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Nonlinear electrodynamic effect of light bending by charged astronomical objects

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Summary. — A nonuniform electric or magnetic field can induce a continually varying index of refraction. A strong electromagnetic field can change the vacuum index of refraction by the nonlinear electrodynamic effect. We calculate the bending angle of a light ray under strong electric and magnetic field of charged black hole and neutron star according to the nonlinear electrodynamics of the Euler-Heisenberg interaction. We estimate that the electrical bending angle is negligibly small compared with the gravitational bending. The bending of light by the magnetic field of a neutron star is also significantly smaller than the bending angle by gravitation.

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1. – Introduction

In the geometric optic approximation, the light path can be bent by a continually changing index of refraction. Since the theory of classical electrodynamics is linear, the vacuum of electrodynamics defined by the absence of charged matter is unique and trivial. This means that the speed of light is constant and the index of refraction does not change. Therefore the light path cannot be bent by electric or magnetic field. However, the Euler-Heisenberg interaction [1, 2] that reflects the nonlinear interaction by quantum electrodynamics can cause the vacuum index of refraction to be nontrivial. In the presence of a nonuniform electric or magnetic background field the index of refraction can vary continually over the light path. Thus the light can be bent when it passes the neighborhood of an electrically or magnetically charged object.

It seems very difficult to test the bending directly in a terrestrial laboratory since the maximum available field is of the order B, $E/c \sim 10^2\,\mathrm{T}$ [3-7]. Alternatively, the bending of light has been studied in astronomical scale by several authors. For instance, De Lorenci *et al.* [8] studied the bending of a high-energy photon when it passes around a charged black hole and Denisov *et al.* [9-14] studied the light bending by the magnetic field of a neutron star.

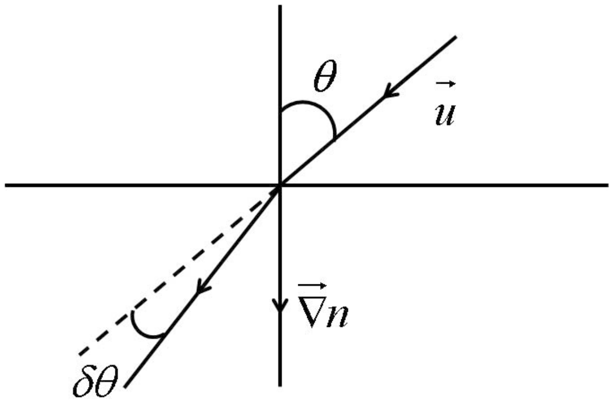

Fig. 1. – Differential bending by nonuniform refractive index.

In this paper, we consider the propagation of a high-energy photon when it passes the electric or magnetic field induced by astronomical objects like charged black hole or magnetized neutron star. From the photon trajectory equation based on Snell's law [15], we will calculate the bending angle of light ray when it passes the electric field induced by a charged black hole or magnetic field by a neutron star.

2. – Photon trajectory under a continually varying index of refraction

In geometric optics the index of refraction in matter is defined as the speed of light in vacuum divided by the speed of light within matter, *i.e.*, $n = c/v$. The gradient of the index of refraction can cause the bending of the light ray. The bending can be calculated by simple geometric optics.

The infinitesimal bending of the photon trajectory over $\delta\vec{\mathbf{r}}$ can be obtained from Snell's law as

$$\delta\theta = \tan\theta\frac{\delta n}{n} = \frac{1}{n}|\nabla n \times \delta\vec{\mathbf{r}}\,|, \tag{1}$$

where $\delta n = \nabla n \cdot \delta\vec{\mathbf{r}}$ and θ denotes the angle between the unit vector $\mathbf{u}$ in the direction of photon propagation and ∇n (see fig. 1). We can write the bending in a vector form as

$$\delta\mathbf{u} = \frac{1}{n}(\delta\vec{\mathbf{r}} \times \nabla n) \times \mathbf{u}, \tag{2}$$

which leads to the trajectory equation

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}s} = \frac{1}{n}(\mathbf{u} \times \nabla n) \times \mathbf{u}, \tag{3}$$

where s denotes the distance parameter of the light trajectory with $\mathrm{d}s = |\mathrm{d}\vec{\mathbf{r}}\,|$ and $\mathbf{u} = \mathrm{d}\vec{\mathbf{r}}/\mathrm{d}s$.

