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On the kinetic treatment of pair productionin strong electric fields

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Summary. — We investigate the behavior of the electron-positron plasma created by a strong electric field using a kinetic approach. Assuming a uniform and unbound field, the system under consideration is uniform and homogeneous in the physical space and axially symmetric in the momentum space with the axis of symmetry given by the direction of the initial field. The relativistic Boltzmann-Vlasov equation for pairs is solved numerically for different starting values of the field with particular attention to the momentum distribution of pairs produced from the field. Then we solve the system of coupled Vlasov-Boltzmann equation for pairs and Boltzmann equation for photons including collision terms for all the two-particle interactions between pairs and photons: electron-positron annihilation into two photons and its inverse process, Compton and Coulomb scatterings. We compare the two cases and discuss the role of the interactions.

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1. – Introduction

Electron-positron pairs can be produced by vacuum breakdown in a strong electric field E, if it exceeds the critical value

$$E \geq E_c = \frac{m_e^2 c^3}{e\hbar}\,, \tag{1}$$

where m_e is the electron mass, c is the speed of light, $\hbar$ is Planck constant [1].

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In [2] we addressed the problem of pair creation in a strong electric field by a treatment based on continuity and energy-momentum conservation equations. A second-order ordinary differential equation has been worked out and all the other physical quantities of interest can be obtained from its solution. According to previous works [2-5], the pairs move back and forth; the electric field oscillates as well with the same frequency but with a shifted phase. This approach allowed us to study the behavior of the system beyond the asymptotic time τ_a, when the pairs oscillation frequency is close to the plasma frequency; at this stage almost all the energy density of the initial electric field was found to be converted into rest energy density of the pairs. Estimating the optical depth, we also found that the role of the interactions should be taken into account in the long run. Besides, we assumed that pairs are originally produced at rest even though the most general rate of pair production already gives a specific distribution in momentum space. In particular they are in a momentum state such that there is motion only in the orthogonal direction to the initial electric field.

We now extend and generalize the results of the previous work [2] with the intention to investigate the effects of pairs interactions and the dynamical role of a most general rate of pair production. Collisions can be naturally described within the kinetic approach. There is invariance under rotations around the direction of the electric field. In this perspective we solve numerically the relativistic Boltzmann-Vlasov equation in a uniform and homogeneous physical space but with an axially symmetric momentum space. The pairs interaction is accounted for by collision integrals computed from the exact QED cross-sections for the two particle interactions, namely electron-positron annihilation into two photons and its inverse process, Bhabha, Möller and Compton scatterings. As a consequence, also photons are described by the relativistic Boltzmann equation. The three particle interactions are not taken into account because their cross-section is roughly α times smaller than the two particle ones.

Since it is well known that a pair plasma thermalizes in a very short time scale [6], we are also interested in this characteristic time for different initial conditions. The temperature in kinetic equilibrium may be found from the total energy and number densities of the plasma [7]. Moreover, we already know that the electric field accelerates electrons and positrons up to high Lorentz factors of the order of hundreds. Along the direction orthogonal to that, pairs are produced with momenta up to $m_e\, c\, (E/E_c)$. It means that the distribution of electrons and positrons in the momentum space is strongly anisotropic being elongated in the direction of the initial electric field.

2. – Coordinates in the momentum space and units

Based on the symmetry of the problem, we consider an axially symmetric momentum space. Hence, the momentum of the particle is described by two components, one parallel ($p_\parallel$) and one orthogonal ($p_\perp$) to the direction of the initial electric field, and the angle (ϕ) between this preferred direction and the actual momentum direction. These momentum space coordinates are defined in the intervals

$$\phi \in [0, 2\pi], \qquad p_\perp \in [0, +\infty), \qquad p_\parallel \in (-\infty, +\infty). \tag{2}$$

Within the chosen phase space configuration, the prescription for the integral over the entire momentum space is

$$\int \mathrm{d}^3\mathbf{p} = \int_0^{2\pi} \mathrm{d}\phi \int_{-\infty}^{+\infty} \mathrm{d}p_\parallel \int_0^{+\infty} \mathrm{d}p_\perp\, p_\perp, \tag{3}$$

and the relativistic energy is given by the following equation:

$$\epsilon = \sqrt{p_\parallel^2 + p_\perp^2 + m^2}, \tag{4}$$

where m is the mass of the considered particle.

3. – The Distribution Function

Instead of using the usual DF f such that the number density is given by the integral over the momentum space

$$n = \int \mathrm{d}^3\mathbf{p}\, f = 2\pi \int_{-\infty}^{+\infty} \mathrm{d}p_\parallel \int_0^{+\infty} \mathrm{d}p_\perp\, p_\perp\, f, \tag{5}$$

we introduce a new DF F such that the energy density is given by the following integral:

$$\rho = \int_{-\infty}^{+\infty} \mathrm{d}p_\parallel \int_0^{+\infty} \mathrm{d}p_\perp\, F. \tag{6}$$

