IL NUOVO CIMENTO Vol. 36 C, N. 1 Suppl. 1 Gennaio-Febbraio 2013
DOI 10.1393/ncc/i2013-11479-4

Colloquia: 12th Italian-Korean Symposium 2011

Kerr rotation as solitonic whirl around the Schwarzschild black hole

V. A. Belinski(1) and H. W. Lee(2)

(1) *ICRANet - Piazzale della Repubblica, 10, 65100 Pescara, Italy*
(2) *Institute of Basic Science and Department of Computer Aided Science Inje University - 197 Inje-ro, 621-749 Gimhae, Korea*

ricevuto il 9 Marzo 2012

Summary. — We propose the new derivation of the Kerr solution by adding to the Schwarzchild black hole the solitonic vortex made from the pure gravitational field. With this method, we can figure out how rotational energy can contribute to the mass of the resulting Kerr black hole. This suggests a new mode to think on the relation between the mass and angular momentum of a Kerr black hole.

PACS 04.20.-q – Classical general relativity.
PACS 04.20.Jb – Exact solutions.
PACS 04.70.Bw – Classical black holes.

1. – On the new derivation of Kerr solution

As was mentioned by L. D. Landau and E. M. Lifshitz [1], there is no constructive derivation of the Kerr metric in the literature that could be adequate to its physical content as the gravitational field of a rotating body. Here we propose the derivation the mathematical procedure of which may corresponds to the physical process of applying an angular twist to a Schwarzschild black hole. It is known that both Schwarzschild and Kerr black holes are gravitational solitons, they can be derived as stationary solitonic states on the flat space-time background [2, 3]. Consequently, an assumption that Kerr solution can be obtained also by adding to the Schwarzschild background a solitonic vortex is plausible. This vortex should be defined in such a way that it brings to the static background only the rotation together with the mass-energy generated by it but not any arbitrary contribution to the total mass independent of rotation. We show how to do this in the framework of the so-called "w - soliton generating technique" developed by G. Alekseev [4] and described in details in the book [5]. This technique is generalization of the pure gravity Inverse Scattering Method for the case when an electromagnetic field is present. However, one can use this technique also for the pure gravity case simply taking the electromagnetic potentials to be zero. Here we use namely this way following the procedure described in chapter 3 of the book [5].

The first non-trivial task is to solve the equations of the corresponding linear spectral problem for the Schwarzschild metric written in the cylindrical Weyl coordinates ρ, z to obtain the background spectral matrix $\hat{\varphi}^{(0)}(w, \rho, z)$ (that is the solution of the matrix equation (3.48) in the book [5]). This was done by one of us [6] in relation to the quite different problem but the result can successfully be used here.

Using this background matrix $\hat{\varphi}^{(0)}$ we constructed (following instructions from point 8 to point 11 of the subsection 3.3.4 of the book [5]) exact solution representing the Schwarzschild black hole under the influence of a gravitational whirl which forced the black hole to rotate. Such perturbation corresponds to the one-pole solitonic excitation of the special choice of the arbitrary constants entering the dressing matrix $\hat{\chi}$, which matrix brings about the transition ($\hat{\varphi} = \hat{\chi}\hat{\varphi}^{(0)}$) from the background to the perturbed space-time. We should eliminate from $\hat{\chi}$ those parameters not related to a rotation, that is the constant generating the electric charge, the constant generating a new rest mass and the constant introducing the NUT-parameter. The analysis shows that the pure rotation will be generated only by the one real parameter σ, namely that one which fixes the position of the pole of the dressing matrix in the complex plane of the spectral parameter w, that is $w_{pole} = i\sigma$ (we should locate this pole on the imaginary axis of the complex plane of the spectral parameter since the real part of w_{pole} produces the NUT-parameter). After calculations we obtain the Kerr metric in which the Kerr mass m_k and specific angular momentum parameter a are the following functions of the background Schwarzschild mass m_s and pole position parameter σ:

$$m_k = m_s \frac{m_s^2 + \sigma^2}{m_s^2 - \sigma^2}\,, \qquad a = \frac{2m_s^2\sigma}{m_s^2 - \sigma^2}\,.$$

From these formulas our main result follows:

$$m_k^2 = m_s^2 + a^2.$$

The physical interpretation of this relation between mass and angular momentum of the Kerr black hole and its possible link to the Christodoulou-Ruffini theory [7, 8] of the black hole irreducible mass need further investigation.

* * *

This research was supported by the International Research & Development Program of the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology(MEST) of Korea(Grant number: K2011.00007, FY 2011)

REFERENCES

[1] Landau L. D. and Lifshitz E. M., *The Classical Theory of Fields*, 4th edition (Elsevier) 1975, p. 346.
[2] Belinski V. A. and Zakharov V. E., *Sov. Phys. JETP*, **48** (1978) 985.
[3] Belinski V. A. and Zakharov V. E., *Sov. Phys. JETP*, **50** (1979) 1.
[4] Alekseev G. A., *JETP Lett.*, **32** (1981) 277.
[5] Belinski V. and Verdaguer E., *Gravitational Solitons*, 1st edition (Cambridge University Press, Cambridge) 2001.
[6] Belinski V., unpublished.
[7] Christodoulou D., *Phys. Rev. Lett.*, **25** (1970) 1596.
[8] Christodoulou D. and Ruffini R., *Phys. Rev. D*, **4** (1971) 3552.