When the correction to the index of refraction is small, the trajectory equation can be approximated to the leading order as

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}s} = (\mathbf{u}_0 \times \nabla n) \times \mathbf{u}_0, \tag{4}$$

where $\mathbf{u}_0$ denotes the initial direction of the incoming photon. Throughout the paper we shall assume the photon comes in from $x = -\infty$ and moves to the $+x$ direction so that

$$\mathbf{u}_0 = (1, 0, 0). \tag{5}$$

Defining $\nabla n = (\eta_1, \eta_2, \eta_3)$, the trajectory equation can be written as

$$\frac{\mathrm{d}^2 x}{\mathrm{d}s^2} = 0, \qquad \frac{\mathrm{d}^2 y}{\mathrm{d}s^2} = \eta_2, \qquad \frac{\mathrm{d}^2 z}{\mathrm{d}s^2} = \eta_3. \tag{6}$$

The first equation shows that $\mathrm{d}s = \mathrm{d}x$ at leading order and the trajectory equations for $y(x)$ and $z(x)$ are given by

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \eta_2, \qquad \frac{\mathrm{d}^2 z}{\mathrm{d}x^2} = \eta_3. \tag{7}$$

If one knows the functional form of the index of refraction, one can calculate the trajectory of light ray from the above equation.

3. – Bending angle by a charged black hole

In the presence of an electric field, the correction to the speed of light due to the nonlinear interaction is given by [16-26]

$$\frac{v}{c} = 1 - \frac{a\alpha^2\hbar^3\epsilon_0}{45m^4c^5}(\mathbf{u} \times \mathbf{E})^2, \tag{8}$$

where $\mathbf{u}$ denotes the unit vector in the direction of photon propagation, $a = 14$ for the perpendicular mode in which the photon polarization is perpendicular to the plane spanned by $\mathbf{u}$ and $\mathbf{E}$, and $a = 8$ for the parallel mode where the polarization is parallel to the plane. Throughout the paper all units are in MKS. The index of refraction due to the background electric field is given by

$$n = \frac{c}{v} = 1 + \frac{a\alpha^2\hbar^3\epsilon_0}{45m^4c^5}(\mathbf{u} \times \mathbf{E})^2. \tag{9}$$

We consider the bending of photon trajectory by a spherically symmetric charged object of total charge Q. For this case the bending angle can be calculated in the same way as in the Coulombic case with the electric field

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}. \tag{10}$$

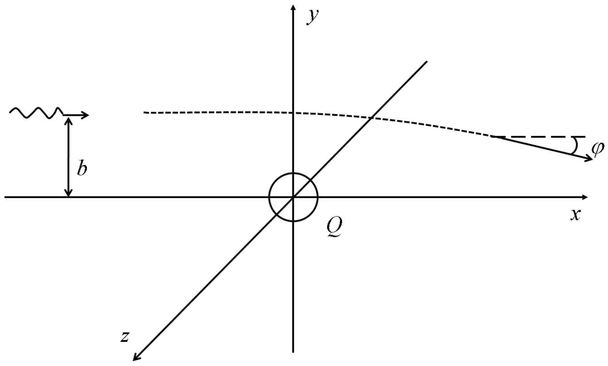

Fig. 2. – Schematic of light bending by a Coulombic charge Q located at the origin.

The index of refraction to the leading order can be written explicitly as

$$n = 1 + \frac{a\alpha^2\hbar^3 Q^2}{720\pi^2\epsilon_0 m^4 c^5}\frac{(y - xy')^2}{r^6(1+y'^2)}\,, \tag{11}$$

where prime is the derivative with respect to x.

