ modes would help for discriminating the possible new physics beyond the SM.

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