In isotropic momentum space this DF is reduced to the spectral energy density $\mathrm{d}\rho/\mathrm{d}\epsilon$. Hence we can recover f from F as

$$f = \frac{F}{2\pi\, \epsilon\, p_\perp}\,. \tag{7}$$

Besides, because of the axial symmetry, F does not depend on ϕ, it depends only on the two components of the momentum, that is

$$F = F(p_\parallel, p_\perp)\,. \tag{8}$$

4. – Boltzmann equations

The relativistic Boltzmann equation for electrons and positrons can be written in our framework as follows:

$$\begin{aligned} &\frac{\partial F_\pm(p_\parallel, p_\perp)}{\partial t} \pm e\, E\, \frac{\partial F_\pm(p_\parallel, p_\perp)}{\partial p_\parallel} = \\ &\eta_\pm(p_\parallel, p_\perp) - \chi_\pm(p_\parallel, p_\perp)\, F_\pm(p_\parallel, p_\perp) + S(p_\parallel, p_\perp, E), \end{aligned} \tag{9}$$

where E is the electric field, $\eta_\pm$, $\chi_\pm$ are the emission and absorption coefficients due to the interactions, and S is the rate of pair production. In particular the electron-positron DF in eq. (9) varies due to the acceleration by the electric field, the pairs creation due to vacuum polarization and the creation or annihilation of particles generated by the interactions. The rate of pair production already distributes particles in the momentum space according to

$$S(p_\parallel, p_\perp, E) = -\frac{|e\, E|}{m_e^3 (2\pi)^2}\, \epsilon\, p_\perp \log\left[1 - \exp\left(-\frac{\pi(m_e^2 + p_\perp^2)}{|e\, E|}\right)\right] \delta(p_\parallel). \tag{10}$$

From the previous equation it is clear that if $E < E_c$ this rate is exponentially suppressed.

Then the Boltzmann equation for photons is

$$\frac{\partial F_\gamma(p_\parallel, p_\perp)}{\partial t} = \eta_\gamma(p_\parallel, p_\perp) - \chi_\gamma(p_\parallel, p_\perp)\, F_\gamma(p_\parallel, p_\perp), \tag{11}$$

because their DF changes according to the collisional term only. In more detail, photons must be produced first from the pairs annihilation, then they affect the electron-positron DF through Compton scattering. They can annihilate producing one electron-positron pair.

5. – Fundamental quantities

We define the maximum achievable pairs number density as

$$n_{\max} = \frac{E^2}{8\,\pi\, m_e\, c^2}\,, \tag{12}$$

which corresponds to the maximum conversion of the initial energy density into the electron-positron rest energy density. The ratio between the pairs number density and $n_{\max}$ corresponds the efficiency of this conversion process.

The total energy density of the pairs ρ is related to the actual and initial electric fields E and E_i by the energy conservation law

$$\rho = \frac{E^2 - E_i^2}{8\pi}\,. \tag{13}$$

The rest energy density of the pairs, namely the sum of the electrons and positrons rest energy densities, is

$$\rho_{\mathrm{rest}} = \rho_- + \rho_+ = (n_- + n_+)\, m_e\, c^2, \tag{14}$$

where n_- and n_+ are the electrons and positrons number densities. Then we define the kinetic energy density as

$$\rho_{\mathrm{kin}} = (n_- + n_+)\, m_e\, c^2 \left(\sqrt{\left(\frac{\bar{p}_\parallel}{m_e\, c}\right)^2 + 1} - 1 \right), \tag{15}$$

where $\bar{p}_\parallel$ is the magnitude of the pairs bulk parallel momentum. Therefore ρ_{kin} is the energy of an ensamble of particles all moving with $p_\parallel = \bar{p}_\parallel$ and $p_\perp = 0$. The difference between the total energy density and all the others defined above is taken to be the internal energy density

$$\rho_{\mathrm{in}} = \rho - \rho_{\mathrm{rest}} - \rho_{\mathrm{kin}}. \tag{16}$$

We call this "internal" since it is related to the dispersion of the DF in the momentum space.

6. – Results and conclusions

There are many analogies between the results we obtain using the analytical and kinetic approaches, namely the time dependence during the first half oscillation of all the quantities involved as pairs number density, bulk parallel momentum and electric field. However, after this short period, the two methods give substantially different results.

In the non-interacting cases and for all the considered parameter sets we find that after several oscillations the magnitude of the electric field becomes much less than the starting value; as a consequence, acceleration and pair production are strongly suppressed. The number density of the pairs saturates to a small fraction of the maximum achievable one, roughly a few percent for all cases considered. Due to the small number of pairs, even if the electric field is small it accelerates them up to a bulk parallel momentum much larger than that obtained using the simplified treatment.

We find that a substantial part of the total energy, initially stored in the electric field is converted after few oscillations into internal energy. This effect could not be obtained within the treatment we used before, since all particles were assumed to have single momentum (delta-function distribution in momentum space).

In the interacting case, the production of photons occurs and their DF is perfectly symmetric with respect to the plane with null parallel momentum at any instant of time. If the energy density of photons is small compared to the pairs one, the dispersion of their DF in the momentum space follows the pairs one: it has a large dispersion along the parallel momentum and a small one along the orthogonal component. This feature follows directly from the initial distribution of pairs given by the rate given by eq. (10). Since the higher the initial electric field, the larger the number density of pairs, interactions becomes important earlier for high initial fields. The energy density of photons becomes comparable to the pairs one only after many oscillations for all the considered cases. As soon as it happens, the plasma should start to thermalize. Therefore the anisotropy in the momentum space is expected to be reduced with time, reaching the thermal distribution function characterized only by its temperature.

We stress that the results presented above come out only when the kinetic treatment is adopted, and consequently distribution of particles in momentum space is accounted for.

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