For a photon incoming from $x=-\infty$ with impact parameter b (see fig. 2), the initial condition reads

$$y(-\infty) = b, \qquad y'(-\infty) = 0, \tag{12}$$

and from the first of the trajectory equation (7) to the leading order we have

$$y'' = \eta_2 = \frac{a\alpha^2 Q^2 \lambda_e^4}{360\pi^2\epsilon_0\hbar c}\left(\frac{y}{r^6} - \frac{3y^3}{r^8}\right), \tag{13}$$

where $\lambda_e = \hbar/mc$ is the Compton length of the electron. The total bending angle $\varphi_{\rm e}$ can be obtained by integration

$$y'(\infty) = \int_{-\infty}^{\infty}\eta_2 {\rm d}x = \tan\varphi_{\rm e} \simeq \varphi_{\rm e}. \tag{14}$$

By putting $y=b$ in η_2, for the leading order solution, we obtain

$$\varphi_{\rm e} = -\frac{a\alpha^2 Q^2}{640\pi\epsilon_0\hbar c}\left(\frac{\lambda_e}{b}\right)^4. \tag{15}$$

The bending always occurs toward the center of the charged object as in the bending by gravitational field.

To compare the bending by electric field with the bending by gravitation, let us consider a charged non-rotating black hole with mass $\mathcal{M}$ and charge Q. The bending by gravitational field is well known:

$$\varphi_{\rm g} = \frac{4G\mathcal{M}}{bc^2}\,. \tag{16}$$

Note that, from $\varphi_{\rm g} \propto 1/b$ and $\varphi_{\rm e} \propto 1/b^4$, the bending by the electric field can be important at short distance. The charge and angular momentum per unit mass ($J/\mathcal{M}$) is constrained by the mass of the black hole, in Planck units, as [27]

$$Q^2 + (J/\mathcal{M})^2 \leq \mathcal{M}^2. \tag{17}$$

For non-rotating ($J = 0$) charged black hole, restoring the physical constants, the total electric charge is constrained by the condition

$$\frac{Q^2}{4\pi\epsilon_0} \leq G\mathcal{M}^2. \tag{18}$$

We can parameterize the charge as

$$Q = \sqrt{4\pi\epsilon_0 G}\mathcal{M}\xi, \tag{19}$$

with $0 \leq \xi \leq 1$. Then the magnitude of the bending angle by electric field can be written as

$$\varphi_{\rm e} = \frac{a\alpha^2\xi^2}{160}\frac{G\mathcal{M}^2}{\hbar c}\left(\frac{\lambda_e}{b}\right)^4 = \frac{a\alpha^2\xi^2}{640}\frac{bc\mathcal{M}}{\hbar}\left(\frac{\lambda_e}{b}\right)^4 \varphi_{\rm g}. \tag{20}$$

For the numerical estimation, compare the possible maximal bending ($\xi = 1$ and $a = 14$) with the gravitational bending for stellar black hole of ten times the solar mass $\mathcal{M} = 10\mathcal{M}_{\rm sun} = 2 \times 10^{31}\,$kg. Since our formalism is based on flat spacetime, not on general relativity, the impact parameter should be large enough. We consider the case when the impact parameter is ten times the Schwarzschild radius, $b = 10r_{\rm sh} \sim 300\,$km, at which

$$\varphi_{\rm g} = 1.98 \times 10^{-1}\,\text{rad}; \qquad \varphi_{\rm e} = 5.47 \times 10^1 \varphi_{\rm g} = 1.08 \times 10^1\,\text{rad}. \tag{21}$$

The bending by electric charge dominates the gravitational bending. Even for a non-extremal charged black hole with $\xi = 0.1$, the electrical bending, $\varphi_{\rm e} = 1.08 \times 10^{-1}\,$rad, is comparable to the gravitational bending.

However, we should not accept this naive estimation. Because the Euler-Heisenberg Lagrangian is a low-energy effective action of QED presented by asymptotic series, the application is limited to a weak-field approximation. Thus, our formalism can be applied only to the region where the field strength is not as strong as the QED critical field $E_{\rm c} = m^2c^3/e\hbar = 1.3 \times 10^{18}\,$V/m. In the region where the electric field is of the order of or higher than $E_{\rm c}$, the vacuum is strongly unstable and the electric field is highly screened by the electron-positron pair creation [2, 28]. Only photons entering the region with electric field below $E_{\rm c}$ can have a chance to be observed.

Let us estimate the field strength for the case $\xi = 0.1$ and $b = 10r_{\rm sh}$, where the electrical bending has the same order as the gravitational bending. Since the electric field at a distance b is given by

$$E = \sqrt{\frac{G}{4\pi\epsilon_0}}\frac{\mathcal{M}\xi}{b^2}\,, \tag{22}$$

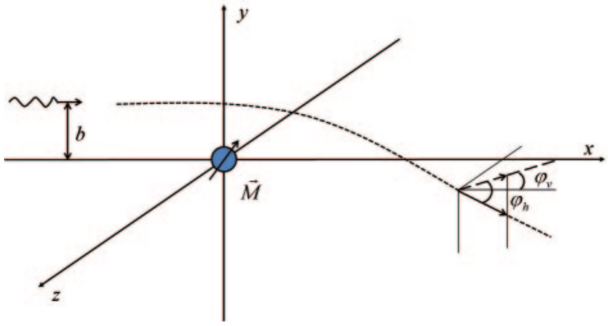

Fig. 3. – Schematic of light bending by a magnetic dipole located at the origin. The dotted line is the photon trajectory, the dashed line is the projection of the outgoing photon path on the xz-plane, and ϕ_h (ϕ_v) is the bending angle of horizontal (vertical) direction.

the field strength at $b = 10r_{\mathrm{sh}}$ and $\xi = 0.1$ is estimated as $1.7 \times 10^{19}\,\mathrm{V/m}$. This number is one order of magnitude above E_{c}. Even for $\xi = 0.01$, where $\varphi_{\mathrm{e}} \sim 10^{-3}\varphi_{\mathrm{g}}$, the field strength is about the order of the critical field. So we conclude that the bending of light by the electric field of a charged black hole is negligibly small compared with the gravitational bending.

4. – Bending by a magnetic dipole

Now we consider the bending of the photon trajectory by a magnetic dipole. Obviously, the bending by a magnetic dipole should depend on the orientation of dipole relative to the direction of the incoming photon. We consider the bending by a magnetic dipole located at the origin with arbitrary orientation (see fig. 3). We take the direction of the incoming photon as the x-axis, the horizontal direction as the y-axis, and the vertical direction as the z-axis. For the magnetic dipole $\mathbf{M}$ located at origin, we define the directional cosines $\alpha = \hat{M} \cdot \hat{x}$, $\beta = \hat{M} \cdot \hat{y}$, $\gamma = \hat{M} \cdot \hat{z}$ such that $\mathbf{M} = M\hat{M}$. The magnetic field by the dipole can be written as

$$\mathbf{B} = (B_x, B_y, B_z) = \frac{\mu_0 M}{4\pi}\left(\frac{3(\hat{M}\cdot\vec{r})\vec{r}}{r^5} - \frac{\hat{M}}{r^3}\right), \tag{23}$$

and the index of refraction due to this background magnetic field is given by

$$n = \frac{c}{v} = 1 + \frac{a\alpha^2\hbar^3\epsilon_0}{45m^4c^3}(\mathbf{u}\times\mathbf{B})^2. \tag{24}$$

Taking the unit vector in the direction of the photon propagation as

$$\mathbf{u} = \frac{1}{\sqrt{1+y'^2+z'^2}}(1, y', z'), \tag{25}$$

the index of refraction can be written explicitly as

$$n = 1 + \frac{a\alpha^2\hbar^3\epsilon_0}{45m^4c^3}\left(\frac{\mu_0 M}{4\pi}\right)^2 \frac{1}{r^{10}}\left[\Big\{\beta(2y^2 - x^2 - z^2) + 3\alpha xy + 3\gamma yz\Big\}^2 + \Big\{\gamma(2z^2 - x^2 - y^2) + 3\alpha xz + 3\beta yz\Big\}^2\right]. \tag{26}$$

The bending angle can be obtained by integrating the leading-order trajectory equation (7) with the boundary condition $y(-\infty) = b$ and $z(-\infty) = 0$. The results for horizontal ($\varphi_h = y'(\infty)$) and vertical ($\varphi_v = z'(\infty)$) deflections are

$$\varphi_h = -\frac{\pi}{3\cdot 2^7}\frac{a\alpha^2\epsilon_0 c}{\hbar}\left(\frac{\mu_0 M}{4\pi}\right)^2 \frac{\lambda_e^4}{b^6}(15\alpha^2 + 41\beta^2 + 16\gamma^2), \tag{27}$$

$$\varphi_v = \frac{5\pi}{3\cdot 2^6}\frac{a\alpha^2\epsilon_0 c}{\hbar}\left(\frac{\mu_0 M}{4\pi}\right)^2 \frac{\lambda_e^4}{b^6}\beta\gamma. \tag{28}$$

Let us now consider three special cases where the relative orientations are so simple that the bending occurs in one particular direction by symmetry. First, we consider the case when the photon path is perpendicular to the dipole moment and traveling on the equator of the dipole. Assume that the magnetic moment directs along $\hat{z}$ and the incident photon is coming from $x = -\infty$. This is the specific case considered by Denisov *et al.* [9]. Taking $\alpha = \beta = 0$ and $\gamma = 1$, the horizontal (y) bending angle is given by

$$\varphi_h = -\frac{\pi}{24}\frac{a\alpha^2\epsilon_0 c}{\hbar}\left(\frac{\mu_0 M}{4\pi}\right)^2 \frac{\lambda_e^4}{b^6}. \tag{29}$$

This result agrees with Denisov *et al.* (see the eqs. (4) and (5) in [9]). There is no vertical (z) bending by symmetry, $\varphi_v = 0$. It is obvious that there is no gradient for the index of refraction in that direction. Second, consider the case when the photon path is parallel or antiparallel to the dipole axis. Assume that the dipole at the origin directs along the $-x$-axis in the xy-plane. Taking $\alpha = -1$ and $\beta = \gamma = 0$, the bending angle in the y-direction is given by

$$\varphi_h = -\frac{5\pi}{2^7}\frac{a\alpha^2\epsilon_0 c}{\hbar}\left(\frac{\mu_0 M}{4\pi}\right)^2 \frac{\lambda_e^4}{b^6}. \tag{30}$$

Also, there is no bending in the z-direction, $\varphi_v = 0$. Finally, consider the case when the photon path is perpendicular to dipole moment and passes the north or south pole. Locate the dipole at the origin directing along the y-axis in the xy-plane. Taking $\alpha = \gamma = 0$ and $\beta = 1$, the horizontal bending angle is given by

$$\varphi_h = -\frac{41\pi}{3\cdot 2^7}\frac{a\alpha^2\epsilon_0 c}{\hbar}\left(\frac{\mu_0 M}{4\pi}\right)^2 \frac{\lambda_e^4}{b^6}. \tag{31}$$

This configuration gives the largest bending since the gradient of the index of refraction is maximal along this direction.

Now we compare the bending by a magnetic field with the gravitational bending. We consider the possible maximum bending given by eq. (31) for a strongly magnetized neutron star with solar mass $M = M_{\rm sun} = 2 \times 10^{30}\,$kg and radius $r_0 = 10\,$km. For strongly magnetized neutron stars the magnetic field at the surface can be as strong as $B_{\rm s} = 10^8$–$10^{11}\,$T. Parameterizing the impact parameter in units of the radius $b = \zeta r_0$ with $\zeta > 1$, the bending angle by a magnetic field can be written as

$$\varphi_{\rm m} = \frac{41\pi}{3 \cdot 2^7} \frac{a\alpha^2 \epsilon_0 c}{\hbar} B_{\rm s}^2 \frac{\lambda_e^4}{\zeta^6}\,, \tag{32}$$

where we have used $B_{\rm s} = \mu_0 M / 4\pi r_0^3$, the magnetic field strength at the neutron star surface.

As discussed for the electrical bending, the field strength should not be above the QED critical field $B_{\rm c} = 4.4 \times 10^9\,$T. The possible maximum ($\zeta = 1$) value of the bending angle when the magnetic field on the surface of the neutron star is of the order $B_{\rm s} = 10^9\,$T is given by

$$\varphi_{\rm g} = 5.93 \times 10^{-1}\,\text{rad}; \quad \varphi_{\rm m} = 1.40 \times 10^{-4}\,\text{rad}, \tag{33}$$

for the ray passing the north or south pole. The field strength of $B_s = 10^9\,$T is not strong enough to compete with the gravitational bending. Thus the bending of light by the magnetic field of a neutron star will also be negligibly small compared with the gravitational bending.

5. – Summary

We have studied how photons can be bent when they travel through the strong electric or magnetic field of a compact object like a charged black hole or a neutron star. We calculated the bending angles according to the nonlinear electrodynamics of the Euler-Heisenberg interaction. Since the Euler-Heisenberg Lagrangian is a low-energy effective action of QED presented by asymptotic series, the application is limited to the weak-field approximation. Thus our formalism is valid only for regions where the field strengths are not as strong as $E_{\rm c}$ and $B_{\rm c}$. Our estimation shows that the bending by an electrically charged black hole is negligibly small compared with the bending by gravitation. We found a general formula for light bending valid for any orientation of the magnetic dipole. In summary, the light bending by both the electric and magnetic fields of astronomical objects is significantly smaller than the gravitational bending.

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REFERENCES

[1] Heisenberg W. and Euler H., *Z. Phys.*, **98** (1936) 714, physics/0605038.
[2] Schwinger J. S., *Phys. Rev.*, **82** (1951) 664.
[3] Iacopini E. and Zavattini E., *Phys. Lett. B*, **85** (1979) 151.
[4] Bakalov D. *et al.*, *Nucl. Phys. B Proc. Suppl.*, **35** (1994) 180.
[5] Boer D. and van Holten J.-W., hep-ph/0204207.

[6] Di Piazza A., Hatsagortsyan K. Z. and Keitel C. H., *Phys. Rev. Lett.*, **97** (2006) 083603.
[7] King B., Di Piazza A. and Keitel C. H., *Nature Photon.*, **4** (2010) 92.
[8] De Lorenci V. A., Figueredo N., Fliche H. H. and Novello M., *Astron. Astrophys.*, **369** (2001) 690.
[9] Denisov V. I., Denisova I. P. and Svertilov S. I., *Dokl. Akad. Nauk. Ser. Fiz.*, **380** (2001) 435.
[10] Denisov V. I., Denisova I. P. and Svertilov S. I., *Dokl. Phys.*, **46** (2001) 705.
[11] Denisov V. I. and Svertilov S. I., *Astron. Astrophys.*, **399** (2003) L39.
[12] Denisov V. I., Denisova I. P. and Svertilov S. I., *Theor. Math. Phys.*, **140** (2004) 1001.
[13] Denisov V. I. and Svertilov S. I., *Phys. Rev. D*, **71** (2005) 063002.
[14] Vshivtseva P. A., Denisov V. I. and Krivchenkov I. V., *Theor. Math. Phys.*, **150** (2007) 73.
[15] Kim J. Y. and Lee T., *Mod. Phys. Lett. A*, **26** (2011) 1481.
[16] Bialynicka-Birula Z. and Bialynicka-Birula I., *Phys. Rev. D*, **2** (1970) 2341.
[17] Adler S. L., *Ann. Phys. (N.Y.)*, **67** (1971) 599.
[18] Heyl J. S. and Hernquist L., *J. Phys. A*, **30** (1997) 6485.
[19] Gies H. and Dittrich W., *Phys. Lett. B*, **431** (1998) 420.
[20] Dittrich W. and Gies H., *Phys. Rev. D*, **58** (1998) 025004, hep-ph/9806417.
[21] Heyl J. S. and Hernquist L., *Phys. Rev. D*, **58** (1998) 043005.
[22] Novello M., De Lorenci V. A., Salim J. M. and Klippert R., *Phys. Rev. D*, **61** (2000) 045001.
[23] De Lorenci V. A., Klippert R., Novello M. and Salim J. M., *Phys. Lett. B*, **482** (2000) 134.
[24] Gies H., hep-ph/0010287.
[25] Rikken G. L. J. A. and Rizzo C., *Phys. Rev. A*, **63** (2001) 012107.
[26] De Lorenci V. A. and Goulart G. P., *Phys. Rev. D*, **78** (2008) 045015.
[27] Misner C. W., Thorne K. S. and Wheeler J. A., *Gravitation* (W. H. Freeman and Company, New York) 1970.
[28] Itzykson C. and Zuber J.-B., *Quantum Field Theory* (McGraw Hill) 1980.